A 1*2*3 Open Economy Model with R23 Data

cgemod

1. Introduction

This model is a variant of the 1*2*3 model (Devarajan, *et al.*, 1994) that has been extended so that it can be calibrated with macroeconomic data for single regions that have been drawn from the R23 database. The extensions include the addition of factor demands, intermediate inputs, savings and investment, a range of government tax instruments, and net transfers from the rest of the world; this model is an extension of Devarajan *et al*., (1997). This implementation is designed as a platform for learning about CGE models, **not** as a tool for conducting (policy) experiments designed to influence policy choices. However, there is no *a priori* reason why it could not be used in the same way as the World Bank's 123PRSP and R23 models[1]. This model (r_123) uses an aggregated version of the R23 database (see McDonald *et al*., 2015) and will operate easily within the bounds of student/demo GAMS.[2]

The technical documentation for this model proceeds by first describing the data used by the model. These data are laid out as a SAM. The SAM structure is also used to identify the behavioural and transaction relationships of the model. These are followed by schematic representations of the price and quantity systems of the model, and then a formal algebraic statement of the model's equations. The equations are then summarised in a table that also identifies the equation names used in the GAMS code and the equation and variable counts for the model.

1 For details on the 123PRSP model see http://poverty.worldbank.org/files/12937_TKWeb_Chap_13_(Rev).pdf. For details on the R23 model see www.cgemod.org.uk

2 Earlier versions of this model used an aggregated version of the GTAP database. It was necessary to augment this database with IMF and World Bank data to relax serious limitations impose by the GTAP database (see McDonald and Sonmez, 2004, for details). The R23 database was developed explicitly to relax constraints imposed by the GTAP database.

2. R_123 Model and Database

Social Accounting Matrix

The R23 database was developed to provide a globally complete and consistent database of macroeconomic/aggregates to support analyses by the World Bank and to provide a database that could be used by GTAP to enhance the control totals used in the construction of the GTAP database. This aggregated version of the R23 database has been stripped of all bi lateral trade data, such that for each region there is a single trade account with exports valued free on board (*fob*) and imports valued carriage, insurance and freight (*cif*) paid. There are single commodity and activity accounts for each country, but the factor accounts have been left disaggregated. The VAT and factor use tax instruments have been removed (by aggregation), to make the model more tractable; otherwise the taxes are left unchanged. A strong feature of this databases are the data on inter regional transactions; this increases the range of policy issues that can be simulated with the model, although the treatment is kept simple to ensure the model is tractable. All transactions are in millions of US dollars.

Table 1 A Macro Social Accounting Matrix for a R_123 CGE Model

	Commodities	Activities	Factors	Households	Govt	Capital	Rest of World	Totals
Commodities	0.0	8,963.8	0.0	4,865.4	1,481.2	2,401.3	1,809.7	19,521.4
Activities	17,069.2	0.0	0.0	0.0	0.0	0.0	0.0	17,069.2
Factors	0.0	8,002.7	0.0	0.0	0.0	0.0	140.1	8,142.8
Households	0.0	0.0	7,083.1	0.0	0.0	0.0	-65.4	7,017.7
Government	428.6	102.7	0.0	1,560.4	0.0	0.0	20.3	2,112.0
Capital	0.0	0.0	706.4	592.0	610.5	0.0	492.4	2,401.3
Rest of World	2,023.6	0.0	353.3	0.0	0.0	0.0	0.0	0.0
Totals	19,521.4	17,069.2	8,142.8	7,017.7	2,112.0	2,401.3	0.0	0.0

Source: Aggregated R23 data for Australia

The macro SAM for Australia reported in Table 1 indicates the coverage of transactions provided by these SAMs: the model encompasses transactions for all the active cells, i.e., those with non-zero values. Non-zero transactions in other cells will cause the model to fail although such entries do not appear in the R23 database. There are five types of commodity demand – intermediate inputs, final demand by households, government and investment, and export demand – that are supplied from two sources – domestic productions and imports. There are a variety of tax instruments on the commodity accounts; import duties, export taxes and sales taxes.

Domestic production activities use intermediate and primary inputs and pay production taxes to the government. Factor incomes are earned by the sale of factor services to domestic activities and the rest of the world, with the incomes from factor services accruing either to domestic households or the rest of the world. In addition to incomes from the sale of factor services households also receive income in the form of net remittances from the rest of the world, and these incomes are used by the household to pay direct (income) taxes, to save and to pay for commodities.

The government receives income from five types of tax – import duties, export taxes, sales taxes, production taxes and income taxes; this can be supplemented by net transfers from the rest of the world (aid and grants)[3]. There are four sources of savings; factors (through capital depreciation), households, the government (internal balance) and the rest of the world (external balance): in the form of the balance on the capital account. All savings are spent to purchase commodities for investment.

The R23 database has 204 countries, one commodity and one activity account, 2 factor accounts, seven tax instruments, 4 sources of savings and gross transfers between countries for all factors and institutions, and transfer from multi-lateral sources. The database, model and documentation are freely available from www.cgemod.org.uk.

Behavioural Relationships

The behavioural relationships are simple in this variant of the model. The activity is assumed to maximise profits using technology characterised by a Cobb-Douglas production function between primary inputs and Leontief technology between aggregate primary inputs and intermediate inputs. The household maximises utility subject to consumption of quantities of the composite commodity; those quantities are determined by the income available for consumption expenditures, after paying income taxes and saving from after tax income, and the prices of the composite commodity.

3 Note that if this entry is negative it means that there is a net expenditure on transfers.

The Armington assumption is used for trade. Domestic output is distributed between the domestic market and exports according to a constant elasticity of transformation (CET)function, while domestic absorption is satisfied from domestic production and imports that are mixed to provide a composite commodity according to a constant elasticity of substitution (CES) function. The optimal ratio of imports to the domestic commodity and exports to the domestic commodity are determined by first order conditions based on relative prices. The small country trade assumption applies to all imports and exports, i.e., world prices in world currency units are fixed. The exchange rate is fixed or flexible according to the closure rule chosen for the foreign exchange account.

The commodity and activity taxes are expressed as simple *ad valorem* tax rates, while the income tax rate is defined as a fixed proportion of household income. Import duties and export taxes apply to imports and exports, while sales taxes are applied to all domestic absorption, i.e., imports are subject to sequential import duties and sales taxes. Production taxes are levied on the value of output by each activity. Income taxes are taken out of household income and then the households are assumed to save a proportion of disposable income. This proportion is either fixed or variable according to the closure rule chosen for the capital account.

Government expenditure consists of commodity (final) demand, which is assumed to be fixed in real terms, and expenditure on export subsidies. Hence government saving, or the internal balance, is defined as a residual. However, the closure rules for the government account allow for various permutations. In the base case, it is assumed that the tax rates and volume of government demand is fixed, and government savings are calculated as a residual. However, the tax rates are all declared as variables; hence for instance the value of government savings can be fixed and one of the tax rates can be allowed to vary thereby producing an estimate of the constrained optimal tax rate. Equally, the volume of government consumption can be changed by adjusting the closure rule with respect to the volume of government consumption.

Table 2 Behavioural Relationships for a R_123 CGE Model

	Commodities	Activities	Factors	Households	Government	Capital	Rest of World	Totals	
Commodities	0	Leontief Input-Output Coefficient	0	Consumption expenditure	Fixed Exogenously	Fixed Exogenously	Commodity Export	Commodity Demand	Consumer Commodity Price
Activities	Total Supply from Domestic Production	0	0	0	0	0	0	CD Production Function	Activity Price
Factors	0	Factor Demands	0	0	0	0	Factor Income from RoW	Factor Income	
Households	0	0	Fixed Shares of Factor Income	0	0	0	Net Remittance to Households from RoW	Household Income	
Government	Sales Tax, Import Duty and Export Tax	Production Taxes	0	Direct Taxes on Household Income	0	0	Net Remittance to Government from RoW	Government Income	
Capital	0	Depreciation	0	Household Savings	Government Savings (Residual)	0	Current Account 'Deficit	Total Savings	
Rest of World	Commodity Imports	0	Fixed Shares of Factor Income	0	0	0	0	Total 'Expenditure' Abroad	
Totals	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad		
	Producer Commodity Price Domestic and World Price for Imports	Value Added Prices							

Table 3 Transactions Relationships for a for a R_123 CGE Model

	Commodities	Activities	Factors	Households	Government	Capital	RoW
Commodities	0	$(PQD*QINTD)$	0	$(PQD*QCD)$	$(PQD*QGD)$	$(PQD*QINVD)$	$\begin{pmatrix} pwe*QE \\ *ER \end{pmatrix}$
Activities	$(PDS*QDS)$	0	0	0	0	0	0
Factors	0	$(WF_f * FD_f)$	0	0	0	0	$\begin{pmatrix} factwor_f \\ *ER \end{pmatrix}$
Households	0	0	$\sum_f \begin{pmatrix} hovash_f \\ *YF_f \end{pmatrix}$	0	0	0	$(howor*ER)$
Government	$\begin{pmatrix} TM * pwm \\ *QM * ER \end{pmatrix}$ $\begin{pmatrix} TE * pwe \\ *QE * ER \end{pmatrix}$ $(TS*PQ*QQ)$	$(TX*PX*QX)$	0	$(TYH*YH)$	0	0	$(govwor*ER)$
Capital	0	0	0	$\begin{pmatrix} YH*(1-TYH) \\ *SHH \end{pmatrix}$	$(YG-EG)$	0	$(KAPWOR*ER)$
Rest of World	$\begin{pmatrix} pwm*QM \\ *ER \end{pmatrix}$	0	$(factwor_f * YF_f)$	0	0	0	0
Total	$(PQ*QQ)$	$(PX*QX)$	YF_f	YH	YG	$INVEST$	0

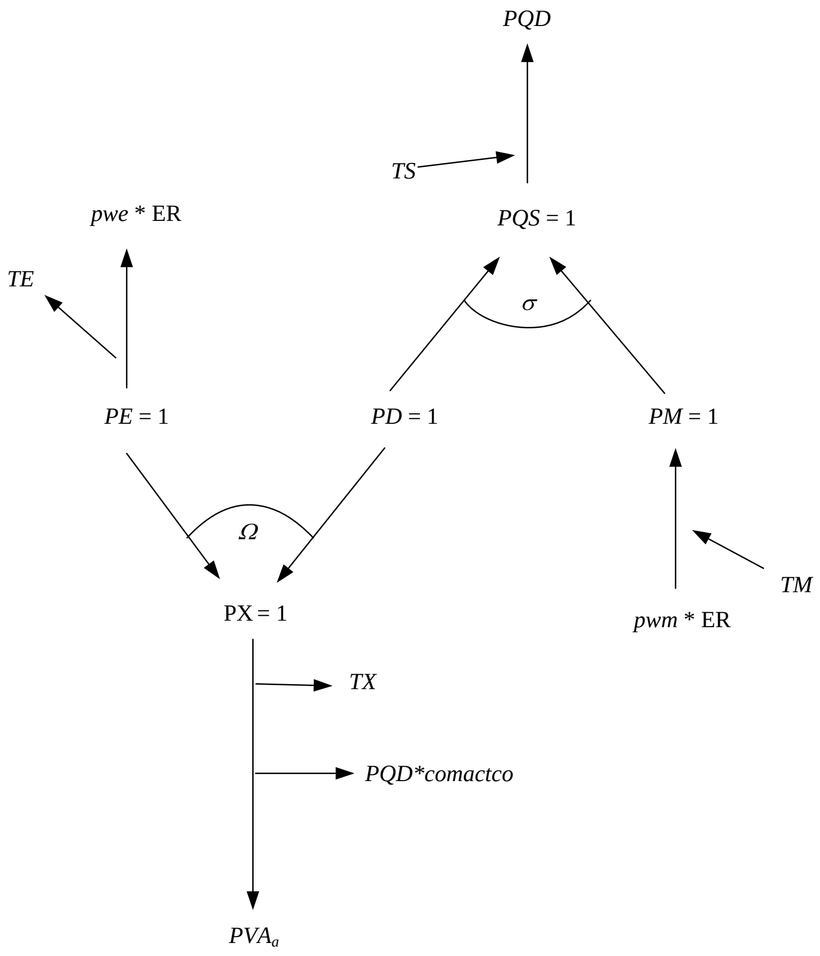
Total savings come from the households, enterprises, the internal balance on the government account and the external balance on the trade account. The external balance is defined as the difference between the value of total exports and total imports, converted into domestic currency units using the exchange rate. In the base model it is assumed that the exchange rate is fixed and hence that the external balance is variable. Alternatively, the exchange rate can be made variable and the external balance can be set at a sustainable level of deficit/surplus. Expenditure by the capital account consists solely of commodity demand for investment. In the base solution, it is assumed that the volume of investment adjusts so that total expenditure on investment is equal to total savings, i.e., the closure rule presumes that savings drive investment expenditures. It is possible to fix the volume of real investment and then allow the savings rate by households to vary; thereby maintaining equilibrium in the capital account.

Price System

The price system is built up using the principle that the components of the ‘price definitions’ are the entries in the columns of the SAM. Hence there are a series of explicit accounting identities that define the relationships between the prices and thereby determine the processes used to calibrate the tax rates for the base solution. However, the model is set up using a series of linear homogeneous relationships and hence is only defined in terms of relative prices. Consequently, as part of the calibration process it is necessary set some of the prices equal to one (or any other number that suits the modeler) – this model adopts the convention that prices are normalised at the level of the CES and CET aggregator functions (see Figure 1).

The relationships between the various prices in the model are illustrated in Figure 1. The domestic consumer price (*PQD*) is determined by the domestic price of the domestically supplied commodity (*PD*) and the domestic price of the import (*PM*), and by the sales tax (*TS*) levied on all domestic demand. The import price is determined by the world price (*pwm*), the exchange rate (*ER*) and the import tariff rat (*TM*). The activity price (*PX*) is determined by the domestic price (*PD*) and the export price (*PE*). The export price being determined by the world price (*pwe*), the exchange rate (*ER*) and the export subsidy rate (*TE*). Finally, the value-added price (*PV*) is determined by the activity price (*PX*), the production tax rate (*TX*), the input-output coefficient and the commodity price (*PQD*).

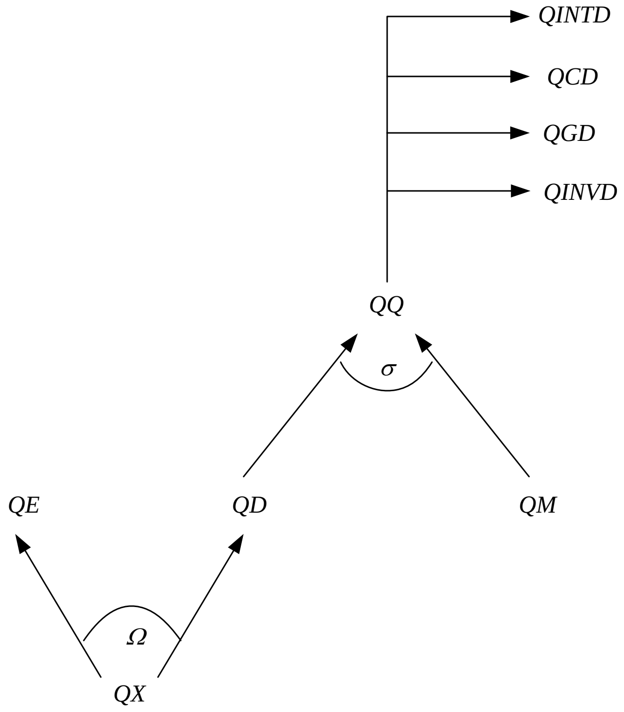
Figure 1 **Price Relationships for a R_123 CGE Model**



Quantity System

Figure 2 illustrates the quantity relationships. Domestic demand (*QQ*) is made up of intermediate demand (*QINTD*), household consumption (*QCD*), government consumption (*QGD*) and investment (*QINVD*). This is satisfied by domestic supply (*QD*) and imports (*QM*). Domestic supply (*QD*) is part of domestic production (*QX*), which has been divided between the domestic market and exports (*QE*).

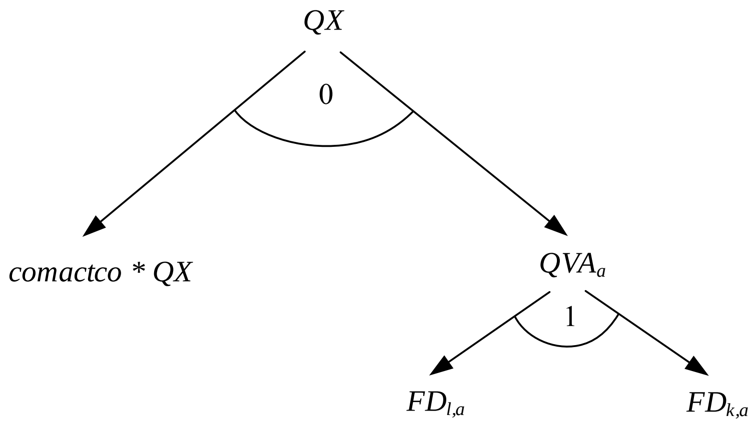
Figure 2 Quantity Relationships for a 123 CGE Model



Production System

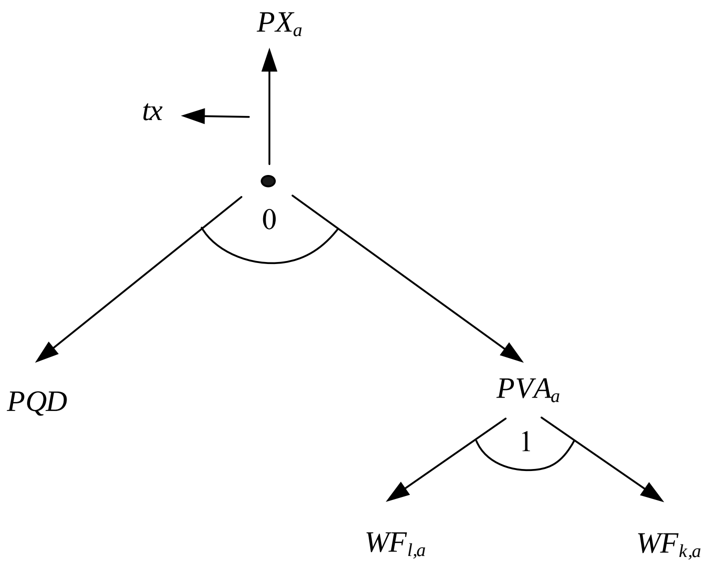
The domestic production relationships are illustrated in Figures 3 and 4 under the simplifying assumptions of two intermediate inputs and the two primary inputs. The intermediate input is used according to the Leontief assumption of a fixed quantity (*comactco*) per unit of output – hence the ‘0’ elasticities of substitution between the intermediate input and ‘aggregate’ primary inputs. Primary inputs are aggregated using a Cobb-Douglas technology such that σ_{22} is equal to one. The factor demands (FD_f) identify total factor demand which must be met by factor supplies (FS_f).

Figure 3 Production Quantity Relationships for a 123 CGE Model



There is a matching price system. The amount available to pay inputs is the output price (*PX*) less the indirect/production tax (*TX*). Because of the Leontief assumption the amount available to pay primary inputs (*PVA*) is determined by the technical coefficient (*comactco*) and the price of the intermediate input (*PQD*). The optimal quantities of each factor are then determined by first order conditions based on factor prices (WF_f).

Figure 4 Production Price Relationships for a 123 CGE Model



3. Algebraic Statement of a 123 CGE Model

For this model, there is limited capacity to make profitable use of sets to abbreviate the number of equations that must be written. However, there is still a requirement for a set that defines membership of the SAM and profitable use can be made of two subsets; one for the factor accounts and the other for the government accounts. The sets for the model are therefore

$$\begin{aligned} sac &= \{\text{ccom, acom, } f\text{, hous, } g\text{, } i\_s\text{, row, total}\} \\ f &= \{\text{flnd, fskl, fuskl, fcap}\} \\ g &= \{\text{imptax, exptax, saltax, prodtax, dirtax, govt}\} \end{aligned} .$$

A macro SAM that can be used to check model calibration is very useful. This needs another set

$$ss = \{commdty, activity, valuad, hholds, govtn, kapital, world, totals\} .$$

The equations for the model are set out in ‘blocks’. The order in which you proceed is largely a matter of personal preference; for this model the blocks used are for ‘exports’, ‘imports’, ‘commodity prices’, ‘production’, ‘factor’, ‘household’, ‘taxes’, ‘government’, ‘kapital (savings and investment)’, ‘market clearing’, ‘GDP’ and ‘model closure’. A series of conventions are adopted for the model.

- all VARIABLES are in upper case;
- all parameters are in lower case, except those used to intialise variables;
- names for parameters are derived using account abbreviations with the row account first and the column account second, e.g., actcom** is a parameter referring to the activitiy:commodity (supply or make) sub-matrix;
- parameter names have a two-character suffix which distinguishes their definition, e.g., ***sh* is a share parameter and ***av* is an average;
- all parameter and variable names have less than 10 characters.

Exports Block Equations

The domestic prices of exports (*PE*) are defined as the product of world prices of exports (*pwe*), the exchange rate (*ER*) and one minus the *ad valorem* export subsidy rate[4] (*TE*).

$$PE = pwe * ER * (1 - TE) \tag{X1}$$

Notice how the world price of exports (*pwe*) is defined as a parameter; this means that embedded in the model is the classic small country trade assumption whereby the country is a price taker on export markets.

Domestic commodity outputs (*QX*) are either exported (*QE*) or supplied to the domestic market (*QD*). The allocation of output between the domestic and export markets is determined by an output transformation function, Constant Elasticity of Transformation (CET) function, (X2)

$$QX = at_c \left(\gamma * QE^{rhot} + (1-\gamma) * QD^{rhot} \right)^{\frac{1}{rhot}} \tag{X2}$$

with the optimum combinations of *QE* and *QD* determined by first-order conditions (X3)

$$\frac{QE}{QD} = \left[\frac{PE}{PD} * \frac{(1-\gamma)}{\gamma} \right]^{\frac{1}{(rhot-1)}}. \tag{X3}$$

Imports Block Equations

The domestic prices of imports (*PM*) are defined as the product of world prices of imports (*pwm*), the exchange rate (*ER*) and one plus the *ad valorem* import tariff rate (*TM*).

$$PM = pwm * ER * (1 + TM) \tag{M1}$$

notice how the world price of imports (*pwm*) is defined as a parameter; this means that embedded in the model is the classic small country trade assumption.

However, both domestic and foreign producers can supply commodities to the domestic market. The composite (consumption) commodities (*QQ*) are a mixture of imports (*QM*) and domestic demand (*QD*). The mixture between the domestic and import supplies is determined by a substitution function, Constant Elasticity of Substitution (CES) function, i.e.,

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[4] Defining export taxes as negative subsidies means that there is symmetry between the treatment of import duties and export subsidies when coding the model in GAMS.

$$QQ = ac\left(\delta * QM^{-rhoc} + (1-\delta) * QD^{-rhoc}\right)^{-\frac{1}{rhoc}} \tag{M2}$$

with the optimal combination of *QM* and *QD* being determined by first-order condition (M3)

$$\frac{QM}{QD} = \left[\frac{PD}{PM} * \frac{\delta}{(1-\delta)}\right]^{\frac{1}{(1+rhoc)}}. \tag{M3}$$

Commodity Price Block Equations

The composite price definition equations (CP1, CP2 and CP3) are derived from the first order conditions for tangencies to consumption and production possibility frontiers. By exploiting Euler's theorem for linearly homogeneous functions the composite prices can be expressed as expenditure identities rather than dual price equations for export transformation and import aggregation, such that *PQS* is the weighted average of the producer price of a commodity, when *PD* is the producer price of the domestically produced commodity and *PM* the domestic price of the imported commodity, i.e.,

$$PQS = \frac{PD * QD + PM * QM}{QQ}. \tag{CP1}$$

where *QD* the quantity of the domestic commodity demanded by domestic consumers, *QM* the quantity of imports and *QQ* the quantity of the composite commodity. Notice how the commodity quantities are the weights. This composite commodity price (CP1) does not include the *ad valorem* sales tax, which creates a price wedge between the purchaser price of the commodity (*PQD*) and the producer price (*PQS*). Hence the purchaser price is defined as the producer price plus the sales taxes, i.e.,

$$PQD = PQS * (1 + TS). \tag{CP2}$$

This formulation means that the sales tax is levied on all sales on the domestic market, irrespective of the origin of the commodity concerned.

The composite output price for a commodity is also derived by exploiting Euler's theorem for linearly homogeneous functions, and is given by

$$PX = \frac{PD * QD + PE * QE}{QX}. \tag{CP3}$$

where PD is the domestic producer price for the output of commodities supplied to the domestic market, QD is the supply of output to the domestic market, QE is the quantity exported by activities, PX is the composite output price by commodity and QX is the quantity of domestic production by commodity.

Production Block Equations

The presence of an intermediate input and production taxes in this system allows the definition of the value-added price (PVA). This is defined as the output price (PX) less the payments for the intermediate inputs and the *ad valorem* production tax on output (TX). The assumption of a fixed proportion in the use of the intermediate input, i.e., Leontief style input-output coefficients, means that the payment for the intermediate input is the price of the intermediate input times the input-output coefficient, i.e.,

$$PVA = \left(PX * \left(1 - TX\right)\right) - \left(PQD * comactco\right). \tag{P2}$$

The production sub-block consists of the production function (P3), a first order conditions for profit maximization (P4) and an intermediate demand equation. The production function is a Cobb-Douglas aggregation function over the factors that are demanded (FD_f),

$$QX = adcd * \prod_f \left(FD_f\right)^{\alpha_f} . \tag{P3}$$

The production function is calibrated for an efficiency parameter ($adcd$) and factor specific elasticities of output (α_f). The matching first order condition for profit maximisation exploits the properties of Euler's theorem and the relationship between the elasticity of output and factor shares in long-run competitive equilibrium, i.e., the elasticity of output for input f is the value share of output received by factor f, i.e.,

$$FD_f = \frac{QX * PVA * \alpha_f}{WF_f} . \tag{P4}$$

Note how because of the intermediate input, and the Leontief assumption for intermediate input use, the value of a unit of output is the value-added price (PVA).

Since production uses an intermediate input it is also necessary to specify the demand for the intermediate input ($QINTD$), i.e.,

$$QINTD = comactco * QX . \tag{P5}$$

Factor Block Equations

The total income received by each factor account (YF_f) is defined as the summation of the earnings of the factor plus factor income from abroad ($factwor_f$) expressed in domestic currency units (F1),

$$YF_f = \left(WF_f * FD_f\right) + \left(factwor_f * ER\right). \tag{F1}$$

Only a proportion of total factor income is available for distribution to the domestic institutional accounts because of payments to foreign owners of the factors used in the economy. It is assumed that a fixed proportion ($worvash_f$) of domestic factor earnings are remitted to foreign owners ($YFWOR_f$),[5] hence

$$YFWOR_f = worvash_f * YF_f. \tag{F2}$$

Although implemented over all factors, this equation is only implemented in this model for income to the factor capital, because of the information in the database.

Household Block Equations

Households acquire income from two sources in this model. The first source is from the factor accounts in terms of a fixed proportion ($hvash_{h,f}$) of factor incomes, and the second is in the form of remittances from the Rest of the World ($howor_h$). Therefore, household income (YH) is defined as

$$YH = \left(\sum_f hvash_f * YF_f\right) + \left(howor * ER\right). \tag{H1}$$

Household consumption expenditure is defined as household income after the payment of direct taxes and savings, and, since there is only a single commodity, consumption demand is simply defined as

$$QCD = \frac{\left(\left(YH * \left(1 - TYH\right)\right) * \left(1 - SHH\right)\right)}{PQD}. \tag{H2}$$

Note how the saving rate (SHH) is defined as the proportion of after-tax income that is saved; this is important for the calibration of the income tax and savings parameters.

5 Underlying these fixed value shares is a set of assumptions relating to the ownership of factors used within the economy. In some instances, it is useful to make the distribution of factor ownership an explicit feature of the models; this is the case in various versions of the STAGE and GLOBE models.

Government Block Equations

Government income is defined as the sum of government tax revenues plus transfers from the Rest of the World (*govwor*) (G1), i.e.,

$$\begin{aligned} YG = & \left(TM * pwm * ER * QM\right) + \left(TE * pwe * ER * QE\right) \\ & + \left(TS * PQS * QQ\right) + \left(TX * PX * QX\right) \\ & + \left(TYH * YH\right) + \left(govwor * ER\right) \end{aligned} \quad . \tag{G1}$$

Note how the tax revenues are treated as expenditures by the accounts paying the taxes.

Government consumption expenditure (*EG*) is defined as the sum of the expenditures on commodity consumption, i.e.,

$$EG = \left(QGD * PQD\right). \tag{G2}$$

In the model closure section government consumption, in the default case, will be fixed.

Kapital Account Block Equations

Income to the capital (savings and investment) account comes from household savings, government savings (*KAPGOV*) and the surplus on the capital account of the balance of payments (*KAPWOR*) (K1). Hence total savings are defined as

$$\begin{aligned} TOTSAV = & \left(\left(YH * \left(1 - TYH\right)\right) * \left(SHH\right)\right) \\ & + KAPGOV + \left(KAPWOR * ER\right) \end{aligned} \tag{K1}$$

In this model the household is assumed to save a proportion (*SHH*) of its income after tax. It may be helpful to assign/calibrate (K1) simultaneously with (H2) to ensure the correct calibration of the income tax and saving rate parameters. Government savings are calculated as a residual (see the *KAPGOV* equations below): note that the surplus on the capital account (*KAPWOR*) is defined in terms of the foreign currency and therefore the exchange rate appears in this equation (this is a matter of preference).

Investment demand is modeled in a similar way to government demand, i.e., as the sum of expenditures

$$INVEST = \left(PQD * QINVD\right). \tag{K3}$$

In the model closure section investment consumption, in the default case, will be fixed.

Market Clearing Block Equations

The market clearing, or equilibrium, conditions are straightforward. Factor supplies must equal factor demands:

$$FS_f = FD_f \tag{MC1}$$

and (composite) commodity supply must equal (composite) commodity demand:

$$QQ = QINTD + QCD + QGD + QINVD\,. \tag{MC2}$$

The government account is cleared by defining government savings (*KAPGOV*) as the difference between government income and government expenditure on consumption, hence government savings are explicitly treated as a residual, i.e.,

$$KAPGOV = YG - EG\,. \tag{MC3}$$

The deficit on the current account is defined as expenditure on imports less revenue from exports, all evaluated in terms of the foreign currency (MC4)

$$\begin{aligned} KAPWOR = \left(pwm * QM\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) - \left(pwe * QE\right) \\ - \left(\sum_f factwor_f\right) - howor - govwor \end{aligned}\,. \tag{MC4}$$

Because (MC5) is defined in terms of the 'foreign' currency it is necessary to convert *KAPWOR* into domestic currency terms when it enters any other equation, e.g., the total savings equation (*TOTSAV*). And finally, the equilibrium condition for the capital account includes a slack variable, rather than dropping an equation from the system (MC5), i.e.,

$$TOTSAV = INVEST + WALRAS\,. \tag{MC5}$$

GDP Block Equations

GDP by expenditure is defined as the total value final demand in the economy (Z1).

$$GDP = PQD * \left(QCD + QGD + QINVD\right) + pwe * QE * ER - pwm * QM * ER\,. \tag{Z1}$$

Strictly speaking this equation is not needed for the model since the model is set up as a square problem that is solved as a mixed complementarity problem, but the *GDP* equation is retained in this model for pedagogic reasons.

Table 4 Equation and Variable Counts for a 123 CGE Model

Name	Equation	Number of Equations	Variable	Number of Variables
	EXPORTS BLOCK			
PEDEF	$PE = pwe * ER * (1 - TE)$	*1*	*PE*	*1*
CET	$QX = at_c \left(\gamma * QE^{rhot} + (1-\gamma) * QD^{rhot} \right)^{\frac{1}{rhot}}$	*1*	*QD*	*1*
ESUPPLY	$\frac{QE}{QD} = \left[\frac{PE}{PD} * \frac{(1-\gamma)}{\gamma} \right]^{\frac{1}{(rhot-1)}}$	*1*	*QE*	*1*
IMPORTS BLOCK				
PMDEF	$PM = pwm * ER * (1 + TM)$	*1*	*PM*	*1*
ARMINGTON	$QQ = ac \left(\delta * QM^{-rhoc} + (1-\delta) * QD^{-rhoc} \right)^{-\frac{1}{rhoc}}$	*1*	*QQ*	*1*
COSTMIN	$\frac{QM}{QD} = \left[\frac{PD}{PM} * \frac{\delta}{(1-\delta)} \right]^{\frac{1}{(1+rhoc)}}$	*1*	*QM*	*1*
	COMMODITY PRICE BLOCK			
			PD	*1*
PQDEF	$PQD = PQS * (1 + TS)$	1	*PQD*	1
PQSEF	$PQS = \frac{PD * QD + PM * QM}{QQ}$	1	*PQS*	1
PXCDEF	$PX = \frac{PD * QD + PE * QE}{QX}$	1	*PX*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	PRODUCTION BLOCK			
PVDEF	$PVA=\left(PX*\left(1-TX\right)\right)-\left(PQD*comactco\right)$	1	*PV*	1
PRODFN	$QX=adcd*\prod_{f}\left(FD_{f}\right)^{\alpha_{f}}$	1	*QX*	1
PROFITMAX	$FD_{f}=\frac{QX*PVA*\alpha_{f}}{WF_{f}}$	*f*	FD_f	*f*
QINTDEQ	$QINTD=comactco*QX$	1	*QINTD*	1
	FACTOR BLOCK			
$YFEQ_f$	$YF_{f}=\left(WF_{f}*FD_{f}\right)+\left(factwor_{f}*ER\right)$	*f*	YF_f	*f*
$YFWOREQ_f$	$YFWOR_{f}=worvash_{f}*YF_{f}$	*f*	$YFWOR_f$	*f*
	HOUSEHOLD BLOCK			
$YHEQ_h$	$YH=\left(\sum_{f}hovash_{f}*YF_{f}\right)+\left(howor*ER\right)$	1	*YH*	1
$QCDEQ_c$	$QCD=\frac{\left(\left(YH*\left(1-TYH\right)\right)*\left(1-SHH\right)\right)}{PQD}$	1	*QCD*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	GOVERNMENT INCOME AND EXPENDITURE BLOCK			
YGEQ	$YG = (TM * pwm * ER * QM) + (TE * pwe * ER * QE) + (TS * PQS * QQ) + (TX * PX * QX) + (TYH * YH) + (govwor * ER)$	1	*YG*	1
EGEQ	$EG = (QGD * PQD)$	1	*EG*	1
	KAPITAL ACCOUNT BLOCK			
TOTSAVEQ	$TOTSAV = ((YH * (1 - TYH)) * (SHH)) + KAPGOV + (KAPWOR * ER)$	1	*TOTSAV*	1
INVESTEQ	$INVEST = (PQD * QINVD)$	1	*INVEST*	1
	GDP BLOCK			
GDPEQ	$GDP = PQD * (QCD + QGD + QINVD) + pwe * QE * ER - pwm * QM * ER$	1	*GDP*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MARKET CLEARING BLOCK			
$FMEQUIL_f$	$FS_f = FD_f$	f	FS_f	
$QEQUIL$	$QQ = QINTD + QCD + QGD + QINVD$	1		
$KAPGOVEQ$	$KAPGOV = YG - EG$	1	$KAPGOV$	1
$KAPEQUIL$	$KAPWOR = (pwm * QM) + \left(\sum_f \frac{YFWOR_f}{ER} \right) - (pwe * QE) - \left(\sum_f factwor_f \right) - howor - govwor$	1	$KAPWOR$	1
$WALRASEQ$	$TOTSAV = INVEST + WALRAS$	1	$WALRAS$	1
	MODEL CLOSURE			
			$\overline{ER}$ **or** $\overline{KAPWOR}$	1
			$\overline{SHH}$ **or** $\overline{QINVD}$ **or** $\overline{INVEST}$	1
			All **but** two of $\overline{TM}, \overline{TS}, \overline{TE}, \overline{TX}, \overline{TYH}, \overline{QGD}, \overline{KAPGOV}, \overline{EG}$	6
			$\overline{FS}_f$	f
			$\overline{PQ}$ **or** $\overline{PD}$	1

4. Model Closure Conditions or Rules

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

The model is designed to facilitate changes in model closure with respect to four groups of markets; the foreign exchange market, the capital (investment-savings) market, the Government's account and the factor markets. The model is set up with a series of simple alternative closures. For the foreign exchange and capital markets the alternatives are of the either/or kind. For the government account the alternatives are more complex since they allow for policy changes with respect to each of the tax instruments, for adjustments in the volume of government consumption and changes in the balance on the government's budget. The basic specification of model assumes full employment and fully flexible factor markets; these assumptions can be relaxed so that, *inter alia*, unemployment can be assumed for one or more factors.

The model allows for the exploration of a range of different policy scenarios, although use of this model for real world policy analysis should be limited by recognising the limitations imposed by the data. This model also allows for a range of both general and specific closure rules. The discussion below provides details of the main options available with this formulation of the model by reference to the accounts to which the rules refer.

<u>Foreign Exchange Account Closure</u>

The closure of the rest of the world account can be achieved by fixing either the exchange rate variable (C1a) or the balance on the current account (C1b). In the base specification of the model the current account balance (*KAPWOR*) is fixed and the exchange rate (ER) is the equilibrating variable for the foreign exchange market. Increasingly economies are constrained to operate a flexible exchange rate policy while simultaneously ensuring that the external (current account) balance does not exceed a ‘sustainable’ deficit. This deficit may be positive or negative. Fixing the exchange rate is appropriate for countries with a fixed exchange rate regime whilst fixing the current account balance is appropriate for countries that face restrictions on the value of the current account balance, e.g., countries following structural adjustment programmes.

$$ER = \overline{ER} \tag{C1a}$$

or

$$KAPWOR = \overline{KAPWOR}. \tag{C1b}$$

It is a common practice to fix a variable at its initial level by using the associated parameter, i.e., ***0, but it is possible to fix the variable to any appropriate value.

Capital Account Closure

The base specification for the capital market closure is that the total value of investment is determined by the total value of savings, where savings consist of savings from the household, government and the foreign exchange account. For the household the base specification assumes that the rate of saving out of after-tax income is fixed at its initial rate. *Ceteris paribus* changes in the household savings rate will alter the total value of investment but the volume of investment also depends upon the price of the investment good. As specified the model can operate under the assumption that either the volume of investment or the value of investment can be fixed exogenously. Note how the variable *QGD* determines the ‘level’ of investment volumes while it is the product of the price and quantity of the investment commodity that determines the value of investment.

To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. This is achieved by fixing either the saving rates for households or the volumes of commodity investment. This involves fixing either the savings rate (C2a) or the investment volume (C2b), i.e.,

$$SHH = \overline{SHH} \tag{C2a}$$

or

$$QINVD = \overline{QINVD} \qquad \text{(C2b)}$$

Note that fixing the investment volume (C2b) means that the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*).

Fixing savings, and thus deeming the economy to be savings-driven, can be considered a ‘Neo-classical’ approach. Closing the economy by fixing investment however makes the model reflect a ‘Keynesian’ investment-driven assumption for the operation of an economy.

The model includes a variable for the value of investment (*INVEST*), which can also be used to close the capital account. If *INVEST* is fixed in an investment driven closure, i.e.,

$$INVEST = \overline{INVEST} \qquad \text{(C2c)}$$

then the model will need to adjust the savings rates to maintain equilibrium between the value of savings (*TOTSAV*) and the fixed value of investment. This can only be achieved by changes in the volumes of commodities demanded for investment (*QINVD*) or their prices (*PQD*); but the prices (*PQD*) depend on much more than investment, hence the main adjustment must take place through the volumes of commodities demanded, i.e., *QINVD*.

Government Account Closure

The range of options the closure rules for the government account are trickier, which is appropriate since they are important components of the model that are used to investigate fiscal policy considerations. The model is specified so that there are eight variables under the government’s control that can be fixed or unfixed; five tax instruments, government consumption volume, government expenditure and government saving. The specification of the closure for the government account in the base model presumes that all tax rates are fixed at their initial rates, i.e.,

$$\begin{aligned} TM &= \overline{TM} \\ TE &= \overline{TE} \\ TS &= \overline{TS} \\ TX &= \overline{TX} \\ TYH &= \overline{TYH} \end{aligned} \qquad \text{(C4a)}$$

and that government expenditure volume is fixed, i.e.,

$$QGD = \overline{QGD}. \qquad \text{(C4b)}$$

and therefore, that government saving, the internal balance, is a residual. Alternatively, rather than controlling the volume of government expenditure, the value of government expenditure can be fixed, i.e.,

$$EG = \overline{EG}. \qquad \text{(C4c)}$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not <u>directly</u> control. Hence the equilibrating condition is that government savings, the internal balance, is not fixed.

However, it is a common government policy objective to identify the tax rates that are consistent with some pre-defined level of government saving (positive or negative). This requires that the value of government saving is fixed (in the base model the default fixed value is the initial value), and that at least one other variable under the government's control is unfixed. If government saving is fixed (C4d), i.e.,

$$KAPGOV = \overline{KAPGOV} \qquad \text{(C4d)}$$

then either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

It is critical when implementing such closure changes for the government account to carefully define the set of constraints that are to be imposed on the parameters/variables under the control of the government; an important modeling restriction to remember is that for each variable made flexible at least one variable must be made fixed.

<u>Numéraire</u>

The model specification allows for a choice from two price numéraire; the purchaser/consumer price or the producer price, i.e.,

$$PQD = \overline{PQD} \qquad \text{(C5a)}$$

or

$$PD = \overline{PD}. \qquad \text{(C5b)}$$

A *numéraire* is needed to serve as a base since the model is homogenous of degree zero in prices and hence only defines relative prices.

Factor Market Clearing

In this model the options for factor market clearing are simple and limited, and hence the options considered with this model are restricted. In the next module, using the smod_on model, the options considered will be more extensive and more difficult to implement than many of the other closure rules.

Here consider two alternatives; fixed factor supplies, either at the base or some other level, and ‘surplus labour’.

Fixed Factor Supplies

This factor market closure requires that the total supply of and total demand for factors equate. The total supplies of each factor are determined exogenously and hence

$$FS_f = \overline{FS}_f \qquad \text{(C6a)}$$

defines the first set of factor market clearing conditions. The demands for factor *f* and the wage rates for factors are determined endogenously. However, since there is only one activity factor demand must equal factor supply if the factor markets are to clear, i.e.,

$$FD_f = \overline{FS}_f . \qquad \text{(C6b)}$$

Clearance of the factor markets is achieved by the wage rates being flexible and therefore bounds are placed upon the factor prices, i.e.,

$$\begin{aligned} \text{Min } WF_f &= -\inf \\ \text{Max } WF_f &= +\inf \end{aligned} \qquad \text{(C6c)}$$

so that meaningful results are produced.

If factor supplies are fixed at the base level, then

$$FS_f = \overline{FS}_f = FS0_f \qquad \text{(C6d)}$$

But the user has the option to adjust the quantities of factors supplied, either equiproportionately changing the supplies of capital and labour or by different proportionate changes.

Surplus Factor Clearing

A more general factor market clearing wherein there are surplus factors, commonly termed an 'unemployed labour' assumption, can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as parameters. If factor market clearing rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations for each factor

$$\begin{aligned} FS_{fact} &= \overline{FS_{fact}} \\ \text{Min } WF_{fact} &= -\text{infinity} \\ \text{Max } WF_{fact} &= +\text{infinity} \\ WF_{fact} &= \overline{WF_{fact}} \\ \text{Min } FS_{fact} &= -\text{infinity} \\ \text{Max } FS_{fact} &= +\text{infinity} \end{aligned} \qquad \text{(C6e)}$$

where *fact* indicates the specific factor. The block of equations in (C6e) includes all the relevant variables that were declared for the model with reference to factors. The choice of which equations are binding, and which are not imposed will determine the factor market clearing conditions.

As can be seen the first three equations in the block (C6e) are the same as those in the 'Fixed Factor Supplies' case; hence ensuring that these three equations are operating for each of the factors is a longhand method for imposing the 'Fixed Factor Supplies' condition. Assume that this set of conditions represents the starting points, i.e., the first three equations are binding, and the last three equations are not imposed.

Assume now that it is planned to impose an assumption that there is unemployment of one or more factors in the economy; typically, this would be labour in this model. If the supply of the unemployed factor is perfectly elastic, then as much can be employed at the ruling wage rate as the activity wants. This requires imposing the condition that

$$WF_{fact} = \overline{WF_{fact}} \qquad \text{(C6f)}$$

and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing

$$FS_{fact} = \overline{FS_{fact}} \tag{C6g}$$

It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions

$$\begin{aligned} \text{Min } FS_{fact} &= -\inf \\ \text{Max } FS_{fact} &= +\inf \end{aligned} \tag{C6h}$$

can be imposed.[6]

6 If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.