Two Sector Closed-Economy CGE Model: Model 2

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1. Introduction

While a basic 2 sector closed-economy model provides a good framework for understanding the principles of whole economy CGE model it is limited because the degree of abstraction from real world situations is extreme. It omits consideration of intermediate inputs, government policy instruments (taxes and subsidies), savings and investment and international trade. The model described in this document addresses three of these issues but leaves international trade and hence trade tax instruments until later. The exercises associated with this model assume that it is derived directly from the basic model.

In developing this model three extensions to the basic model are required. First households will be required to distribute their expenditures between consumption, saving and income taxes. Second the government will have three tax instruments that can be varied; these tax instruments will impact upon commodities, activities and households and therefore influence the commodity price formation mechanisms, the incomes available to activities for the payment of factors and the incomes to households for consumption expenditures and savings. And third intermediate inputs are added, which means that the price formation processes from the activity/production side must be redefined to incorporate the effects of intermediate inputs, primary inputs and production taxes. A particularly important dimension of the first two stages is the extension in the degree of complexity of the commodity price system. The first two extensions require the addition of two additional agents in the economy; a government and a savings and investment account; these must be included in the model in such a way that the circular flow is maintained, i.e., the expenditure on investment must equal (total) savings and government income must equal government expenditure.

There are five main aims with this model

1. The development of an understanding of the structure of CGE models.
2. The development of generic (GAMS) programming skills.
3. The provision of an introduction to policy modelling using policy instruments found in typical economies.
4. The development of an understanding of macroeconomic closure ‘rules’.
5. The provision of an introduction to variations in the factor market clearing conditions.

The exercises begin with the development of the code for a 2-sector closed economy model with government and investment. This is described in detail. The exercises relating to policy experiments, macroeconomic closure 'rules' and factor market clearing all assume that the user will conduct the exercises using the 2-sector closed economy model with government, investment and intermediate inputs.

In terms of programming the exercise requires the modification and addition of equations, and the various stages required to add the requisite variables and parameters. To avoid (unnecessary) confusion the initial programming is carried out with a simple 'neoclassical'[1] closure rule; specifically, the total value of investment is assumed to be driven by exogenously determined savings rates.

The approach adopted introduces variations in macroeconomic closure rules and market clearing conditions at an early stage. This is a deliberate decision, which, while it adds to the complexity, is deemed important because of how closure rule selections and market clearing conditions are often critical to the results. The exercises illustrate how different closure rules might influence the results derived from the model.

[1] The term 'neoclassical' is used here because it is commonly used in the literature to describe this closure. It is arguable that a more appropriate term is 'new classical'. It should be noted that as the models are developed the restrictions associated with this closure setting are relaxed and the models become increasingly 'agnostic' with respect to macroeconomic closure conditions by allowing users to impose their own views about macroeconomic closure.

2. Two Sector Closed Economy Model: Model 2

This model is concerned with extending the basic two sector closed economy model to include investment and government accounts and intermediate inputs. The extension is carried out in one stage.

Social Accounting Matrix

The SAM for Model 2 is detailed in Table 1. The differences between this SAM and the SAM for Model 1, other than the values of transactions, are the inclusion of intermediate inputs, i.e., a matrix of inter industry transactions[2], an account for savings and investment (capital) and an account for the government. The presence of intermediate inputs ensures that changes in one activity's outputs/inputs feed back to the other activity via intermediate inputs, while the various tax instruments require changes in the price formation processes and the presence of savings and investments means that there are 'injections' and 'withdrawals' from the system.

Table 1 Social Accounting Matrix for a Model with Investment, Government & Intermediate Inputs

		Commodities		Activities		Factors		Households		Govt	Investment	
		Primary	Secondary	Agriculture	Industry	Labour	Capital	Urban	Rural			Total
Commodities	Primary			30	50			50	70	20	15	235
	Secondary			50	100			90	60	60	40	400
Activities	Agriculture	215										215
	Industry		375									375
Factors	Labour			60	140							200
	Capital			65	75							140
Households	Urban					100	90					190
	Rural					100	50					150
Government		20	25	10	10			25	5			95
Savings								25	15	15		55
Total		235	400	215	375	200	140	190	150	95	55	

2 In this SAM, the matrix of inter industry transactions satisfies the conditions necessary for it to be described as part of an input-output table. In this chapter and subsequent chapters, the more general terms USE matrix will be used to describe commodity demand transactions; this is because the code subsequently developed will work with either a SUPPLY and USE matrix or input-output representation of an economy.

The SAM retains a structure whereby the production relationships, household preferences and patterns of factor ownership differ between the activity, commodity, factor and household accounts.

Table 2 Behavioural Relationships for a Model with Investment, Government & Intermediate Inputs

	Commodities	Activities	Factors	Households	Government	Investment	Total	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Cobb-Douglas Utility Functions	Fixed in Real Terms	Fixed Shares of Savings	Commodity Demand	Consumer Commodity Prices
Activities	Cobb-Douglas Production Functions	0	0	0	0	0	Activity Output	Activity Prices
Factors	0	Factor Demands	0	0	0	0	Factor Income	Factor Prices
Households	0	0	Fixed Shares of Factor Income	0	0	0	Household Income	
Government	*Ad valorem* GST	*Ad valorem*	0	Average Income Tax rates	0	0	Government Income	
Savings	0	0	0	Household Savings	Government Savings (Residual)	0	Total Savings	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Government Expenditure	Total Investment		
	Producer Commodity Prices	Value Added Prices						

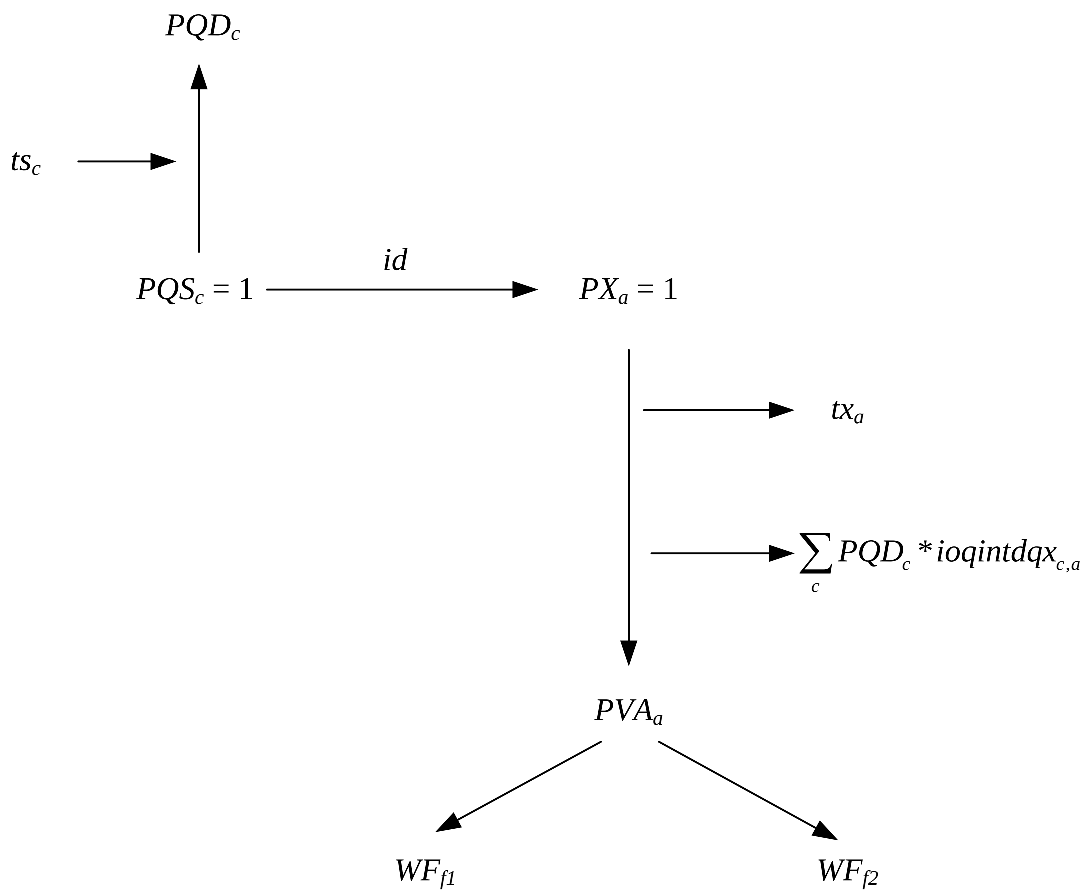
The behavioural SAM, Table 2, sets out the assumed behavioural relationships. For simplicity, it is assumed that households save fixed proportions of their income, and that total saving is expended on investments in fixed proportions; the level of savings therefore drives investment in this model. However, it is easy to modify this model so that the savings rates are variables and the levels of investment are exogenously fixed. This is achieved by changing the model closure equations. The addition of a government account with three tax instruments makes appreciable differences, especially to the price system, while at the same time adding an additional source of savings – the government's internal balance. The inclusion of general sales taxes (GST) means that the basic (producer) and purchaser (consumer) prices of commodities are no longer the same; these differences are achieved by means of simple *ad valorem* (proportionate) price wedges. Similarly, the activity prices are no longer the amount received by factors per unit of activity output; these differences arise because there are production taxes, which again are defined as simple *ad valorem* (proportionate) price wedges. These price wedges are illustrated in Figure 1.

Price and Quantity Systems

The sales taxes (ts_c) cause the purchaser prices (PQD_c) to exceed the basic prices (PQS_c), while intermediate inputs and production taxes (tx_a) cause the activity prices (PX_a) to be greater than the value-added prices (PVA_a). These taxes require modifications to the equations in the price block. Payments for each unit of activity output must now be divided between value added and payments for intermediate inputs and production taxes with the resultant value added being paid to factors. Intermediate inputs are treated very simply; it is assumed that a unit of activity output uses a fixed proportionate quantity of each intermediate input, i.e., Leontief input-output technologies are assumed, and hence expenditure on intermediate inputs per unit of output is the sum of the intermediate input prices multiplied by the technical coefficients derived from the Use matrix. This is still a common presumption in the literature, and represents a convenient and simple starting point, especially since it allows a simple redefinition of the price of value added (*PVA*). This means that there is a 'wedge' between activity prices (PX_a) and value-added prices (PVA_a) that must be included in the model.

An overview of how these changes impact upon the model's quantity systems is given in Figure 2. The addition of intermediate, investment and government demand means that the total demand for commodities (QQ_c) is divided between intermediate demand ($QINTD_c$) household consumption ($QCD_{c,h}$), investment ($QINVD_c$) and government (QGD_c) demand. The lack of clear behavioural relationships, e.g., as with utility functions for households, means that investment and government are treated relatively simply, e.g., the relative quantities of commodities demanded by each agent are fixed, with the level of demand determined via the choice of closure rule. For investment, the simplest rule is that the total value of expenditures on investment is determined by the total value of savings in the economy, with household savings rates fixed, while for government demand it can be assumed that the total consumption expenditure by the government is fixed in nominal terms, with all tax rates fixed, and thence government saving is determined as a residual, i.e., income less commodity expenditure. In both cases the model allows for variations in these simple behavioural assumptions through the choice of model closure rule – these are discussed in detail below. Notice how the total demand for intermediate inputs in the economy is only defined over the commodity accounts and not as the volume of intermediate demand for each commodity by each activity. This is simply a way of expressing demand that reduces the number of variables and equations in the model; it is elementary to alter the code so that *QINTD* is defined over *c* and *a*, and then effecting summations over *a* to derive the same result as achieved in this version.[3]

Figure 1 Price System for Model 2 Closed Economy with Investment, Government & Intermediate Inputs

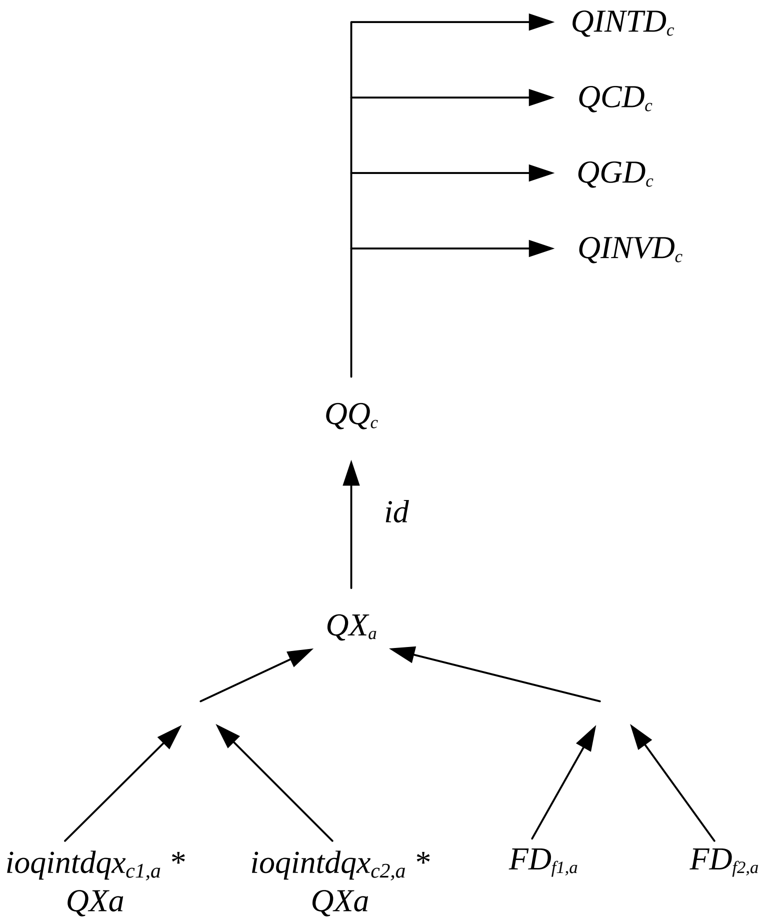


The other changes needed are for household expenditures. Again, a simple assumption is made; household savings and consumption expenditure are determined after the payment of direct taxes. The behavioural assumptions remain simple: fixed proportions of after tax

3 In the 2-sector model this only 'saves' two equations and variables, but in a 10-sector model this 'saves' 90 equations and variables.

incomes are saved and thereafter expenditure patterns are determined by Cobb-Douglas utility functions.

Figure 2 **Quantity System for Model 2 Closed Economy with Investment, Government & Intermediate Inputs**



The price and quantity systems have developed as far as is worthwhile within the context of closed economy models. It is of course possible to conceive of additional tax instruments, which would modify the price system, but these would only be variations on the themes already explored. Similarly, greater richness could be added to the institutional arrangements, e.g., inter-household transfers, government transfers to households, etc., but again these would be little more than variations on themes already explored. Furthermore, if left in the model they would simply severe to complicate the process of adding trade relationships to the model, and thereby distract attention from subsequent developments. They can be added easily to make a more fully developed open economy model.

Formal Approach to the Price System

The price system is a straightforward extension of that in the basic model, and as with that model the definitions can be derived directly from the system of prices that underpin the SAM database. Start with the accounting definitions for the commodity accounts. Define PQS_c as the basic price for commodity *c*, i.e., the price received by producers. Start with the accounting definitions for the commodity accounts.

$$PQD_c * \left(QINTD_c + \sum_h QCD_{c,h} + QINVD_c + QGD_c \right) = PQS_c * QX_a + SAM_{g,c}$$
$$PQD_c * QQ_c = PQS_a * QX_a + SAM_{g,c}$$
$$\forall c = a$$

where $SAM_{g,c}$ is the value of expenditures by the commodity account to the government, i.e., it is the revenue from taxes levied on commodities. At first sight this may seem strange because the RHS of the equation has the price defined over *c* and the quantities defined over *a*, but note the condition for all $c = a$.

The first consideration is how the basic prices for commodities are defined. From the definitions in Table 1 it can be deduced that since the Supply matrix is a diagonal matrix, there is a unique mapping between commodities and activities,

$$PQS_c * QQ_c = PX_a * QX_a \qquad \forall c = a$$

and hence, by market clearing in the quantities of commodities, (total supply equals total demand $QQ_c = QX_a \quad \forall c = a$)

$$PQS_c = PX_a \qquad \forall c = a .$$

Assume, the behavioural assumption for the tax, that the *ad valorem* general sales tax (GST) rates on each commodity (ts_c) are identical for all domestic purchasers, and noting the market clearing condition, the accounting identity can be re written as

$$PQD_c * QQ_c = PQS_c * QQ_c + \left(ts_c * PQS_c * QQ_c\right)$$
$$PQD_c = PQS_c + \left(ts_c * PQS_c\right) = PQS_c * \left(1 + ts_c\right).$$

where $SAM_{g,c} = \left(ts_c * PQS_c * QQ_c\right)$, i.e., the GST is an *ad valorem* markup on commodities valued at basic prices.

Consider now the activities column. The value of output by each activity is known, and since total income must equal total expenditure the following accounting identity must exist if there are no intermediate inputs

$$PX_a * QX_a = \sum_f WF_f * FD_{f,a} + SAM_{g,a} \qquad \forall a$$

which defining the total value of payments to factors per unit of output as *PVA* and the *ad valorem* production tax rates on each activity as tx_a, can be re written as

$$\begin{aligned} PX_a * QX_a &= PVA_a * QX_a + (tx_a * PX_a * QX_a) \qquad \forall a \\ PX_a &= PVA_a + (tx_a * PX_a) \\ PVA_a &= PX_a * (1 - tx_a) \end{aligned}$$

Adding in intermediate inputs is simple in the case where the activities use intermediate inputs in fixed proportions per unit of output. The relationship between total output and total inputs can now be written as

$$\begin{aligned} PX_a * QX_a &= SAM_{c,a} + \sum_f WF_f * FD_{f,a} + SAM_{g,a} \qquad \forall a \\ PX_a * QX_a &= \left(a^x_{c_1,a} * QX_a * PQD_{c_1} + \ldots + a^x_{c_n,a} * QX_a * PQD_{c_n} \right) \\ &\quad + PVA_a * QX_a + (tx_a * PX_a * QX_a) \\ PX_a &= \sum_c \left(a^x_{c,a} * PQD_c \right) + PVA_a + (tx_a * PX_a) \\ PVA_a &= PX_a * (1 - tx_a) - \sum_c \left(a^x_{c,a} * PQD_c \right) \end{aligned}$$

where $a^x_{c,a}$ is the Leontief input-output coefficient for commodity *c* used in production by activity *a*. This is a useful way to define activity prices. When the production system has Leontief production functions for intermediate inputs, the quantity of any intermediate input used to produce a unit of the activity's output is fixed, when there is no substitution between 'aggregate' intermediate inputs and 'aggregate' value added. It makes the adjustment of the first order conditions for factor use straightforward.

Defining the profits of each activity as

$$\Pi_a = (PX_a * QX_a) - \sum_c (a^x_{c,a} * PQD_c * QX_a) - (tx_a * PX_a * QX_a)$$
$$- \left[(WF_l * FD_{l,a}) + (WF_k * FD_{k,a}) \right]$$
$$= (PVA_a * QX_a) - \left[(WF_l * FD_{l,a}) + (WF_k * FD_{k,a}) \right]$$
$$= \left(PVA_a * \left(\alpha^x_a . FD^{\beta_{l,a}}_{l,a} . FD^{\beta_{k,a}}_{k,a} \right) \right) - \left[(WF_l * FD_{l,a}) + (WF_k * FD_{k,a}) \right]$$

By substituting out $\left\{ (PX_a * QX_a) - \sum_c (a^x_{c,a} * PQ_c * QX_a) - (tx_a * PX_a * QX_a) \right\}$ using the price definition. Then, partially differentiating with respect to the factor quantities, solving for profit maximisation to get the factor prices gives

$$WF_l = \left(PVA_a * \left(\alpha^x_a . \beta_{l,a} . FD^{(\beta_{l,a}-1)}_{l,a} . FD^{\beta_{k,a}}_{k,a} \right) \right)$$
$$= PVA_a * \beta_{l,a} * \frac{QX_a}{FD_{l,a}}$$
$$\Rightarrow WF_l * FD_{l,a} = \beta_{l,a} . (PVA_a * QX_a)$$
$$WF_k = \left(PVA_a * \left(\alpha^x_a . \beta_{k,a} . FD^{\beta_{l,a}}_{l,a} . FD^{(\beta_{k,a}-1)}_{k,a} \right) \right)$$
$$= PVA_a * \beta_{k,a} * \frac{QX_a}{FD_{k,a}}$$
$$\Rightarrow WF_k * FD_{k,a} = \beta_{k,a} . (PVA_a * QX_a)$$

which simply states that the shares of the product of activity *a* received by labour and capital are equal to the respective coefficients on the factors in the production function. This is another application of Eulers theorem made possible by the choice of functional form.

The introduction of taxes on household income and savings by households also need to be dealt with. The assumption made here is that there is an average tax rate on household income and that households save fixed proportions of their after-tax income.

3. Algebraic Statement of Model 2: 2 Sector Closed Economy Model

The sets for commodities, activities, factors and households remain unchanged but the dimensions of the SAM have changed due the addition of a capital account for savings and investment and a government account for tax revenues and government expenditures. It is therefore necessary to extend the global set, *sac*, i.e.,

$$sac = \{c, a, f, h, g, i\_s, total\}$$
$$g = \{govt\}$$

Similarly, the set *ss* for the aggregate SAM also needs extending, i.e.,

$$ss = \{commdty, activity, valuad, hholds, govtn, kapital, totals\}.$$

To indicate those equations that have been changed, or added, in this model relative to the earlier model a letter suffix (d) has been added for equations that have been changed.

Model Equations

This model introduces the assumption that intermediate are used in fixed quantities per unit of activity output; this is a strong Leontief fixed coefficients technology assumption. This assumption about production technology allows for substitution between the two primary inputs, labour and capital, but not between aggregate intermediate and primary inputs.

<u>Price Block Equations</u>

The producer and consumer prices for commodities are no longer the same. Hence the first price equation needs to be re-expressed as a mapping between the producer prices of commodities (P_c) and activity prices, i.e.,

$$PX_a = \sum_c ioqqqx_{a,c} * PQS_c \qquad \text{(P1d)}$$

and then the consumer prices (PQD_c) need to be defined in terms of basic (producer) prices and sales taxes (ts_c). The sales taxes are defined as parameters and this produces

$$PQD_c = PQS_c * (1 + ts_c). \qquad \text{(P3d)}$$

The consumer price index (*CPI*), which is defined as a weighted sum of the commodity prices where the weights are the shares of each commodity in total demand ($comtotsh_c$) remains unchanged.

$$CPI = \sum_{c} comtotsh_{c} * PQD_{c} \qquad \text{(P2)}$$

The production taxes serve to reduce the share of the revenue per unit of activity output that is to be distributed to factors. The value-added prices (PVA_a) are therefore defined as the activity prices less the per unit production taxes (tx_a), from which are subtracted the weighted sum of consumer commodity prices where the weights used are the input-output technical coefficients, i.e.,

$$PVA_{a} = \left(PX_{a} * \left(1 - tx_{a}\right)\right) - \left(\sum_{c} PQD_{c} * ioqintdqx_{c,a}\right). \qquad \text{(P3)}$$

It may help to examine equation E6d below to appreciate the treatment of production taxes in this model. Note how value added prices define, implicitly, the proportions of the output prices that are to be distributed between the primary inputs. Hence, it is the value-added prices that will enter the first order condition for profit maximisation in the production block equations.

<u>Production Block Equations</u>

The production block consists of the production function, a mapping of activity outputs (QX_a) into commodity supplies and a first order condition for profit maximisation. The production function is a Cobb-Douglas aggregation function over the two factors that are demanded by each activity ($FD_{f,a}$). Each production function has been calibrated for an efficiency parameter (ad_a) and two elasticities of output ($\alpha_{f,a}$)

$$QX_{a} = ad_{a} \prod_{f} \left(FD_{f,a}\right)^{\alpha_{f,a}} . \qquad \text{(X1)}$$

Since each activity produces only a single output the mapping between activity outputs and commodity supplies is via an identity matrix whose elements are denoted $actcomsh_{a,c}$

$$QQ_{c} = \sum_{a} ioqqqx_{a,c} * QX_{a} . \qquad \text{(X2)}$$

The first order condition for profit maximisation exploits the properties of Euler's theorem and the relationship between the elasticity of output and factor shares in long-run competitive equilibrium, i.e., the elasticity of output for input *f* in activity *a* is the share of output from activity *a* received by factor *f*. But the 'products' to be distributed among the factors are now defined by reference to the value-added prices rather than activity prices, i.e.,

$$FD_{f,a} = \frac{QX_a * PVA_a * \alpha_{f,a}}{WF_f}. \tag{X3}$$

This formulation for factor demands appears to be adequate, but it does inhibit policy experiments and restricts the assumptions that can be made about the operation of the factor market. Assume, for instance, that there were data on factor quantities and these data indicated that the wage and rental rates for labour and capital, respectively, differed by activity.[4] Then the wage rate would need to be specified by factor (*f*) and activity (*a*). Alternatively, we can define the factor-activity specific factor price as the product of the average price for a factor (WF_f) and a set of weights that reflect differences in relative productivities ($WFDIST_{f,a}$) and then we write (X3) as

$$FD_{f,a} = \frac{QX_a * PVA_a * \alpha_{f,a}}{WF_f * WFDIST_{f,a}}. \tag{X3b}$$

In the initial formulation of the model the relative productivities ($WFDIST_{f,a}$) will be assumed to be fixed (at a value of one), i.e., treated as parameters; this is useful because it makes the factor market clearing condition, which is indexed on *f*, consistent. It will however also turn out to be useful (LATER) to be able to relax this assumption when running simulations that involve different assumptions about how the factor markets clear.

The production block also needs extending to include an equation for the (commodity) demand for intermediate inputs by commodity ($QINTD_c$). This is defined as the level of activity outputs multiplied by the quantities of inputs used per unit of output[5], i.e.,

$$QINTD_c = \sum_a ioqintdqx_{c,a} * QX_a. \tag{X4}$$

There is no need to model intermediate input demand from the perspective of the activity. This consideration is handled by the inclusion of an allowance for intermediate input demand through the price of value added equations and thereby in the first order conditions for profit maximization.

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4 This implies that the factors are not homogenous and/or perfectly mobile. For now, we will ignore the economics of this issue and simply respond to the empirical evidence.

5 It could be argued that since intermediate input demand is defined by commodities this equation should be included in the expenditure block.

Income Block Equations

The equations in the income block are those specified from the perspective of income received. It is a matter of personal preference whether they are defined from the income or expenditure perspective, but it is important to ensure that each relationship is only specified once.

This block consists of four equations. The first two are unchanged. Total income received by each factor account (YF_f) is defined as the summation of the earnings of that factor across all activities, i.e.,

$$YF_f = \sum_a WF_f * FD_{f,a} \ . \tag{Y1}$$

The total incomes for each household type (YH_h) are defined as a fixed mapping of factor incomes to households summed across all factors owned by that household type, i.e.,

$$YH_h = \sum_f hvash_{h,f} * YF_f \ . \tag{Y2}$$

The equation for income to the savings-investment account needs to allow for two sources of savings – households and government. Since government savings (*KAPGOV*) are a model variable – see below – this requires including the government savings in the definition of total savings. Household savings are slightly more complex since these are defined as fixed shares (s*hh*) of household income net of direct taxes on household income where the taxes are levied at average rates (*ty*). This produces the following expression for total savings

$$TOTSAV = \sum_h \left(\left(YH_h * (1 - ty_h) \right) * \left(SADJ * shh_h \right) \right) + KAPGOV \ . \tag{Y3}$$

Note how $\left(YH_h * (1 - ty_h) \right)$ defines income after tax. This is important. The modeling of savings determines how the parameters *ty* and *shh* must be assigned/calibrated in the GAMS programme. (See also E7 below.). In the initial formulation of the model *SADJ* is a variable set equal to one, and then fixed, i.e., it operates as a parameter equal to one and leaves the savings rates (shh_h) fixed. For now, it can be ignored, later we can relax this assumption when running simulations that involve different assumptions about how the savings-investment account clears.

The addition of a government account requires an equation for government income (*YG*). Government income is made of three components; the revenues from sales taxes (*COMTAX*), production taxes (*INDTAX*) and household taxes (*HTAX*), i.e.,

$$YG = COMTAX + INDTAX + HTAX \ . \qquad \text{(Y4)}$$

The government income equation collects together the tax revenues, which are modeled as expenditures by the accounts paying the taxes, and therefore are defined in the expenditure block. This equation is therefore a simple adding up condition, for which the 'real' equations appear elsewhere. Although this approach adds equations it has the (arguable) advantage of being more transparent and easier to modify.

Expenditure Block Equations

The expenditure equations require appreciable changes. Households now divide their income three ways, income taxes, savings and consumption expenditure. It is therefore convenient to add an equation that defines expenditures by households on commodities ($HEXP_h$), which is defined as household incomes after the payment of taxes and savings, i.e.,

$$HEXP_h = \left(YH_h * \left(1 - ty_h\right)\right) * \left(1 - \left(SADJ * shh_h\right)\right). \qquad \text{(E1)}$$

Note how savings are defined as being out of after tax incomes. The household commodity demand equations can then be written in terms of household consumption expenditures rather than household incomes, i.e.,

$$QCD_{c,h} = \frac{comhav_{c,h} * HEXP_h}{PQD_c}. \qquad \text{(E2)}$$

This is matter of 'style' and preference rather than substance: an additional equation and variable have been declared and assigned with the primary intention of making the process easier to follow and debug.

Government demand for commodities is assumed to be fixed in (proportionate) real terms, i.e., the volume is fixed, but can be scaled or allowed to vary using an adjustment factor (*QGDADJ*). The precise specification depends upon the choice of closure rule (see below).

$$QGD_c = qgdconst_c \cdot QGDADJ \ . \qquad \text{(E3)}$$

which given the purchaser prices means that government expenditure can be defined as

$$EG = \sum_{c} PQD_{c} * QGD_{c} \text{ .}^{6} \qquad \text{(E4)}$$

Similarly, the model contains no behavioural assumptions that will determine the patterns of investment. Hence it is assumed that investible funds, *INVEST*, are exactly equal to expenditures on investments distributed in fixed proportions across all commodities, i.e.,

$$INVEST = \sum_{c} PQD_{c} * QINVD_{c} \qquad \text{(E4)}$$

but this only defines total investment expenditure. If the composition of commodities purchased for investment is fixed then the quantities of commodities demanded for investment are

$$QINVD_{c} = IADJ * qinvdconst_{c} \qquad \text{(E5)}$$

where $qinvdconst_{c}$ is a vector of the quantities of investment commodities purchased in the base period. In the initial formulation of the model *IADJ* is a variable, whose initial value is one; this is the equilibrating variable that ensures the savings-investment account clears. When the model is replicating the base data the value of this variable will be one. But when running simulations, it is free to change, e.g., if household incomes increase then, *ceteris paribus*, household savings will increase, *IADJ* then changes so the volume of investment increases. This amounts to an assumption that investment is determined by the volume of investible funds (savings). Later, we can relax this assumption when running simulations that involve different assumptions about how the savings-investment account clears.

There are three equations for expenditure by accounts on taxes, E6d, E7d and E8d. Each adopts the same basic structure: the expenditure on, or revenue from, a tax is defined as the tax rate multiplied by a value which is then adjusted using a variable scaling factor. The sales tax equation defines the revenue from sales taxes as the per-unit value of the commodity multiplied by the value of sales multiplied by the tax rate, and then summed over all commodities, i.e.,

$$COMTAX = \sum_{c} \left(ts_{c} * PQS_{c} * QQ_{c} \right). \qquad \text{(E6)}$$

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[6] Strictly, the inclusion of this equation could be avoided, but it adds flexibility for certain simulations so it can be useful.

Similarly, production tax revenue is defined as the per-unit value of the activity output multiplied by the value of production multiplied by the tax rate, and then summed over all activities, i.e.,

$$INDTAX = \sum_a \left(tx_a * PX_a * QX_a \right). \tag{E7}$$

Finally, household direct taxes are defined simply as household income multiplied by the direct tax rate, and then summed over all households, i.e.,

$$HTAX = \sum_h \left(ty_h * YH_h \right). \tag{E8}$$

Thus, the average and marginal income tax rates are assumed to be equal.

Market Clearing Block Equations

Market clearing requires the simultaneous clearing of all markets. In this model, there are four relevant markets, the factor and commodity markets and the government and capital accounts. The presumption of a full employment equilibrium requires that factor demands ($FD_{f,a}$) and factor supplies (FS_f) equate, i.e.,

$$FS_f = \sum_a FD_{f,a} \tag{M1}$$

while the commodity equilibrium requires that commodity supply (QQ_c) and commodity demands equate, i.e.,

$$QQ_c = QINTD_c + \sum_h QCD_{c,h} + QGD_c + QINVD_c \,. \tag{M2}$$

Note how the commodity demands have been extended to include intermediate inputs (*QINTD*), government demand (*QGD*) and investment demand (*QINVD*) in addition to household demand (*QCD*).

The government budget must, *ex-post*, balance, and therefore an equation must be added to ensure ‘market’ clearing for the government account. The difference between government income (*YG*) and government expenditure on commodities is government savings (*KAPGOV*), which are defined implicitly as a residual, i.e.,

$$YG = EG + KAPGOV \,. \tag{M3}$$

This equation defines government savings, but has been written in terms of equality between government income and expenditure. This is a matter of individual preference.

It is also necessary to add a market clearing equation for the capital market; specifically, the condition that total savings (*TOTSAV*) are equal to total investments (*INVEST*). This also provides an opportunity to relocate the slack variable (*WALRAS*) from the real GDP equation[7], and therefore the market clearing equation for the capital market can be written as

$$TOTSAV = INVEST + WALRAS\ . \tag{M4}$$

Strictly moving the *WALRAS* variable was not necessary, BUT it is common to express certain components of savings as residuals, and thus it is arguably consistent to include the slack variable in the capital market equilibrium condition.

<u>GDP Block Equations</u>

The macroeconomic aggregate equation need a minor change. The real GDP equation can now be dispensed with and GDP can be derived from the expenditure side by including all domestic final (commodity) demand, i.e.,

$$GDP = \sum_{c,h} QCD_{c,h} * PQD_c + \sum_c \left(\left(QGD_c + QINVD_c \right) * PQD_c \right). \tag{G1}$$

<u>Model Closure Equations</u>

This model consists of 51 variables, of which one is a slack variable, and 42 equations. This means that 9 variables need to be fixed as part of the model closure. In the base version of the model this is achieved by fixing the scaling factors for household savings rates (1), sales taxes (1), production taxes (1), and direct taxes (1), government expenditure (1), factor supplies (2) and the consumer price index (1).

For the base version of this model the supplies of labour and capital (FS_f) are assumed to be exogenously determined, i.e.,

$$FS_f = \overline{FS}_f\ . \tag{C1}$$

In addition, the consumer price index is fixed and thereby serves as the price normalisation equation, i.e.,

$$CPI = \overline{CPI}\ . \tag{C2}$$

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[7] It is possible to compute GDP from the income side. But this calculate requires changing if the indirect taxes in the model change so it is easier to make the calculation from the expenditure side

The *numéraire* is needed because the model is homogenous of degree zero in prices and hence only defines relative prices.

The savings-investment account is cleared by assuming fixed savings rates, hence

$$SADJ = \overline{SADJ} \text{ .} \tag{C3}$$

and that the volume of investment is flexible, i.e., *IADJ* is left flexible and the quantities of the commodities used for investment would have increased in the same ratio. Alternatively, the value of investment (*INVEST*) could have been left flexible, in which case the changes in the prices of investment commodities would enter into the market clearing mechanism.

It is also necessary to close the government accounts; it is assumed that the value of government savings/borrowings is fixed, i.e.,

$$KAPGOV = \overline{KAPGOV} \tag{C4}$$

and therefore, that the government account is cleared by scaling the volume of government expenditure, *QGDADJ* adjusts.

Table 3 Equation and Variable Counts for Model

Name	Equation	Number of Equations	Variable	Number of Variables
			PQS_c	2
$PQDEF_c$	$PQD_c = PQS_c * (1 + ts_c)$	2	PQD_c	2
$PXDEF_a$	$PX_a = \sum_c ioqqqx_{a,c} * PQS_c$	2	PX_a	2
$PVDEF_a$	$PVA_a = (PX_a * (1 - tx_a)) - \left(\sum_c PQD_c * ioqintdqx_{c,a} \right)$	2	PVA_a	2
$CPIDEF$	$CPI = \sum_c comtotsh_c * PQD_c$	1	CPI	1
$PRODFN_a$	$QX_a = ad_a \prod_f FD_{f,a}^{\alpha_{f,a}}$	2	QX_a	2
$PROFITMAX_{f,a}$	$FD_{f,a} = \frac{QX_a * PVA_a * \alpha_{f,a}}{WF_f * WFDIST_{f,a}}$	4	$FD_{a,f}$	4
$QINTDEQ_c$	$QINTD_c = \sum_a ioqintdqx_{c,a} * QX_a$	2	$QINTD_c$	2
$COMOUT_c$	$QQ_c = \sum_a ioqqqx_{a,c} * QX_a$	2	WF_f	2

Table 3 (cont)Equation and Variable Counts for Model

Name	Equation	Number of Equations	Variable	Number of Variables
$YFEQ_f$	$YF_f = \sum_a WF_f * FD_{f,a}$	2	YF_f	2
$YHEQ_h$	$YH_h = \sum_f hvash_{h,f} * YF_f$	2	YH_h	2
$YGEQ$	$YG = COMTAX + INDTAX + HTAX$	1	YG	1
$TOTSAVEQ$	$TOTSAV = \sum_h ((YH_h * (1 - ty_h)) * (SADJ * shh_h)) + KAPGOV$	1	$TOTSAV$	1
$HEXPEQ_h$	$HEXP_h = (YH_h * (1 - ty_h)) * (1 - SADJ * shh_h)$	2	$HEXP_h$	2
$QCDEQ_{c,h}$	$QCD_{c,h} = \frac{comhav_{c,h} * HEXP_h}{PQD_c}$	4	$QCD_{c,h}$	4
$QGDEQ_c$	$QGD_c = qgdconst_c \cdot QGDADJ$	2	QGD_c	2
$EGEQ$	$EG = \sum_c PQD_c * QGD_c$	1	EG	1
$QINVDEQ_c$	$QINVD_c = IADJ * qinvdconst_c$	2	$QINVD_c$	2
$INVESTEQ$	$INVEST = \sum_c PQD_c * QINVD_c$	1	$INVEST$	1
$COMTAXEQ$	$COMTAX = \sum_c (ts_c * PQS_c * QQ_c)$	1	$COMTAX$	1
$INDTAXEQ$	$INDTAX = \sum_a (tx_a * PX_a * QX_a)$	1	$INDTAX$	1
$HTAXEQ$	$HTAX = \sum_h (ty_h * YH_h)$	1	$HTAX$	1

Table 3 (cont)Equation and Variable Counts for Model

Name	Equation	Number of Equations	Variable	Number of Variables
$FMEQUIL_f$	$FS_f = \sum_a FD_{f,a}$	2	FS_f	2
$QEQUIL_c$	$QQ_c = QINTD_c + \sum_h QCD_{c,h} + QGD_c + QINVD_c$	2	QQ_c	2
$KAPGOVEQ$	$YG = EG + KAPGOV$	1	$KAPGOV$	1
$WALRASEQ$	$TOTSAV = INVEST + WALRAS$	1	$IADJ$	1
$GDPEQ$	$GDP = \sum_c \left(\left(\sum_h QCD_{c,h} + QGD_c + QINVD_c \right) * PQD_c \right)$	1	GDP	1

Table 3 (cont)Equation and Variable Counts for Model

Name	Equation	Number of Equations	Variable	Number of Variables
			FS_f	2
			QGDADJ	1
			CPI	1
			SADJ	1
			KAPGOV	1
			WALRAS	1
		45		53+1