



Production Relationships Part 2



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Outline

- Introduction
- *CES functions*
 - Theory and calibration
- *ANARRES_t Production*
 - Top level production (QX)
 - Primary - CES
 - Intermediates – CES/Leontief
 - Value added nest (QVA)
 - Level 3 nest (FD_{fag})
 - Aggregate output
 - CES form
 - Leontief form



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Value Added Nest



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Value Added Nest

$$\begin{array}{c}
 PVA_a \\
 \swarrow \quad \searrow \\
 \sigma_{va} \\
 \swarrow \quad \searrow \\
 \bullet \quad \bullet \\
 \downarrow \quad \downarrow \\
 tf_{l,a} \leftarrow \quad \rightarrow tf_{k,a} \\
 \downarrow \quad \downarrow \\
 WFA_{l,a} \quad WFA_{k,a}
 \end{array}$$

Assume this is $PFD(f,a)$

**BUT the prices/costs faced by activities
when choosing optimal input combinations
are inclusive of factor use taxes/subsidies**

NOT $WFA(f,a)$



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Value Added – General CES Form

Primal function

$$QVA_a = \alpha_a^{va} \left[\sum_f \delta_f^{va} * FD_f^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}}$$

Profit function

$$\begin{aligned} \pi_a^{va} &= PVA_a * QVA_a - \sum_f WFA_{f,a} * FD_f \\ &= PVA_a * \alpha_a^{va} \left[\sum_f \delta_f^{va} * FD_f^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}} - \sum_f WFA_{f,a} * FD_f \end{aligned}$$

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Value Added – General CES Form

Without factor use taxes

$$\begin{aligned} \frac{\partial \pi_a^{va}}{\partial FD_f} &= PVA_a \cdot \alpha_a^{va} \cdot \left[\sum_f \delta_f^{va} \cdot FD_f^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}-1} \cdot \delta_f^{va} \cdot FD_f^{-\rho_a^{va}-1} - WFA_{f,a} = 0 \\ WFA_{f,a} &= PVA_a \cdot \alpha_a^{va} \cdot \left[\sum_f \delta_f^{va} \cdot FD_f^{-\rho_a^{va}} \right]^{-\left(\frac{1+\rho_a^{va}}{\rho_a^{va}}\right)} \cdot \delta_f^{va} \cdot FD_f^{-(\rho_a^{va}+1)} \\ &= PVA_a \cdot \alpha_a^{va} \cdot QVA_a \left[\sum_f \delta_f^{va} \cdot FD_f^{-\rho_a^{va}} \right]^{-1} \cdot \delta_f^{va} \cdot FD_f^{-(\rho_a^{va}+1)} \end{aligned}$$

With factor use taxes

$$(WFA_{f,a}) * (1 + t_{f,a}^f) = PVA_a \cdot \alpha_a^{va} \cdot QVA_a \left[\sum_f \delta_f^{va} \cdot FD_f^{-\rho_a^{va}} \right]^{-1} \cdot \delta_f^{va} \cdot FD_f^{-(\rho_a^{va}+1)}$$

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Value Added Nest – in GAMS

```

QVAPRODFN(a,r)$[rhova(a,r) AND rgn(r) AND SUM(ff, deltava(ff,a,r))].
QVA(a,r)=E=ADVA(a,r)
    *{SUM(ff$[map_va_ff(ff,a) AND deltava(ff,a,r)],
    deltava(ff,a,r)*(ADFD(ff,a,r)*FD(ff,a,r))
    **(-rhova(a,r)))}
    **(-1/rhova(a,r)) ;

QVAFOC(ff,a,r)$[map_va_ff(ff,a) AND deltava(ff,a,r) AND rgn(r)].
WFA(ff,a,r)*(1 + TF(ff,a,r)) =E= PVA(a,r)*QVA(a,r)
    *{SUM[ffp$deltava(ffp,a,r),deltava(ffp,a,r)
    * [ADFD(ffp,a,r)*FD(ffp,a,r)]
    **(-rhova(a,r))]**(-1)
    *deltava(ff,a,r)*ADFD(ff,a,r)**(-rhova(a,r))
    *FD(ff,a,r)**(-rhova(a,r)-1) ;

```

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Value Added Nest – Calibration

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rhova(a,r) = (1/mod_elastva(a,r)) - 1 ;

* share parameters WITH factor use taxes - tf02(f,a,r)

deltava(ff,a,r)$[map_va_ff(ff,a) AND FD0(ff,a,r)]
    = {[WFA0(ff,a,r)]*[1+tf02(ff,a,r)]*[FD0(ff,a,r)]
    **[1+rhova(a,r)]}
    / SUM{ffp$map_va_ff(ffp,a),
    [WFO(ff,r)*WFDIST0(ff,a,r)]*[1+tf02(ffp,a,r)]
    *[FD0(ffp,a,r)]**[1+rhova(a,r)]} ;

ADVA0(a,r)$SUM(ff$map_va_ff(ff,a),deltava(ff,a,r)*FD0(ff,a,r))
    = QVA0(a,r)
    / (SUM(ff$map_va_ff(ff,a),deltava(ff,a,r)
    *FD0(ff,a,r)**(-rhova(a,r)))**(-1/rhova(a,r)));

```

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The WFA(f,a) Problem 1

- LOOP – one price per row
- $WFA(f,a)$ – implies ‘ a ’ prices for every ‘ f ’ rows
- **Solution 1:**
 - If the quantities of a factor used by each activity are known then $WFA(f,a)$ are known
 - BUT there is only one price definition for each f

SOLUTION 1: Assume differences in MP for factor f used by activity a are factor f and activity a specific

$$WFA_{f,a} = WF_f * wfdist_{f,a}$$

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The WFA(f,a) Problem 2

- If the quantities of a factor used by each activity are **NOT** known then $FD(f,a)$ are unknown
- **Solution 2:**
 - If $WFA(f,a) = 1$ for all f,a then $FD(f,a) = VFD(f,a)$
 - BUT there is only one price definition for each f

SOLUTION 2: Assume NO differences in MP for factor f used by activity a and therefore factors are homogeneous

$$WFA_{f,a} = WF_f * wfdist_{f,a}$$

and $WF_f = wfdist_{f,a} = 1$

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Third Level Nest



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Aggregate Factor – General CES Form

Without factor use taxes

$$\frac{\partial \pi_{f,a}^{fd}}{\partial FD_{f,a,r}} = WFA_{fag,a,r} \cdot \alpha_{a,r}^{fd} \cdot \left[\sum_f \delta_{fag,f,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}} \right]^{-1/\rho} \cdot \delta_{f,a,r}^{fda} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}-1} - WFA_{f,a} = 0$$

$$WFA_{f,a} = WFA_{fag,a,r} * \alpha_{a,r}^{fd} * \left[\sum_f \delta_{fag,f,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}} \right]^{\left(\frac{1+\rho_{a,r}^{fd}}{\rho_{a,r}^{fd}} \right)} * \sum_f \delta_{fag,f,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}}$$

$$= WFA_{fag,a,r} * \alpha_{a,r}^{fd} * FD_{fag,a,r} * \left[\sum_f \delta_{fag,f,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}} \right]^{-1} * \sum_f \delta_{fag,f,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{a,r}^{fd}}$$

With factor use taxes

$$(WFA_{f,a,r}) * (1 + tf_{f,a,r})$$

$$= WFA_{fag,a} * FD_{fag,a,r} + (1 + TF_{fag,a,r}) \left[\sum_f \delta_{f,fag,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{f,a,r}^{fd}} \right]^{-1} \cdot \delta_{f,fag,a,r}^{fd} \cdot FD_{f,a,r}^{-\rho_{f,a,r}^{fd}+1}$$

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Third Level Production Nest - GAMS

```

FDPRODFN(ff,a,r)$[SUM[ffp,map_fagg_ff(ff,ffp,a)] AND rgn(r)]..
  FD(ff,a,r) =E= ADFAG(ff,a,r)
    *{SUM[ffp${map_fagg_ff(ff,ffp,a) AND deltafd(ff,ffp,a,r)},
      deltafd(ff,ffp,a,r) *FD(ffp,a,r)**(-rhofd(ff,a,r))]}
      **[-1/rhofd(ff,a,r)] ;

* FOC for input ffp used to produce aggregate input ffp by activity a
* NOTE ffp can consist of natural or aggregate factors

FDFOC(ff,ffp,a,r)$[map_fagg_ff(ff,ffp,a) AND deltafd(ff,ffp,a,r) AND rgn(r)]..
WFA(ffp,a,r)*[1 + TF(ffp,a,r)]
  =E= {WFA(ff,a,r) *[1 + TF(ff,a,r)] * FD(ff,a,r)}
    *{SUM[ffpp${map_fagg_ff(ff,ffpp,a)
      AND deltafd(ff,ffpp,a,r)},
      deltafd(ff,ffpp,a,r)*FD(ffpp,a,r)
      **(-rhofd(ff,a,r))]}**(-1)
      *deltafd(ff,ffp,a,r)
      *FD(ffp,a,r)**[-rhofd(ff,a,r)-1] ;

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Third Level Nest - Calibration

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rhofd(fag,a,r) = (1/ELASTFD(fag,a,r)) - 1 ;

deltafd(ff,ffp,a,r)$[map_fagg_ff(ff,ffp,a) AND SUM[ffpp$map_fagg_ff(ff,ffpp,a),FD0(ffpp,a,r)] }
  = { [WFA0(ffp,a,r)]*[1+tf02(ffp,a,r)]*(FD0(ffp,a,r))
      **[1+rhofd(ff,a,r)] }
    /{ SUM(ffpp$map_fagg_ff(ff,ffpp,a),
      [WFA0(ffpp,a,r)]*[1+tf02(ffpp,a,r)]*(FD0(ffpp,a,r))
      **[1+rhofd(ff,a,r)] } } ;

adfaf(ff,a,r)$[SUM[ffp,map_fagg_ff(ff,ffp,a)] AND SUM[ffpp$map_fagg_ff(ff,ffp,a),FD0(ffp,a,r)] }
  = FD0(ff,a,r)
    /{ SUM[ffpp$map_fagg_ff(ff,ffpp,a),
      deltafd(ff,ffp,a,r)*FD0(ffp,a,r)**(-rhofd(ff,a,r))] }
      **[-1/rhofd(ff,a,r)] ;

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Production Relationships
Part 2

The End



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