A Basic 1*2*3 Open Economy Model[1]

cgemod

[1] This model owes to, and the text draws on, Devarajan *et al.*, (1990).

1. Introduction

The basic, or traditional, neoclassical trade model presumes that all commodities are tradeable and that all commodities are perfect substitutes and hence a ‘law of one price’ must hold, i.e., **all** commodities should have the same price in **all** markets. Consequently, it is possible to invoke a simple border price paradigm (see Timmer, 1986) whereby the domestic prices of all commodities, for a small open economy, are determined ultimately by world prices. This presumption has been extensively adopted for partial equilibrium analyses of trade (e.g., Francois and Hall, 1997).

The Salter-Swan (or Australian) trade model pointed up the extreme limitations imposed by a presumption that all commodities are tradable, and hence initiated a series of models within which a dichotomy between traded (tradables) and non-traded (non-tradables) commodities was imposed. This approach has much to recommend it, but in part the problem presented by the presumption that all commodities are traded remains; for the subset of commodities that are tradable a ‘law of one price’ must hold. Thus, even in a Salter-Swan type model it would be expected that we would observe:

- prices of all traded commodities would be set by world prices;
- minor shifts in policy instruments periodically producing extreme fluctuations in trade patterns;
- complete specialisation; and
- no-cross hauling.

But, even with extremely disaggregated trade data, we rarely if ever observe any of these features. Thus, when such trade models (basic and/or Salter-Swan) find ‘corner solutions’, e.g., complete specialisation, and/or extreme fluctuations in relative prices, the empirical validity of the models is questionable. Similar problems existed with the empirical open economy input-output models (see Leontief and Strout, 1963).

A resolution to the problem came from two papers by Armington (1969a and 1969b) wherein the assumption that imports and domestic demand are imperfect substitutes was proposed, i.e., commodities are ‘semi-tradable’, which has also been extended to encompass the substitutability between exports and domestic supply.[2] Armington was primarily

[2] Anyone familiar with undergraduate microeconomics will quickly recognise that the presumption of imperfect substitution is implicit to the vast majority of orthodox microeconomic theory except in this

concerned with providing a theoretically consistent basis for the 'modified-shares' approach to modeling trade. This had largely originated with concerns about the results from trade models that presumed the trade shares by partners were approximately constant. This debate resulted in models that forecast trade patterns using a two-stage approach: in the first stage trade shares were forecast using the constant-shares method, in the form of a matrix of trade shares, and in the second stage the matrix was modified to reflect factors expected to cause changes in shares (see Taplin, 1967). It seems that, to a greater or lesser extent, the early applications of the modified-shares method involved ill-defined arguments to justify changes in the matrices of shares.

The CGE literature has, virtually without exception, adopted the, so-called, Armington 'insight' (Dervis *et al.*, 1982). The properties of trade models that adopt the Armington assumption have been extensively analysed (de Melo and Robinson, 1981 and 1989), although there are questions about the universal appropriateness of the assumption (Alston, *et al*., 1989; Brown, 1987). The model developed below ignores any debates about the appropriateness of the Armington insight, rather the focus is on understanding the properties and mechanics of CGE models that use the Armington insight.

The model developed in this paper is an adaptation[3] of the stylised models devised by de Melo and Robinson, (1981 and 1989) and synthesized by Devarajan *et al*., (1990) in which there is **one** country with **two** production sectors and **three** commodities – the 1*2*3 model. The model examined here has **no** factor markets, government or investment; subsequent variants of this model relax these restrictive assumptions, but for the moment they serve a valuable pedagogic purpose by allowing the student to concentrate on the treatment of trade without, albeit interesting, distractions. The data used in the empirical exercises are purely hypothetical. This is arguably the simplest CGE model that incorporates trade and yet can produce useful insights.

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case the distinction is between varieties of a commodity while in the standard economic theoretic case it is between two distinct homogenous commodities.

3 The adaptations are designed so that this basic 123 model fits pedagogic needs and is open to extensions that allow real data to be used. It is presumed that a simple 2 sector closed economy model has been studied.

2. A Basic 1*2*3 Open Economy CGE Model

This model has **one** country with **two** production sectors and **three** commodities. The objective of the model is to demonstrate the incorporation of the Armington 'insight' into a CGE model in the simplest possible context, and then to develop some exercises that demonstrate the consistency of the model with (neoclassical) trade theory. Hence the model contains no factor markets, government or investment, and can be expressed in 14 equations.

Economic Theory

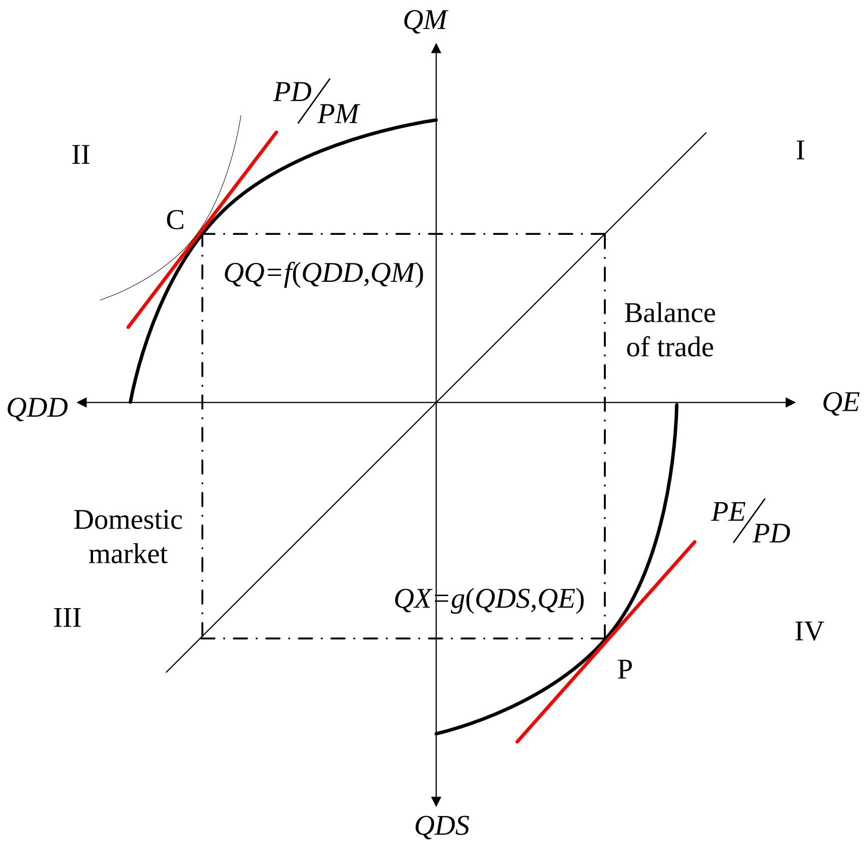
The basic mechanics of the model can be illustrated in a four-quadrant trade diagram (Figure 1), where all the dimensions are positive. The bottom right-hand quadrant of this diagram illustrates how, in this model, domestic production, represented by the production possibility frontier (*ppf*), can either be consumed domestically, commodity *QDS*, or exported, commodity *QE*. The commodities *QDS* and *QE* are presumed to be imperfect substitutes; hence the *ppf* is concave to the origin. Given the relative prices of *QDS* and *QE*, i.e., *PE*/*PD*, represented by the budget constraint, the producer decides the optimal (relative) quantities of *QDS* and *QE* to produce to maximise 'profit', i.e., at the point *P*.

If the exchange rate is the *numéraire*, and it is assumed (for the moment) that the balance of trade is zero, and the export, *PE*, and import, *PM*, prices are equal, then the relationship between exports and imports is one to one, and the balance of trade condition, in quadrant I, is represented by a 45^{o} line through the origin.[4]

Similarly, the transformation of domestic commodity production, *QDS*, into the domestic commodity for consumption, *QDD*, is a one to one mapping, and hence the domestic market is characterised by a 45^{o} line through the origin in quadrant III

[4] Changing the relative prices, PE and PM, simply changes the slope while the line still passes through the origin. The 1:1 ratio simplifies the illustration.

Figure 1 **A Basic 1*2*3 Model**



The information in quadrants III, IV, and I can now be used to map out the consumption possibility frontier (*cpf*) in the remaining quadrant, II. Well-being/utility is maximised where the line representing the relative prices of the domestic commodity and the imported commodity is tangential to the *cpf*, *C*, which is also where the indifference curve is at a tangent to the *cpf*. In this case where world prices are equal, and trade is balanced. The *cpf* is a mirror image of the *ppf*; this is a special case chosen for simplicity of exposition.

The model that follows is an 'empirical' implementation of the economic ideas encapsulated by the model in Figure 1.

<u>Social Accounting Matrix</u>

The Social Accounting Matrix (SAM) for a basic 1*2*3 CGE model is reported in Table 1. The value of commodities supplied to the domestic market consists of 80 units from domestic producers and 20 units of imports. The representative consumer, who is also the sole recipient of value added, consumes all these commodities. The final 20 units of domestic production are exported. Hence although this economy is open, it is not very dependent on trade.

Table 1 Social Accounting Matrix for a Basic 1*2*3 Model

	Commodity	Activity	Household	Rest of World	Total
Commodity	0	0	100	0	100
Activity	80	0	0	20	100
Household	0	100	0	0	100
Rest of World	20	0	0	0	20
Total	100	100	100	20	0

Note how in this SAM all income is spent on consumption in the current period and how there is balanced commodity trade. The former assumption cannot be relaxed because of the absence of a capital account, but the latter assumption can be relaxed by allowing the exogenously determined current account of the balance of payments to be in surplus or deficit via a transfer to the household from the rest of the world.

<u>Transactions Relationships</u>

The transaction relationships are specified in Table 2. For this model a small country assumption is adopted: hence the world prices of exports (*pwe*) and imports (*pwm*) are defined in terms of the world currency and fixed. Consequently, the domestic prices of exports (*PE*) and imports (*PM*) are determined by the free on board (*fob*) and carriage, insurance and freight paid (*cif*) world prices multiplied by the exchange rate (*ER*). These world prices enter the determination of domestic consumer prices.

An important feature of the Armington 'insight' is the determination of activity and consumer prices. Armington based trade models presume the existence of 'composite' commodities.[5] The concept of such composites is adopted throughout standard economics; welfare/wellbeing from a utility function is a composite derived by aggregating heterogenous

5 While all CGE models that use the Armington insight presume the existence of 'composite' commodities the precise formulation used will differ between models.

commodities, while the output from a production function is a composite derived by aggregating heterogenous inputs. The Armington 'insight' uses the same approach to the aggregation of heterogenous commodities; the difference is that instead of aggregating apples and oranges the Armington 'insight' is used to aggregate varieties of oranges.

In this model the domestic commodity (*QDD*) is combined with the imported commodity (*QM*) to form the supply of a 'composite' commodity (*QQS*) that is consumed on the domestic market. The price of the composite commodity is determined by the relative prices of the imported commodity (*PM*) and the domestic commodity (*PDD*) and the degree to which they are substitutes. The same principal applies to the distribution of domestic production between the domestic (*QDS*) and export markets (*QE*). In this instance the price (*PX*), which is paid for the composite output (*QX*), is determined by the relative prices of the exported commodity (*PE*) and the domestic commodity (*PDS*) and the degree to which they are substitutes.

Table 2 Transaction Relationships for a Basic 1*2*3 Model

	Commodity	**Activity**	**Household**	**Rest of World**	**Prices**
Commodity	0	0	$PQ*QCD$	0	PQ
Activity	$PDD*QDD$ $PDS*QDS$	0	0	$pwe*QE*ER$	PX
Household	0	$PX*\overline{QX}$	0	$KAPWOR$	YH
Rest of World	$pwm*QM*ER$	0	0	0	-
Total	$PQ*QQS$	GDP	YH	-	

The difference between trade models with and without the Armington 'insight' is therefore the ability for domestic and foreign variants of commodities to be (imperfect) substitutes for each other.[6] This allows for product differentiation and for situations in which

[6] Bread wheat and biscuit wheat is an example. Both are wheat and in the same 4-digit category in the HS of trade codes (1001); but they are not perfect substitutes for each other. Even at the 6-digit HS level the difference between bread and biscuit wheat is not recorded.

the classifications of commodities and activities are not sufficiently fine to create homogeneous categories.

Behavioural Relationships

The remaining behavioural assumptions are very simple for this model. In the absence of factor markets domestic production is fixed and the quantity of the composite commodity consumed is determined by income (*YH*) and the consumer price (*PQ*). Total value added is distributed to the domestic consumer, but while the production possibility frontier is fixed, value added can vary with changes in the activity price (*PX*), which will be influenced by domestic and export prices and the exchange rate. Total income can also be varied by allowing for the possibility of a sustainable deficit on the current account (*KAPWOR*); this can be conceived of as an unrequited inflow, e.g., aid or remittances.

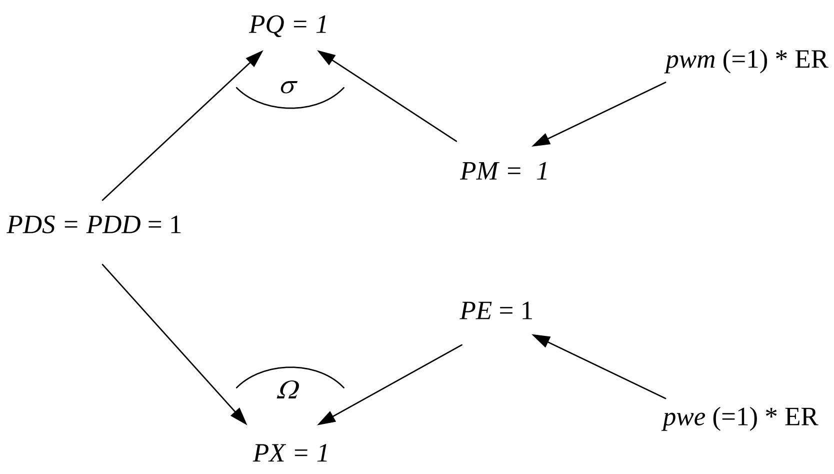
The simplicity of the other relationships allows attention to focus on the key relationships; the Constant Elasticity of Transformation (CET) between domestic supply (*QDS*) and exports (*QE*) and the Constant Elasticity of Substitution (CES) between domestic demand (*QDD*) and imports (*QM*). Details of these functions and the FOC for optima are given below.

Table 3 Behavioural Relationships for a Basic 1*2*3 Model

	Commodity	**Activity**	**Household**	**Rest of World**	**Total**
Commodity	0	0	Consumption at consumer price	0	Commodity Demand
Activity	Domestic Production	0	0	Armington (CET) Function	Fixed Production Possibility Frontier
Household	0	Domestic Factor Income	0	Balance of Trade (exogenous)	Household Income
Rest of World	Armington (CES) Function	0	0	0	Import 'Expenditure'
Total	Commodity Supply	GDP	Household Expenditure	Export 'Revenue'	0

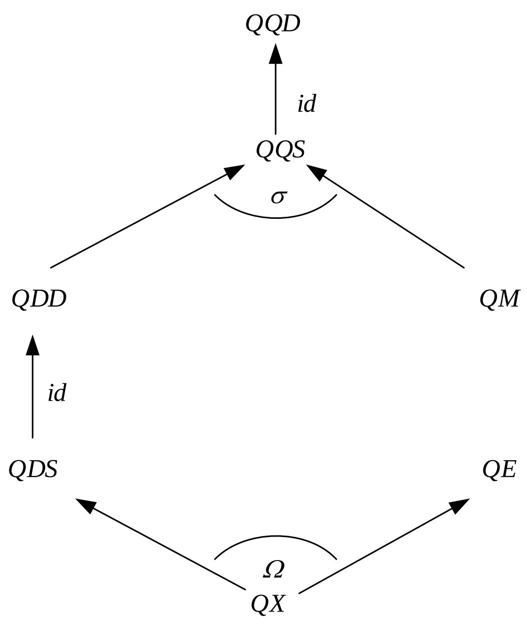
The price relationships are illustrated simply in Figure 2. The only factors influencing consumer (*PQ*) and activity (*PX*) prices are the world prices of imports (*pwm*) and exports (*pwe*), the exchange rate (*ER*) and the degree of substitutability. When there is no government there are no tax instruments and/or any other endogenous restrictions on trade.

Figure 2 **Price Relationships for a Basic 1*2*3 Model**



Similarly, the quantity relationships are very simple (see Figure 3). Domestic commodities (*QDD*) and imports (*QM*) are mixed to produce supply to the domestic market (*QQS*), while domestic production (*QX*) is divided between the domestic market (*QDS*) and exports (*QE*).

Figure 3 **Quantity Relationships for a Basic 1*2*3 Model**



3. Armington 'Insight' and CES Functions

Standard neoclassical models presume that all commodities are tradable, and all tradable commodities are perfect substitutes. This is scarcely plausible, since such models have a propensity to yield extreme specialisation and wild swings in relative prices when world prices or trade policies change. Moreover, it does not accord with observed patterns of trade, even when the data are recorded with very high degrees of disaggregation (several thousand categories of commodities). The Salter (1959) and Swan (1960) models recognised this by distinguishing between 'tradables' and 'nontradables', which resolved the problem at least at a theoretical level, but had little impact on empirical work. In the Salter-Swan model the link between the domestic and world prices depends solely upon whether a commodity/sector is traded. If a commodity is traded the domestic price equals the world price (*c.i.f* or *f.o.b*). If a commodity is non-traded the domestic price is determined by the interaction of demand and supply on the domestic market. Thus, tradables and nontradables are treated asymmetrically.

The Armington insight extends the Salter-Swan approach by assuming imperfect substitutability between tradable commodities and hence treating all tradable commodities symmetrically. The link between domestic and world prices then depends upon the trade shares and the elasticities of substitution. For any given elasticity, the domestic price will be closer to the world price the greater the trade share, and similarly for any given trade share the domestic price will be closer to the world price the greater the elasticity (see de Melo and Robinson, 1985).

This ensures there are links between domestic and world prices, as in the standard neoclassical model, but these are less strong than in the standard model. Also, the model can accommodate two-way trade within the same sector, intra-industry trade; a feature of trade flows that is commonly observed. The approach is therefore consistent with a view of trade, imports and exports, as taking place in differentiated products and is consistent with the Salter-Swan model in that it gives rise to normally shaped offer curves. The exchange rate is also appropriately defined: if the domestic commodity is the numéraire, i.e., *PD* is set equal to one, then the exchange rate, *ER*, is the real exchange rate of neoclassical theory (the relative price of tradables (*QM & QE*) to nontradables (*QDD & QDS*)). If *ER* is set equal to one then *PD* defines the real exchange rate, while for other choices of *numéraire*, *ER* is a monotonic transformation of the real exchange rate.

Despite their limitations CES/CET functions are used. (CES(ubstitution) is used for consumption, CET(ransformation) for production). Specifically, we can write a general (primal) form, *QX*, of the CES/CET specification as

$$QX = \bar{A}\left[(1-\gamma).QDS^{\rho} + \gamma.QE^{\rho}\right]^{1/\rho} \tag{1}$$

and the price dual, *P*, as

$$P = \bar{A}^{-1}\left[(1-\gamma)^{1/(1-\rho)}.P_1^{\rho/(\rho-1)} + \gamma^{1/(1-\rho)}.P_2^{\rho/(\rho-1)}\right]^{\rho-1/\rho} \tag{2}$$

where the substitution/transformation elasticity is given by

$$\sigma \textbf{ or } \Omega = \frac{1}{(1-\rho)} \qquad -\infty < \rho < +1 \tag{3}$$

The export supply and import demand functions are then given by

$$\frac{QE}{QDS} = \left[\frac{(1-\gamma)}{\gamma}.\frac{PE}{PD}\right]^{\Omega} \tag{4}$$

$$\frac{QM}{QDD} = \left[\frac{\delta}{(1-\delta)}.\frac{PD}{PM}\right]^{\sigma} \tag{5}$$

where δ and γ are the respective share parameters and σ and Ω are the elasticities.

Exploiting Euler's theorem for linearly homogeneous functions, we can replace the dual price equations for export transformation and import aggregation by expenditure identities, i.e.,

$$PQ = \frac{(PM.QM)+(PD.QDD)}{QD} \tag{6}$$

$$PX = \frac{(PE.QE)+(PD.QDS)}{QX} \tag{7}$$

4. Programming Problem

The 1-2-3 model can be presented as a programming problem where the objective is to maximise utility subject to constraints imposed by technology, the balance of trade and domestic supply and demand. This emphasises the fact that the 1-2-3 model (and in fact all standard CGE models) are simply applications of the standard microeconomic/Walrasian concept of constrained optimisation.

Writing this as a constrained optimisation, or programming problem, is relatively straightforward.

Max $QQ = F\left(QM, QDS, \sigma\right)$ (absorption)

with respect to QM, QE, QDS, QDD

subject to

(1) $G\left(QE, QDS, \Omega\right) \leq \overline{QX}$ (technology)

(2) $pw^{m}.QM \leq pw^{e}.QE + \overline{KAPWOR}$ (balance of trade)

(3) $QDD \leq QDS$ (domestic supply and demand)

and the respective shadow prices are

(1a) $\lambda^{x} = \dfrac{PX}{PQ}$

(2a) $\lambda^{b} = \dfrac{ER}{PQ}$

(3a) $\lambda^{d} = \dfrac{PD}{PQ}$

The constraints correspond to equations in the 1-2-3 model.

Note that QQ is the *numéraire* commodity and hence PQ = 1.

5. Algebraic Statement of a Basic 1*2*3 Open Economy Model

This model does not make use of the set facility in GAMS, and therefore each equation needs to be fully specified. This is not an extra burden because there are not multiple commodities, activities, factors or consumers.

As with the closed economy models it is convenient to define the model in 'blocks'. The order in which you proceed is largely a matter of personal preference.

Conventions

It is also advisable to adopt a series of conventions for the naming of variables and parameters. The conventions adopted are a matter of personal preferences. In all these models the following conventions are adopted:

- all VARIABLES are in upper case;
- all parameters are in lower case, except those used to initialise variables, which use the variable name plus a 0 (zero) suffix;
- names for parameters are derived using account abbreviations with the row account first and the column account second, e.g., actcom** is a parameter referring to the activity:commodity (supply or make) sub-matrix;
- parameter names have a two character suffix which distinguishes their definition, e.g., ***sh* is a share parameter and ***av* is an average;
- all parameter and variable names have less than 10 characters.

Price Block

The price block consists of five equations. The first equation (P1) defines the domestic price of the imported commodity in domestic currency terms (*PM*) as the product of the world price (*cif*) of imports (*pwm*), which is a fixed (exogenous) variable, and the exchange rate (*ER*), which is declared as a variable.

$$PM = pwm * ER \qquad \text{(P1)}$$

In the same manner, the domestic price of exports (*PE*) is defined as the product of the world price (*fob*) of exports (*pwe*), which is a fixed (exogenous) variable, and the exchange rate (*ER*).

$$PE = pwe * ER \quad \text{(P2)}$$

The composite price equations (P3 and P4) are derived from the first-order conditions (FOC) for tangencies to consumption and production possibility frontiers. By exploiting Euler's theorem for linearly homogeneous functions, the composite prices can be expressed as expenditure identities rather than dual price equations for export transformation and import aggregation. Thus the activity price (*PX*) is a weighted average of the prices received for domestically produced commodities on the domestic (*PDS*), i.e., the supply price for the domestic commodity, and export (*PE*) markets, where the weights are the volume shares of total output (*QX*) sold on the domestic (*QDS*) and export (*QE*) markets, i.e.,

$$PX = \frac{(PDS * QDS) + (PE * QE)}{QX}. \quad \text{(P3)}$$

Similarly the price of the composite commodity supplied to the domestic market (*PQ*) is a weighted average of the prices paid for domestically produced commodities on the domestic market (*PDD*), i.e., the demand price for the domestic commodity, and import (*PM*), where the weights are the volume shares of total supply of the composite commodity (*QQ*) purchased on the domestic (*QDD*) and import (*QM*) markets, i.e.,

$$PQ = \frac{(PDD * QDD) + (PM * QM)}{QQ}. \quad \text{(P4)}$$

Finally, the producer activity and commodity prices for the domestic commodity are identical, i.e.,

$$PDS = PDD. \quad \text{(P5)}$$

This equation is not strictly essential, but if this equation is dropped then one or the other of the variables *PDD* or *PDS* must be dropped from the model. It has been included to ensure this model is very explicit; strictly it is redundant.

<u>Supply Block</u>

Domestic production is fixed in the absence of factor markets. Producers must therefore decide upon the distribution of output between the domestic market and the export market. Assuming producers seek to maximise profits they will choose an efficient combination of exports (*QE*) and domestic sales (*QDS*) on the basis of relative prices. The transformation function, in CET form, is given by (X1), where *rhot* is the elasticity parameter; *gamma* is the share parameter and *at* the shift parameter.

$$QX = at\left(\gamma * QE^{rhot} + (1-\gamma) * QDS^{rhot}\right)^{\frac{1}{rhot}} \tag{X1}$$

and the efficient combination of exports and domestic supply is given by the FOC

$$\frac{QE}{QDS} = \left(\frac{PE}{PDS} * \frac{(1-\gamma)}{\gamma}\right)^{\frac{1}{(rhot-1)}}. \tag{X2}$$

Similarly, if consumers seek to maximise utility they will choose an efficient combination of imports (*QM*) and domestic demand (*QDD*) on the basis of relative prices. The substitution function, in CES form, is given by (X3), where *rhoc* is the elasticity parameter, *delta* is the share parameter and *ac* the shift parameter.

$$QQ = ac\left(\delta * QM^{-rhoc} + (1-\delta) * QDD^{-rhoc}\right)^{-\frac{1}{rhoc}}. \tag{X3}$$

The efficient combination of imports and domestic demand is given by the FOC

$$\frac{QM}{QDD} = \left(\frac{PDD}{PM} * \frac{\delta}{(1-\delta)}\right)^{\frac{1}{(1+rhoc)}}. \tag{X4}$$

<u>Income Block</u>

In the absence of factor markets all value added is distributed to the representative consumer (Y1). However, this economy can receive unrequited income, from the rest of the world, in the form of a (sustainable) deficit on the current account (*KAPWOR*). Hence total household income (*YH*) is defined as value added plus the surplus on the capital account of the balance of payments.

$$YH = (PX * QX) + (KAPWOR * ER). \tag{Y1}$$

Note how the surplus on the capital account of the balance of payments is defined in terms of the foreign currency and converted, by the exchange rate (*ER*), into domestic currency terms.[7]

[7] In the base data for the model the surplus on the capital account of the balance of payments is zero. However, it is necessary to include a term for the surplus on the capital account of the balance of payments in the model since it will be regarded as 'variable' in the simulation exercises.

Expenditure Block

With no alternate outlet for income, all income (*YH*) must be spent on consumption. The volume of consumption (*QCD*) is therefore defined by income divided by the consumer price (C1).

$$QCD = \frac{YH}{PQ} . \quad \text{(C1)}$$

Market Clearing Block

The equilibrium conditions are straightforward; the only complication is that with the possibility of trade two equilibrium conditions are needed for output. In the first condition, supply of the composite commodity to the domestic market (*QQ*) and domestic consumption demand on the domestic market (*QCD*) must equate (E1).

$$QQ - QCD = 0 \quad \text{(E1)}$$

In the second condition, total domestic commodity demand (*QDD*) and total domestic commodity supply (*QDS*) must equate (E2).

$$QDD - QDS = 0 \quad \text{(E2)}$$

Finally it is necessary to ensure that the trade account is balanced. The deficit on the current account or surplus on the capital account (*KAPWOR*) is defined as the value of imports less the value of exports.

$$KAPWOR = \left(pwm * QM - pwe * QE \right) + WALRAS . \quad \text{(E3)}$$

By defining *KAPWOR* in terms of the foreign currency E3 can be written without reference to the exchange rate, and consequently it is necessary to include the exchange in Y1.

Model Closure Block

The formulation of the model in this instance has 3 more variables than equations. But the model satisfies Walras's Law and therefore one equation is redundant. It is therefore necessary to fix 4 variables. In this case it is assumed that output, *QX*, which defines the capacity of the economy; the composite commodity price, *PQ*, which serves as the numéraire; and the exchange rate, *ER*, are fixed exogenously (M1, M2 and M3).

$$QX = \overline{QX} \tag{M1}$$

$$PQ = \overline{PQ} \tag{M2}$$

$$ER = \overline{ER} \tag{M3}$$

or

$$KAPWOR = \overline{KAPWOR} \tag{M3a}$$

Note how the small country trade assumption means that the world prices can be declared as parameters. An alternative would have been to declare the world prices of imports and exports as variables, and then fix them in the model closure or make them variables with associated equations, e.g., if the world price of export depends upon the country's exports of a commodity then an export demand equation could be included.

Table 4 Equation & Variable Counts: Basic 1*2*3 Model

Name	Equation	Number of Equations	Variable	Number of Variables
	PRICE BLOCK			
PMDEF	$PM = pwm * ER$	1	*PM*	1
PEDEF	$PE = pwe * ER$	1	*PE*	1
PQEF	$PQ = \frac{(PDD * QDD) + (PM * QM)}{QQ}$	1	*PDD*	1
PXDEF	$PX = \frac{(PDS * QDS) + (PE * QE)}{QX}$	1	*PX*	1
PDSDEF	$PDS = PDD$		*PDS*	1
	SUPPLY BLOCK			
CET	$QX = at\left(\gamma * QE^{rhot} + (1-\gamma) * QDS^{rhot}\right)^{\frac{1}{rhot}}$	1	*QDS*	1
ESUPPLY	$\frac{QE}{QDS} = \left(\frac{PE}{PDS} * \frac{(1-\gamma)}{\gamma}\right)^{\frac{1}{(rhot-1)}}$	1	*QE*	1
ARMINGTON	$QQ = ac\left(\delta * QM^{-rhoc} + (1-\delta) * QDD^{-rhoc}\right)^{-\frac{1}{rhoc}}$	1	*QQ*	1
COSTMIN	$\frac{QM}{QDD} = \left(\frac{PDD}{PM} * \frac{\delta}{(1-\delta)}\right)^{\frac{1}{(1+rhoc)}}$	1	*QM*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	INCOME and EXPENDITURE BLOCKS			
YHEQ	$YH = (PX * QX) + (KAPWOR * ER)$	1	YH	1
CDEQ	$QCD = \frac{YH}{PQ}$	1	QCD	1
	MARKET CLEARING BLOCK			
QEQUIL$_c$	$QQ = QCD$	1		
DOMEQUIL	$QDD = QDS$	1	QDD	1
CAEQUIL	$KAPWOR = (pwm * QM - pwe * QE) + WALRAS$	1	$KAPWOR$	1
			$WALRAS$	1
	MODEL CLOSURE			
			$\overline{QX}$	1
			$\overline{PQ}$	1
			$\overline{ER}$	1
		13		16 + 1

Choice of Numéraire

CGE models are, almost invariably, formulated in terms of relative prices not absolute prices. That is all demand and supply functions are homogeneous of degree zero in all prices and hence will only solve for relative prices. Hence the economic system is assumed to be indifferent to the absolute level of prices, but sensitive to relative prices. Thus, if all prices increase equiproportionately the actual quantities demanded and supplied will remain unchanged. The issue of a *numéraire* is central to these CGE models, and is solved by setting one price or an index equal to some constant, almost always one.

The choice of *numéraire* is important. The three most common are aggregate producer price, aggregate consumer price and the exchange rate. It is important to be aware of how the choice of *numéraire* influences the behaviour of CGE models and thus to ensure the appropriate choice for the policy issue currently being examined.

Equation and Variable Counting

It is important to keep track of the number of equations and variables. A good working rule of thumb is to ensure that the number equations and the number of variables are consistent. Table 5d details the number of equations and variables for this model.

6. Exogenous Changes and a Basic 1*2*3 Model in Diagrams

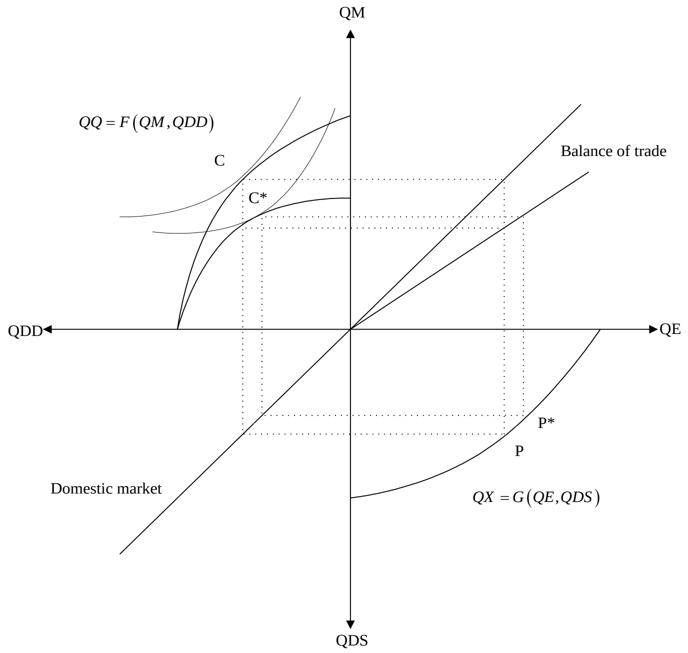
Before proceeding to the exercises it is useful to consider the effects of exogenous changes upon this model using the diagrammatic exposition introduced earlier. This approach provides a means by which policy experiments can be thought about in a more accessible manner, and hence eases the process of formulating policy experiments.

<u>Change in World Price</u>

A change in the world price, actually a change in the relative price of imports and exports, is illustrated in Figure 6e, where the effect considered is an adverse terms of trade shock, i.e., an increase in the (world) price of the imported commodity. This means that more of the export commodity (*QE*) must be supplied for any given quantity of the import commodity (*QM*). Hence, the balance of trade line will pivot to the right around the origin, if $\overline{KAPWOR} = 0$, and the *cpf* will pivot right around its intersection with the horizontal axis.

The case illustrated in Figure 4 is for a depreciation in the real exchange rate, in which case the consumption of both the domestic and import commodities decline, i.e., real income declines, and the production of the export commodity increases at the expense of the domestic commodity. But will the real exchange rate always depreciate? In fact, this depends upon the relative magnitudes of the income and substitution effects. Real income declines but, because of the change in the relative prices of the import and domestic commodities, there will be a substitution effect whose magnitude will be determined by the elasticity of substitution, σ. When $\sigma > 1$ the substitution effect is dominant and the economy reduces export production (and hence imports) and increases domestic commodity production. And when $\sigma < 1$ the income effect is dominant and the economy increases export production (and hence imports) and reduces domestic commodity production.

Figure 4 Change in World Prices



We can see how this depends upon the elasticities of import substitution and export transformation, σ and Ω. Log-linearising the model the FOC for the optimum ratios of *QE* to *QD* and *QM* to *QD* can be written as

$$\frac{QE}{QDS} = k_1 \left[\frac{PE}{PDS} \right]^{\Omega} \tag{1}$$

$$\frac{QM}{QDD} = k_2 \left[\frac{PMS}{PDD} \right]^{\sigma} \tag{2}$$

and noting the balance of payments expression to be

$$pwm.QM - pwe.QE = \overline{KAPWOR} \tag{3}$$

and then log differentiating (1), (2) and (3), noting that the export price is constant, produces

$$\widehat{QE} - \widehat{QDS} = \Omega \widehat{PD} \tag{4}$$

$$\widehat{QM} - \widehat{QDD} = \sigma\left(\widehat{PDD} - \widehat{pwm}\right) \tag{5}$$

$$\widehat{QM} + \widehat{pwm} = \widehat{QE} \tag{6}$$

where '^' denotes the logarithmic differential. Eliminating QM, QDD, QDS and QE, and solving for PD gives

$$\widehat{PD} = \left(\frac{\sigma - 1}{\sigma - \Omega}\right).\widehat{pwm}$$

which relates the proportionate change in the domestic price of the domestic commodity to the world price of the import commodity and the elasticities of import substitution and export transformation, σ and Ω.

Consequently, the magnitude of σ determines whether *PD* increases or decreases, and the direction of change will determine whether the exchange rate appreciates ($\sigma > 1$) or depreciates ($\sigma < 1$).

Foreign Capital Inflow

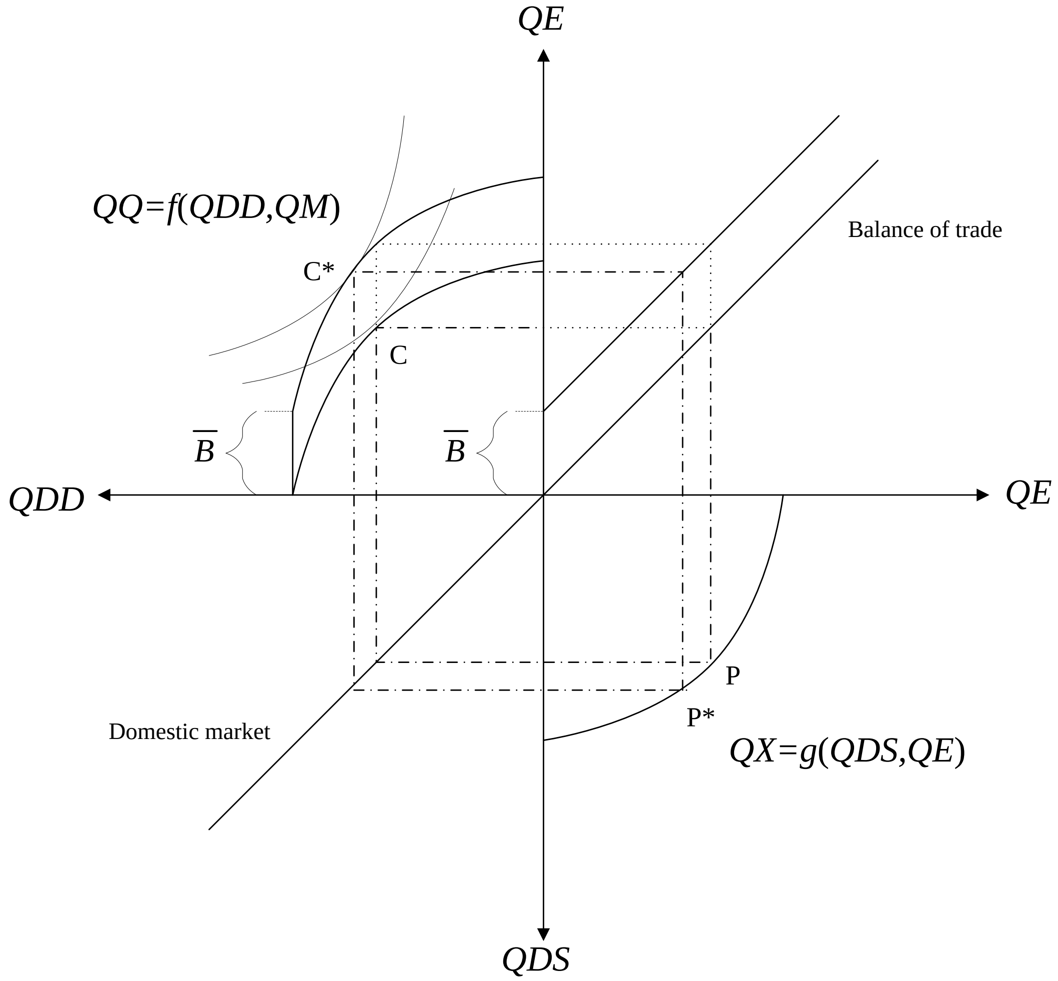
Consider the case in which there is an increase in the inflow of foreign capital, $\overline{KAPWOR} = \overline{B} > 0$, as illustrated in Figure 5, whereas in Figure 1 $\overline{KAPWOR}\left(=\overline{B}\right) = 0$. This could take the form of aid or a primary resource boom where all the additional export earnings are repatriated. In such a case we expect that the price of non-tradables will rise relative to the price of tradables and there will be a change in the sizes of the two sectors, i.e., what is known as the 'Dutch Disease' problem. This is illustrated in Figure 5.

Keeping the exchange rate as the *numéraire* and holding the relative prices of imports and exports constant, the slope of the balance of trade line is left constant at 45°, but the intercept is increased to $\overline{B}$. Similarly, the *cpf* is shifted up by $\overline{CAPWOR}$, and hence the new equilibrium depends upon the import aggregation function/utility function. The fundamental point is that the price of the domestic commodity rises relative to the price of the import and export commodities (note the price/budget lines are omitted in Figure 4).

Typically, this will produce an appreciation of the real exchange rate: is this always the case? The two extremes are an infinite elasticity of substitution between imports and domestic commodities and a zero elasticity of substitution between imports and domestic commodities. In the first case the new equilibrium, *C**, will lie directly above *C*, the indifference curves are 'flat' and the *cpf*s are vertically parallel. Hence all the extra foreign exchange will go on imports, *QM*. In the second case, we have Leontief type indifference curves and the new equilibrium will be on a ray from the origin, i.e., more of *QD* and *QM* will be consumed. But since *PM* is fixed by hypothesis, *PDD* must have increased.

Hence the exchange rate will have appreciated, or in the extreme stayed unchanged. Production of *QD* will stay constant or more likely increase and the production of *QE* (the tradable commodity) will either stay constant or more likely decline.

Figure 4 Increase in Foreign Capital Inflow



7. Model Variables and Parameter

PDD = Consumer price for domestic supply of commodity	*pwe* = World price of exports
PDS = Producer price for domestic output of activity	*pwm* = World price of imports
PE = Domestic price of exports by activity	*ac* = Shift parameter for Armington CES function
PM = Domestic price of competitive imports of commodity	*delta* = Share parameter for Armington CES function
PQ = Consumer price of composite commodity	*predelta* = dummy used to estimated delta
PX = Composite price of output by activity	*rhoc* = Elasticity parameter for Armington CES function
ER = Exchange rate (domestic per world unit)	*at* = Shift parameter for Armington CET function
QDD = Domestic demand for commodity	*gamma* = Share parameter for Armington CET function
QE = Domestic output exported by activity	*rhot* = Elasticity parameter for Output Armington CET function
QM = Imports of commodity	
QX = Domestic production by activity	
QDS = Domestic output supplied to domestic market by activity	
YH = Income to household	
CAPWOR = Current account balance	
QCD = Household consumption by commodity	
QQ = Supply of composite commodity	
WALRAS = Slack variable for Walras's Law	