A Basic Two Sector Closed Economy CGE Model

cgemod

Contents

1. Introduction

Two sector closed economy general equilibrium (GE) models have a long history, not only do they provide the basic structure used for teaching GE to undergraduate students they have also been used extensively in the CGE literature as expositional and teaching tools. An advantage of closed economy models when learning CGE techniques is that they allow the development of many of the techniques used in a CGE model without having to address the complications presented by the modelling of trade relations.

This chapter and the associated exercises concentrate on the classic 2 sector GE model taught to undergraduates (see notes on general equilibrium for more detail). By abstracting from the multitude of real world issues involving governments, investment, intermediate inputs and trade it is possible to develop a simple model that allows the user to examine the properties of a GE model with particular reference to the ‘laws’ of welfare economics. The process is further simplified by choosing functional forms for the behavioural relationships that are easy to implement.

The aims with this model are

1. The development of an appreciation of the processes involved when moving from a theoretical to an applied model.
2. The development of an understanding of the structure of CGE models.
3. The development of generic (GAMS) programming skills.
4. An introduction to model calibration.

2. A Basic Two Sector Closed Economy CGE Model

This is one of the simplest CGE models that can be devised, and which will yield meaningful insights. The model captures the full circular flow of a stylised and very simple economy. But the objective here is not realism; rather the objective is to establish the basic structure of a CGE model. The associated exercises assume that the model is implemented such that the variables are defined in terms of levels using the General Algebraic Modelling System (GAMS) software.[1]

Once the basic structure of the model has been established it will be used to conduct a series of simple policy experiments; these include changes in technology, preferences and factor endowments. These experiments will provide quantitative illustrations of the key properties of the model, thereby verifying the theoretical propositions, while demonstrating the use of a CGE model to conduct policy experiments.

Social Accounting Matrix

The SAM for this economy (Utopia) is reported in Table 1. The Utopian economy is characterised by a full circular flow. Households (urban and rural) provide the demand for commodities (primary and secondary); the transactions are recorded in the sub-matrix commodities:households. The demand for commodities is satisfied by the supply of commodities by activities (agriculture and industry), i.e., the activities:commodities sub-matrix. To produce these commodities the activities employ factors (labour and capital), i.e., the transactions in the factors:activities sub-matrix. Finally households receive incomes by selling factors, i.e., the transactions in the households:factors sub-matrix, and the resultant income allows them to express their preferences for commodities. In this very basic model, there are no intermediate input transactions, and no accounts for a government or investment. Hence the model is 'timeless', in that there is no delayed consumption via savings and investments, and there are no expenditures or revenue raising actions by government so there are no government policy instruments. As such the SAM is consistent with the simplest form of comparative static 2(*2)*2*2 general equilibrium model studied in undergraduate level

[1] The same model could be implemented in several different ways. One important alternative is the modelling of the variables in terms of percentage changes, which is the (main) method used by the GEMPACK software (GEMPACK has developed a levels version of the software, but the solutions are still derived using percentage changes). However, while the method of implementation may differ the basic principles do not differ with the choice of solution method.

microeconomics. As will become clear in the subsequent models it also avoids a vast array of complications associated with the interfacing of Walrasian microeconomics with macroeconomics; these issues will be addressed in subsequent exercises.

In this economy, each activity only produces a single commodity. This type of SAM is consistent with the class of CGE models developed in Dervis *et al*., (1982), and has formed the basis of many CGE models produced since the late 1970s. It is a reduced form of SAM compared to the SAM structure specified in the United Nations (UN) System of National Accounts (SNA) (UN, 1993 and 2008). Among other features it requires that there is the same number of commodities as activities, and that the output dimensions of the commodity and activity accounts are identical. As with any reduced form SAM, the information content of this SAM is less than that provided by a 'full' SAM.

While the structure of the SAM is consistent with that used by Dervis *et al*., (1982), the separate identification of commodity accounts entails minor differences in how the GAMS programme is structured. Although this requires a few more equations the resultant programme provides a more flexible basis for subsequent refinements. Notice also that if the transactions value for a single cell is unreported its value can be derived from the requirement that incomes and expenditures for all accounts must equate.

Table 1 Social Accounting Matrix for Model 1

		Commodities		**Activities**		**Factors**		**Households**		
		Primary	Secondary	Agriculture	Industry	Labour	Capital	Urban	Rural	Total
Commodities	Primary							50	75	125
	Secondary							100	50	150
Activities	Agriculture	125								125
	Industry		150							150
Factors	Labour			62	55					117
	Capital			63	95					158
Households	Urban					60	90			150
	Rural					57	68			125
Total		125	150	125	150	117	158	150	125	

Behavioural Relationships

The SAM in Table 1 is a transactions matrix and hence the entries are products of both price and quantity terms. Table 2 makes this explicit and provides a starting point for setting out the notation.

Table 2 Variable Labels: Basic 2*2*2*2 Closed Economy Model

		Commodities		Activities		Factors		Households	
		Food	Clothes	Agric	Industry	Labour	Capital	Urban	Rural
Commodities	Food	0	0	0	0	0	0	$PQD_c * QCD_{c,h}$	
	Clothes	0	0	0	0	0	0		
Activities	Agriculture	$PX_a * QX_a$		0	0	0	0	0	0
	Industry			0	0	0	0	0	0
Factors	Labour	0	0	$WF_f * FD_{f,a}$		0	0	0	0
	Capital	0	0			0	0	0	0
Households	Urban	0	0	0	0	$hovash_{h,f} * YF_f$		0	0
	Rural	0	0	0	0			0	0

In a fix-price model the concern is with the quantity terms for any given set of prices, *p*, which are determined by the fixed values of factor and commodity. However, in a flex-price model the modeller must be concerned with the determination of both prices and quantities. For this it is also useful to use the SAM as an organisational structure too specify the behavioural relationships for each of the active sub matrices of the SAM; Table 3 details the behavioural relationships chosen for this basic model.[2]

The behavioural relationships for this model assume that households choose their consumption bundles so as to maximise utility, where their preferences are expressed in Cobb-Douglas utility functions. Similarly, activities are presumed to produce their output by combining labour and capital (the factors) using technologies that can be represented by Cobb-Douglas production functions. The activities seek to maximise profits and therefore the combination of the production functions, and factor demand and supply functions yield the quantities of factors employed and the prices of those factors. The incomes to factors are then distributed to households: since there is no reason to assume otherwise it is assumed that each

[2] Note that the transactions recorded in a SAM were generated through the operation of an economy at some point in time. Such transactions could be consistent with many different behavioural relationships: any one SAM can provide the data for many different models. Thus, it is not appropriate to use the definite article when defining any model derived from any single SAM.

household receives the same rate of return on the quantities of factors owned by this household; this is achieved by a simple (linear) mapping of factor incomes to households.

Table 3 Behavioural Relationships: Basic 2*2*2*2 Closed Economy Model

	Commodities	Activities	Factors	Households	Total	Prices
Commodities	0	0	0	Cobb-Douglas Utility Functions	Commodity Demand	Commodity Prices
Activities	Cobb-Douglas Production Functions	0	0	0	Activity Output	Activity Prices
Factors	0	Factor Demand Functions	0	0	Factor Income	Factor Prices
Households	0	0	Fixed Shares of Factor Income	0	Household Income	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure		

A particular feature of Table 3 is the addition of a column for the ‘prices’ in the rows of the SAM. An important feature of any CGE flex-price model is the need to keep track of both quantities and prices. Initially this seems to be awkward but provided the problem is approached systematically it presents no substantive difficulties; fundamental to this is the recognition of the importance of the ‘law’ of one price (LOOP) that dictates that the prices in each row are identical for all transactions reported in that row.

The behavioural relationships set out in Table 3 are those suggested as a representation of society’s programming problem found in intermediate microeconomics, i.e.,

$$\max_{F,C} W = W\left(u^U, u^R\right)$$

subject to

$$\begin{array}{lll} F = F_U + F_R = \alpha_F L_F^{\beta_1} K_F^{\beta_2} & C = C_U + C_R = \alpha_C L_C^{\beta_3} K_C^{\beta_4} & \text{technology} \\ U^U = \alpha_U F_U^{\gamma_1} C_U^{\gamma_2} & U^R = \alpha_R F_R^{\gamma_3} C_R^{\gamma_4} & \text{preferences} \\ \bar{L} = L_F + L_C & \bar{K} = K_F + K_C & \text{factor endowments} \end{array}$$

where *F* is food, *C* is clothes, *U* is urban, *R* is rural, *L* is labour and *K* is capital. These are variables, except in so far as the total stocks of *L* and *K* are fixed. All the Greek characters are parameters that must be identified as part of the calibration process. But all the variables refer to the quantities in the system, using abbreviated notation to save space.

But Table 2 indicates that for a SAM it is also necessary to identify three sets of prices, *PQD*, *PX* and *WF*. Since the model will be price driven an important issue is how the prices and quantities can be derived from the transactions data in the SAM.

Price and Quantity Systems

The price system is based on the principles of the system of price relationships that underpin the SAM database; the existence of a system of prices embodied within a SAM is not an accident or even a fortuitous coincidence, rather it is a consequence of the fundamentals of the design of a SAM. It is the interdependences between prices embodied in the column entries of a SAM that provides the key to understanding the price system. To identify the definitions for prices, use will be made of the accounting requirement that the column and row totals are identical.

Start with the commodity accounts. From the definitions in Table 2, and using that notation, it can be deduced that since the Supply matrix is a diagonal matrix, there is a unique mapping between commodities and activities,

$$\sum_h PQD_c * QCD_{c,h} = PX_a * QX_a \qquad \forall c = a \ . \tag{1}$$

where *PQD* are the purchaser prices, *QCD* are the quantities of commodity *c* purchased by household *h*. *PX* are the basic prices received by activities for their output and *QX* are the outputs by activity *a*.

Defining $QQ_c = \sum_h QCD_{c,h}$ (1) can be written as

$$PQD_c * QQ_c = PX_a * QX_a \qquad \forall c = a \tag{2}$$

and since the market clearing conditions require that the total quantities demanded and supplied are equal for each commodity, i.e., $QQ_c \equiv QX_a \quad \forall c = a$, the purchaser and basic are uniquely related in this model, i.e.,

$$PQD_c = PX_a \qquad \forall c = a \ . \tag{3}$$

One way to proceed would be to identify all the purchaser and basic prices and then calculate the associated quantities. However, this information is rarely if ever available, so an alternative is needed. Noting that the system is linear homogenous, and hence the model will concentrate upon relative prices, a simple way to 'benchmark' the price system is to normalise the system. Hence, if the initial producer prices, *PX*0, are all defined equal to 1 all the remaining prices and quantities for the commodity system can be defined; at the same time the output and value of output for each activity will be defined, i.e., the row totals for the activity accounts.

The utility (*U*) functions for households, and hence commodity demand, can be written as

$$U_h = \phi_h^u . \prod_c QCD_{c,h}^{\gamma_{c,h}} \tag{4}$$

where given the coefficients on the quantities sum to one will ensure that all income is spent and the demand system is complete. The standard reduced form condition, as the ratio of marginal utilities (*MU*) for the utility maximising input ratio, can be written as

$$\frac{QCD_{2,h}}{QCD_{1,h}} = \frac{PQD_1}{PQD_2} . \left(\frac{\gamma_{2,h}}{\gamma_{1,h}} \right) = \frac{PQD_1}{PQD_2} . \left(\frac{(1-\gamma_{1,h})}{\gamma_{1,h}} \right) \tag{5}$$

where $\gamma_{c,h}$ is the expenditure share of commodity *c* by household *h*. Note how the choice of a linear homogenous functional form means that only relative quantities and prices are defined.

Considering this as a utility maximisation problem it is straightforward to demonstrate, by an application of Euler's theorem, that provided the exponents sum to one that the expenditure shares are equal to the values of the components; this is sufficient to ensure that all income, or as will later4 prove useful all disposable income, is spent on consumption. Thus, we can define

$$PQD_1 * QCD_{1,h} = \gamma_{1,h} * YH_h \tag{6a}$$

where YH_h is the income of household *h*, and hence that

$$\sum_c PQD_c * QCD_{c,h} = \sum_c \gamma_{c,h} * YH_h = YH_h \tag{6b}$$

which noting that *PQD* has been previously defined means that if *YH* and the expenditure shares are known then *QCD* is defined.

Consider now the activities columns. The value of output by each activity is known, and since total income must equal total expenditure the following accounting identity must exist

$$PX_a * QX_a = \sum_f WF_f * FD_{f,a} \qquad \forall a \tag{7}$$

where WF_f is the price of factor f and $FD_{f,a}$ is the quantity of factor f used by activity a. In the event of a perfectly competitive industry there will be zero profit. One way to approach this problem is from the perspective a production function and its associated first order conditions for profit maximization. Defining the production function as a Cobb-Douglas function gives

$$QX_a = \alpha_a^x . FD_{l,a}^{\beta_{l,a}} . FD_{k,a}^{\beta_{k,a}} = \alpha_a^x . \prod_f FD_{f,a}^{\beta_{f,a}} \tag{8}$$

and if there are constant returns to scale the β coefficients sum to 1.

Defining the profits of each activity as

$$\begin{aligned} \Pi_a &= \left(PX_a * QX_a\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right] \\ &= \left(PX_a * \left(\alpha_a^x . FD_{l,a}^{\beta_{l,a}} . FD_{k,a}^{\beta_{k,a}}\right)\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right] \end{aligned} \tag{9}$$

and partially differentiating with respect to the factor quantities, setting the partial derivative equal to zero and solving for factor prices gives

$$\begin{aligned} WF_l &= \left(PX_a * \left(\alpha_a^x . \beta_{l,a} . FD_{l,a}^{(\beta_{l,a}-1)} . FD_{k,a}^{\beta_{k,a}}\right)\right) \\ &= PX_a * \beta_{l,a} * \frac{QX_a}{FD_{l,a}} \\ &\Rightarrow WF_l * FD_{l,a} = \beta_{l,a} . \left(PX_a * QX_a\right) \\ WF_k &= \left(PX_a * \left(\alpha_a^x . \beta_{k,a} . FD_{l,a}^{\beta_{l,a}} . FD_{k,a}^{(\beta_{k,a}-1)}\right)\right) \\ &= PX_a * \beta_{k,a} * \frac{QX_a}{FD_{k,a}} \\ &\Rightarrow WF_k * FD_{k,a} = \beta_{k,a} . \left(PX_a * QX_a\right) \end{aligned} \tag{10}$$

which states that the shares of the product of activity a received by labour and capital are equal to the respective coefficients on the factors in the production function. This is a simple application of Euler's theorem.[3]

If either factor quantities or prices are known exogenously then the other can be readily calculated. However, if this information is not available it is possible to adopt a convention,

3 It is common in some models to find an explicit set of equations that define the zero-profit condition for each activity. These are not needed in this approach since the zero-profit condition is embedded in the derivation of the factor prices.

first used by Harberger, whereby the factor prices, *WF*, are set equal to 1 and hence the (normalised) factor quantities, *FD*, can be calculated.

The incomes to the factor accounts, YF_f, can now be defined as

$$YF_f = \sum_a WF_f * FD_{f,a} \tag{11}$$

and since total income must equal total expenditure for each account, the total payments from the factor accounts to the household accounts are known.

But it remains to be determined how these are distributed. Starting from the accounting identities for the factor accounts, it can be seen that

$$\sum_a WF_f * FD_{f,a} = YF_f = \sum_h WF_f * FS_{h,f} \qquad \forall f \tag{12}$$

are equal to the payments from the sale of factor services to the factor accounts, and hence that

$$\begin{aligned} WF_f * \sum_a FD_{f,a} &= WF_f * \sum_h FS_{h,f} \\ \sum_a FD_{f,a} &= \sum_h FS_{h,f} \qquad \forall f \end{aligned} \tag{13}$$

which simply confirms that the total demand for factors by activities, $\sum_a FD_{f,a}$, is equal to the total supply of factors by households, $\sum_h FS_{h,f}$, under the maintained assumption that the factors are homogenous. It is therefore reasonable to expect that payments to households by the factor accounts in respect of the supply of factor services by households are proportionate to the ownership of factor services by households.

The incomes to each household from the sale of factor services are given by

$$YH_h = \sum_f WF_f * FS_{h,f} \qquad \forall h \tag{14}$$

which can also be written as

$$YH_h = \sum_f hvash_{h,f} * YF_f \qquad \forall h \tag{15}$$

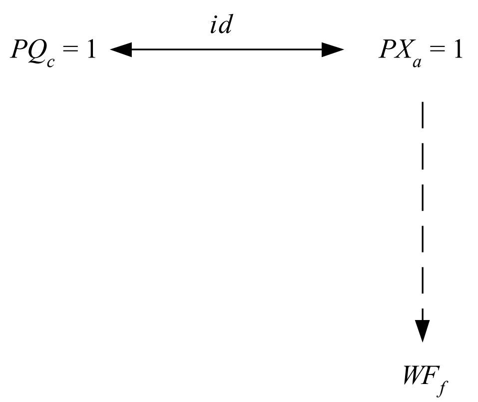
where

$$hvash_{h,f} = \frac{FS_{h,f}}{\sum_h FS_{h,f}} = \frac{WF_f * FS_{h,f}}{\sum_h WF_f * FS_{h,f}} \qquad \forall h, f \tag{16}$$

such that, under the maintained assumption of factor homogeneity, the distribution of factor incomes to households is in fixed proportions of factor incomes and that if there is an unchanged distribution of factor ownership these proportions are constant.

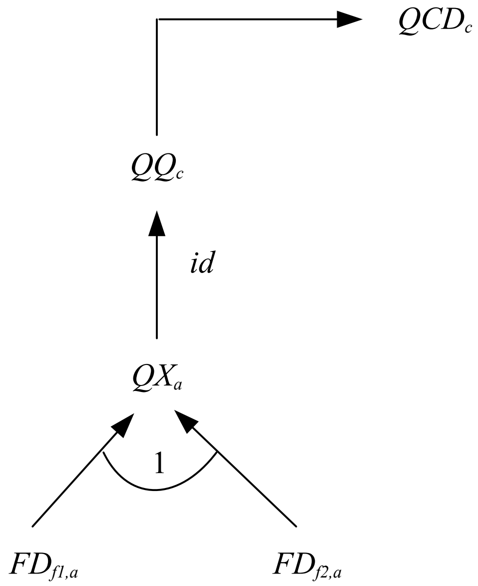
It is useful to consider flow diagram representations of these relationships; this is done in Figure 1. As the price and quantity systems become more complex, flow diagrams are a simple, and common, means of illustrating the relationships between different prices within the economy, so starting with a simple case will subsequently be useful. The identity (*id*) between activity (*PX*) and commodity (*PQD*) prices reflects the presumption, which is embedded in the database, that each activity only produces a single commodity. The dashed line between activity and factor (*WF*) prices indicates that the level of activity prices enters the determination of factor prices. Similarly, the flows of quantities within the economy can be represented in a simple flow diagram, Figure 2. In this case, each activity a employs labour and capital to produce the activity outputs (*QX*), which determine the supplies of commodities (*QQ*) to the economy that must then be allocated to final demand by households (*QCD*). The digit 1 in the arc below QX indicates the elasticity of substitution, which, means, in this case, the production function is Cobb Douglas.

Figure 1 **Price System for a Basic 2*2*2*2 Closed Economy Model**



For this economy, the relationships are simple and immediately understandable, and while the relationships will become more complex, they will remain simple. Again, the key to avoiding confusion is to be systematic. It is now relatively straightforward to progress to an algebraic formulation of this CGE model.

Figure 2 Quantity System for a Basic 2*2*2*2 Closed Economy Model



3. Algebraic Statement of Model 1: A Basic 2*2*2*2 Economy Model

Rather than writing out each, and every, equation in detail it is useful to start by defining a series of sets. Then if a behavioural relationship applies to all members of a set the equation only needs to be specified once. This is also a feature of the GAMS language, i.e., it is a set-based language. The pairs of commodities, activities, factors and households give the natural choice of sets for this model. We can therefore define the following sets

$$c = \{\text{primary, secondary}\}$$
$$a = \{\text{agriculture, industry}\}$$
$$f = \{\text{labour, capital}\}$$
$$h = \{\text{urban, rural}\}$$

It will also prove useful to define a global set, *sac*, as

$$sac = \{c, a, f, h, total\}.$$

<u>Equations for Model 1</u>

When setting out the equations for a model it is convenient to work in ‘blocks’. The order in which you proceed is largely a matter of personal preference. For this, and subsequent, models the blocks used will be for prices, ‘production’, ‘income’, ‘expenditure’, ‘market clearing’, ‘GDP’ and ‘model closure’.

It is also advisable to adopt a series of conventions for the naming of variables and parameters. The conventions adopted are a matter of personal preferences. In all cgemod models the following conventions are adopted:

- all VARIABLES are in upper case;
- the prefixes are *P* for prices and *Q* for quantities, *W* for factor prices and *F* for factor quantities;
- all parameters are in lower case, except those used to initialise variables;
- elasticities and associated parameters are given Greek names;
- parameter names have a two-character suffix/prefix which distinguishes their definition, e.g., ***sh* is a share parameter, *io*** is a coefficient parameter and ***av* is an average;

- all parameter and variable names have less than 10 characters[4].

Price Block Equations

The price block consists of two equations. The first equation specifies the mapping of activity prices (PX_a) to commodity prices (PQD_c) via a set of fixed shares ($ioqqqx_{a,c}$). The second equation is a consumer price index (CPI), which is defined as a weighted sum of the commodity prices where the weights are the shares of each commodity in total demand ($comtotsh_c$). The CPI will be used for price normalisation (see below).

$$PX_a = \sum_c ioqqqx_{a,c} * PQD_c \qquad \text{(P1)}$$

$$CPI = \sum_c comtotsh_c * PQD_c \qquad \text{(P2)}$$

Production Block Equations

The production block consists of the production function, a mapping of activity outputs (QX_a) onto commodity supplies and a first order condition for profit maximisation. The production function is a Cobb-Douglas aggregation function over the two factors that are demanded by each activity ($FD_{f,a}$). Each production function has been calibrated for an efficiency parameter (ad_a) and two elasticities of output ($\alpha_{f,a}$). Since each activity produces only a single output the mapping between activity outputs and commodity supplies is via an identity matrix whose elements are denoted $ioqqqx_{a,c}$. The first order condition for profit maximisation exploits the properties of Euler's theorem and the relationship between the elasticity of output and factor shares in long-run competitive equilibrium, i.e., the elasticity of output for input f in activity a is the share of output from activity a received by factor f.

$$QX_a = ad_a \prod_f \left(FD_{f,a}\right)^{\alpha_{f,a}} \qquad \text{(X1)}$$

$$QQ_c = \sum_a ioqqqx_{a,c} * QX_a \qquad \text{(X2)}$$

$$FD_{f,a} = \frac{QX_a * PX_a * \alpha_{f,a}}{WF_f} \qquad \text{(X3)}$$

4 Originally this was a GAMS constraint, but it has been kept to avoid excessively long names that can clutter up code.

Income Block Equations

The equations in the income block are those specified from the perspective of income received. It is a matter of personal preference whether they are defined from the income or expenditure perspective, but it is important to specify each relationship only once.

This block consists of two equations. The total income received by each factor account (YF_f) is defined as the summation of the earnings of that factor across all activities, i.e.,

$$YF_f = \sum_a WF_f * FD_{f,a} \ . \tag{Y1}$$

The total incomes for each household type (YH_h) are defined as a fixed mapping of factor incomes to households summed across all factors owned by that household type, i.e.,

$$YH_h = \sum_f hvash_{h,f} * YF_f \ . \tag{Y2}$$

This relationship contains an implicit assumption about the ownership of factors that is explored below.

Expenditure Block Equations

The equations in the expenditure block are those specified from the perspective of expenditures by accounts.

This block consists of a single equation. Households are assumed to maximise utility subject to a Cobb-Douglas utility function: hence they allocate fixed proportions of consumption expenditure, in this case household income, to each commodity.

$$QCD_{c,h} = \frac{comhav_{c,h} YH_h}{PQD_c} \tag{E1}$$

Market Clearing Block Equations

Market clearing requires the simultaneous clearing of all markets. In this model there are two relevant markets, the factor and commodity markets. The presumption of a full employment equilibrium requires that factor demands ($FD_{f,a}$) and factor supplies (FS_f) equate, while the commodity equilibrium requires that commodity supply (QQ_c) and commodity demand ($QCD_{c,h}$), summed over households, equate.

$$FS_f = \sum_a FD_{f,a} \tag{M1}$$

$$QQ_c = \sum_h QCD_{c,h} + WALRAS \tag{M2}$$

The addition of the variable WALRAS to the market clearing condition will be explained below.

GDP Block Equations

Equations relating to macroeconomic aggregates may be useful additions, but they not are essential since they simple summary measures that can easily be calculated as part the process of analyzing the results. GDP here is defined from the consumption side as

$$GDP = \left(\sum_{c,h} PQD_c * QCD_{c,h} \right) \tag{G1}$$

Model Closure Equations

The model, as laid out above, has 27 variables (26 + 1), but only 24 equations. A possible solution to this is to reduce the number of variables. This is achieved by the model closure equations whereby 3 of the variables are fixed: in this case the supplies of labour and capital (FS_f) are assumed to be exogenously determined, i.e.,

$$FS_f = \overline{FS}_f . \tag{C1}$$

In addition, the consumer price index is fixed and thereby serves as the price normalisation equation, i.e.,

$$CPI = \overline{CPI} . \tag{C2}$$

The *numéraire* is needed because the model is homogenous of degree zero in prices and hence only defines relative prices.

Equation and Variable Counting

It is important to keep track of the number of equations and variables. A good working rule of thumb is to ensure that the number equations and the number of variables are consistent. Table 4 details the number of equations and variables for this model.

Two features of the model emerge from this counting process. First, the number of variables exceeds the number of equations, and second, the variable *WALRAS* has no obvious connection with any specific equation. The model closure equations identified above resolve the difficulty of different numbers of equations and variables but leave unaddressed the role of the *WALRAS* variable. As specified the model satisfies Walras's Law, whereby if all markets bar one are in equilibrium so will be the final market, i.e., one of the equations is dependent upon the others. Hence, Walras's Law indicates that one of the equations should be dropped.

The alternative adopted here is to add a slack variable, *WALRAS*, rather than drop an equation. The reason for adopting this approach is that if the model is consistent with Walras's Law, the variable *WALRAS* will have a value of zero. If *WALRAS* is not equal to zero, then the model does not satisfy Walras's Law. Hence the value of the variable *WALRAS* provides a quick, if fallible, check on model consistency.

Table 4 Equation and Variable Counts: Basic Closed Economy Model

GAMS Equation Name	Equation Formula	Number of Equations	Variable	Number of Variables
			PQD_c	2
PXDEF	$PX_a = \sum_c ioqqqx_{a,c} * PQD_c$	2	PX_a	2
CPIDEF	$CPI = \sum_c comtotsh_c * PQD_c$	1	CPI	1
PRODFN	$QX_a = ad_a \prod_f FD_{f,a}^{\alpha_{f,a}}$	2	QX_a	2
COMOUT	$QQ_c = \sum_a ioqqqx_{a,c} * QX_a$	2	WF_f	2
PROFITMAX	$FD_{f,a} = \dfrac{QX_a * PX_a * \alpha_{f,a}}{WF_f}$	4	$FD_{a,f}$	4

GAMS Equation Name	**Equation Formula**	**Number of Equations**	**Variable**	**Number of Variables**
YFEQ	$YF_f = \sum_a WF_f * FD_{f,a}$	2	YF_f	2
YHEQ	$YH_h = \sum_f hvash_{h,f} * YF_f$	2	YH_h	2
QCDEQ	$QCD_{c,h} = \frac{comhav_{c,h} * YH_h}{PQD_c}$	4	$QCD_{c,h}$	4
FMEQUIL	$FS_f = \sum_a FD_{f,a}$	2	FS_f	2
QEQUIL	$QQ_c = \sum_h QCD_{c,h} + WALRAS$	2	QQ_c	2
GDPEQ	$GDP = \left(\sum_{c,h} PQD_c * QCD_{c,h} \right)$	1	GDP	1
			$WALRAS$	1
		24		26 + 1

Model Calibration

The fundamental idea behind model calibration is the identification of a set of parameter values that when combined with the pre-defined behavioural relationships ensure that in the solution the variables have the same values as those in the base data set. One way to start is with the prices and quantities of the commodities.

A SAM database only records transaction values; hence unless there are additional data from another (linked) source the first problem is to identify both prices and quantities from transaction values. The trick here is to set one of the price vectors for commodities equal to one and then derive all the other price vectors from the accounting conditions; note that the model will be homogenous of degree zero in prices so the focus of attention is on relative prices and the changes in relative prices so the absolute magnitude of prices does not matter. In this case setting the vector of activity prices (*PX*) equal to one provides a good starting point; although *PX* is an activity price in this, special, case it is also the supply/producer price of a commodity because the supply matrix is diagonal and hence there is a one to one relationship between the activity prices and the producer prices of commodities. Given values for *PX* then a series of initial values for the quantity variables, *QX*, are defined as

```
QX0(a) = SAM("total",a)/PX0(a)
```

and noting $PX0_a = PQD0_c$ for all $a = c$ from the accounting identities, then the quantities of commodities supplied are

```
QQ0(c) = SAM("total",c)/PQD0(c)
```

and that the initial values for the quantities of commodities consumed by households are

```
QCD0(c,h) = SAM(c,h)/PQD0(c).
```

Note the presumption is that the price received by an activity for its commodity is identical to the price paid by the consumer.[5]

Similarly adopting the assumption that factor prices (*WF*) are equal to one means that the initial factor demands (*FD*) are

[5] It would be perfectly legitimate in this model to distinguish between the supply/producer (*PQS*) and demand/purchaser (*PQD*) prices for commodities; this would mean that there was an identity between PX and *PQS*, because of the diagonal supply matrix, and that *PQS* and *PQD* were identical because there are no taxes of other 'price wedges' between the producer and purchaser prices. Although it is not used here this distinction will be valuable when a government sector is introduced to the model.

```
FD0(f,a) = SAM(f,a)/WF0(f)
```

and given full employment that total factor supplies are

```
FS0(f) = SAM("total ",f)/WF0(f)
```

The following definitions can also be derived from the SAM

```
YF0(f) = SAM("total ",f)

YH0(h) = SAM("total ",h)

GDP0 = SUM((c,h),SAM(c,h)/PQD0(c))
```

It is now relatively straightforward to derive the parameters for the production and utility functions. The elasticities of output (share parameters) for the production function are simply the value share of the factors in total activity output, i.e.,

```
alpha(f,a) = SAM(f,a)/SUM(fp,SAM(fp,a))
```

and given the values for the coefficients it is possible to solve the production function equations for the technology parameters, such that

```
ad(a) = QX0(a)/PROD(f, FD0(f,a)**alpha(f,a)).
```

Notice how this method (in effect) works backwards using a piece of exogenous information – in this case the elasticities of output.[6] Typically, and in general, given the elasticity of substitution and the prices and quantities in the initial equilibrium the share parameters can be determined (*alpha*) and then the shift parameter (*ad*) can be determined.

Given the outputs by activities it is necessary to define the supply of commodities to the economy. Given that the supply matrix is diagonal this requires that there is a one to one mapping between activity outputs and commodity supplies, which can be represented as an identity matrix. One way the calculate such a matrix is to divide the elements in each row of the supply matrix by the row sums, i.e.,

```
ioqqqx(a,c) = SAM(a,c)/SAM("total",c).
```

The easy way with the household demand functions is to recognise that given Cobb-Douglas utility functions the expenditures shares are constant – note that the elasticity of substitution is equal to one – and hence the expenditure shares can be defined as

6 Note that the elasticity of substitution is 1. With other functional forms, it is necessary to use explicit exogenous information about the elasticity of substitution; in this case the fact that the elasticity of substitution is equal to one underlies the method.

```
comhav(c,h) = SAM(c,h)/(SUM(cp,SAM(cp,h))).
```

The only parameters that remain to be defined are the matrix of household shares of factor incomes and the weights for the price index. Under the assumptions of full employment and homogenous factors the pattern of ownership of factors is the same as the pattern of incomes from factor sales by households, hence

```
hvash(h,f) = SAM(h,f)/SAM("total",f).
```

And finally defining the weights on the consumer price index as the initial value shares in consumption by households means that

```
comtotsh(c) = SUM(h,SAM(c,h))/(SUM((cp,hp),SAM(cp,hp))).
```