A Basic Closed Economy CGE Model: Part 2

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Outline

- Introduction
- *Economic Theory*
- *Mathematical Model*
- *Data & Behaviour*
  - *Social Accounting Matrix*
  - *Behavioural Relationships*
- The Model in GAMS
  - Formal/Algebraic Statement
- Equation and Variable Counting

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The Model in GAMS

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From Abstract to Concrete

Work from the accounting identities and the behavioural relationships to EQUATIONS

- Production
- Demand
  - Factors
  - Commodities
- Income distribution
- Prices
  - Commodities
  - Factors
- Endowments
- *Numéraire*
- Optimand

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Formal/Algebraic Statement

- VARIABLES are in upper case;
- Prefixes: *P* – prices; *Q* – quantities; *W* - factor prices; *F* - factor quantities;
- parameters are in lower case, except those used to intialise variables;
- elasticities and associated parameters are given Greek names;
- parameter names have a two-character suffix/prefix which distinguishes their definition, e.g., ***sh*, *io*** and ***av*

$c = \{\text{primary, secondary}\}$

$a = \{\text{agriculture, industry}\}$

$f = \{\text{labour, capital}\}$

$h = \{\text{urban, rural}\}$

$sac = \{c, a, f, h, total\}$

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Price Block Equations

$$\sum_h PQD_c * QCD_{c,h} = PX_a * QX_a \qquad \forall c = a$$

Acc^g Identity

$$PQD_c * QQ_c = PX_a * QX_a$$

where $QQ_c = \sum_h QCD_{c,h}$

$$QQ_c \equiv QX_a \qquad \forall c = a$$

Supply = Production

$$PQD_c = PX_a \qquad \forall c = a$$

In GAMS ➡

$$PX_a = \sum_c ioqqqx_{a,c} * PQD_c$$

PX(a) =E= SUM(c,ioqqqx(a,c)*PQD(c));

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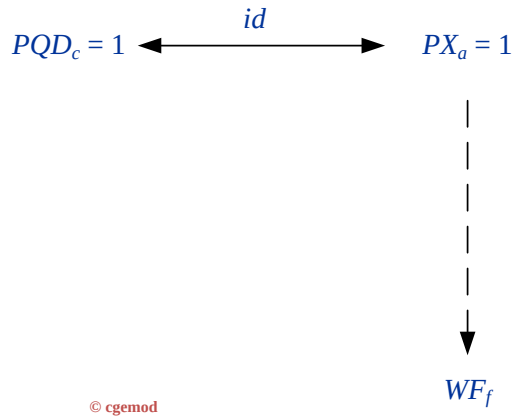
Price Tree

- Linear homogenous functions
  - Relative prices
  - If prices known, they can be used
  - Otherwise use $P = 1$



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Utility Functions & Equations

$$\sum_c PQD_c * QCD_{c,h} = YH_h$$ **Acc^g Identity**

$$U_h = \phi_h^u \cdot \prod_c QCD_{c,h}^{\gamma_{c,h}}$$ **Utility function**

$$\frac{QCD_{2,h}}{QCD_{1,h}} = \frac{PQD_1}{PQD_2} \cdot \left(\frac{\gamma_{2,h}}{\gamma_{1,h}}\right) = \frac{PQD_1}{PQD_2} \cdot \left(\frac{(1-\gamma_{1,h})}{\gamma_{1,h}}\right)$$ **I^st Order Condition**

$$PQD_1 * QCD_{1,h} = \gamma_{1,h} * YH_h$$ **Eulers theorem**

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Utility Functions & Equations

Expenditure of ALL income to each household MUST be accounted for

$$\sum_c PQD_c * QCD_{c,h} = \sum_c \gamma_{c,h} * YH_h = YH_h$$ **Complete demand system**

All income (YH) is spent on consumption

A (implicit) budget constraint

In GAMS ➡

$$QCD_{c,h} = \frac{comhav(c,h) * YH_h}{PQD_c}$$

PQD(c)*QCD(c,h) =E= comhav(c,h)*YH(h)) ;

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Production Block Equations

$$PQD_c * QQ_c = PX_a * QX_a \qquad \forall c = a$$ **Acc^g Identity**

$$QX_a = \alpha_a^x . FD_{l,a}^{\beta_{l,a}} . FD_{k,a}^{\beta_{k,a}} = \alpha_a^x . \prod_f FD_{f,a}^{\beta_{f,a}}$$ **Production function**

In GAMS ➡

$$QX_a = ad_a \prod_f \left(FD_{f,a} \right)^{\alpha_{f,a}}$$

QX(a) =E= ad(a)*PROD(f,FD(f,a)**alpha(f,a)) ;

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Factor Demand Equations

$$PX_a * QX_a = \sum_f WF_f * FD_{f,a} \qquad \forall a$$

Acc^g Identity

$$\Pi_a = \left(PX_a * QX_a\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right]$$
$$= \left(PX_a * \left(\alpha_a^x . FD_{l,a}^{\beta_{l,a}} . FD_{k,a}^{\beta_{k,a}}\right)\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right]$$

Profit function

$$WF_l = \left(PX_a * \left(\alpha_a^x . \beta_{l,a} . FD_{l,a}^{(\beta_{l,a}-1)} . FD_{k,a}^{\beta_{k,a}}\right)\right)$$
$$= PX_a * \beta_{l,a} * \frac{QX_a}{FD_{l,a}}$$
$$\Rightarrow WF_l * FD_{l,a} = \beta_{l,a} . \left(PX_a * QX_a\right)$$

Profit Max.

In GAMS ➡

$$FD_{f,a} = \frac{QX_a * PX_a * \alpha_{f,a}}{WF_f}$$

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WF(f)*FD(f,a) =E= QX(a)*PX(a)*alpha(f,a) ;
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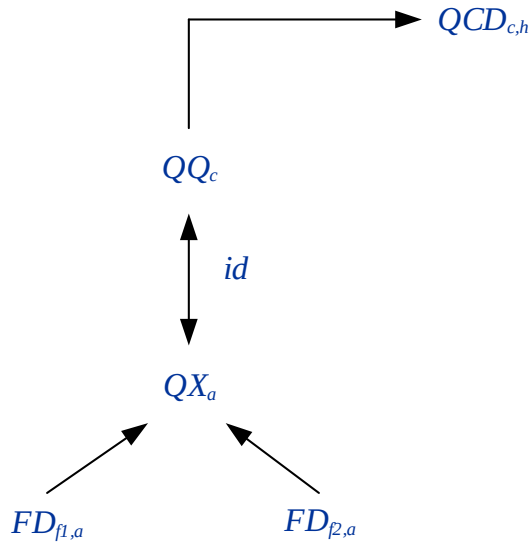
Quantity Tree



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Factor Incomes

$$YF_f = \sum_a WF_f * FD_{f,a}$$ **Acc^g Identity**

In GAMS ➡ $$YF_f = \sum_a WF_f * FD_{f,a}$$

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YF(f) =E= SUM(a,WF(f)*FD(f,a)) ;
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Functional Distribution of Income

$$\sum_a WF_f * FD_{f,a} = YF_f = \sum_h WF_f * FS_{h,f} \quad \forall f$$ **Acc^g Identity**

$$WF_f * \sum_a FD_{f,a} = WF_f * \sum_h FS_{h,f}$$

$$\sum_a FD_{f,a} = \sum_h FS_{h,f} \quad \forall f$$ **Factor Demand = Supply**

$$YH_h = \sum_f WF_f * FS_{h,f} \quad \forall h$$ **Household Income**

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Household Incomes

$$YH_h = \sum_f WF_f * FS_{h,f} \qquad \forall h$$ **Household Income**

In GAMS ➡ $$YH_h = \sum_f hvash_{h,f} * YF_f$$

YH(h) =E= SUM(f,hvash(h,f)*YF(f)) ;

$$YH_h = \sum_f hvash_{h,f} * YF_f \qquad \forall h$$

where **Fixed Endowments**

$$hvash_{h,f} = \frac{FS_{h,f}}{\sum_h FS_{h,f}} = \frac{WF_f * FS_{h,f}}{\sum_h WF_f * FS_{h,f}} \qquad \forall h, f$$

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Market Clearing Equations

$$FS_f = \sum_a FD_{f,a}$$ **Factor Demand = Supply**

In GAMS ➡ FS(f) =E= SUM(a,FD(f,a)) ;

$$QQ_c = \sum_h QCD_{c,h} + WALRAS$$ **Commodity Supply = Demand**

In GAMS ➡ QQ(c) =E= SUM(h, QCD(c,h)) + WALRAS ;

$$QQ_c \equiv QX_a \qquad \forall c = a$$ **Supply = Production**

In GAMS ➡ QQ(c) =E= SUM(a,ioqqqx(a,c)*QX(a)) ;

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Other Equations

$$CPI = \sum_{c} comtotsh_c * PQD_c$$

Numéraire

In GAMS ➡ CPI =E= SUM(c,comtotsh(c)*PQD(c)) ;

$$GDP = \sum_{c,h} PQD_c * QCD_{c,h}$$

Optimand

In GAMS ➡ GDP =E= SUM((c,h), QCD(c,h)*PQD(c)) ;

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Model Closure Equations

$$FS_f = \overline{FS}_f$$

Full employment

In GAMS ➡ FS.FX(f) = FS0(f) ;

$$CPI = \overline{CPI}$$

Numéraire

In GAMS ➡ CPI.FX = CPI0 ;

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What is Walras?

Other than a song by the Beatles!!

- Number of variables < Number of equations
  - $n < (n + 1)$
- Walras's Law
  - if all markets except one are in equilibrium so will be the final market
- Therefore
  - drop an equation?
  - OR add a slack variable
    - if the model is consistent with Walras's Law, the variable *WALRAS* will have a value of zero

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Equation & Variable Counting 1

- CGE models are simultaneous equation systems
  - Mathematically numbers of equations & variables must equate
- Counting equations and variables is critical
  - Economically there will always be more variables than equations
  - Flexibility in policy analyses means there will be even more variables than equations
- Matching equations and variables is important
  - Provides understanding of the economic logic
  - But is not always straightforward, e.g., redundancies
- Endogenous vv Exogenous variables (!!)
  - Some variables are designed to be either endogenous (variable) or exogenous (fixed), i.e., they can be changed
  - Different market clearing conditions require changes
  - Different macroeconomic closure conditions require changes
  - Making such changes requires very careful counting

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Equation & Variable Counting 2

GAMS Equation Name	Equation Formula	No. Equations	Variable	No. Variables
			PQD_c	2
PXDEF	$PX_a = \sum_c ioqqqx_{a,c} * PQD_c$	2	PX_a	2
PROFITMAX	$FD_{f,a} = \frac{QX_a * PX_a * \alpha_{f,a}}{WF_f}$	4	$FD_{f,a}$	4
			WALRAS	1
		24		**26 + 1**

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The End

A Basic Closed Economy CGE Model Pt 2

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