

Consumer Choice and Demand

The most fundamental concept in economics is probably that of *opportunity cost*. That is, the opportunity cost of a choice is the benefits foregone because of not choosing the next best alternative. The concept of opportunity cost underpins the economics of consumer choice and identifies the 3 key components of consumer choice:

- i) objects of choice
- ii) tastes and preferences
- iii) constraints upon choice

The theory of consumer choice is essentially a series logically consistent models based upon assumptions regarding these key components. Hence, the theory is a series of axiom systems.

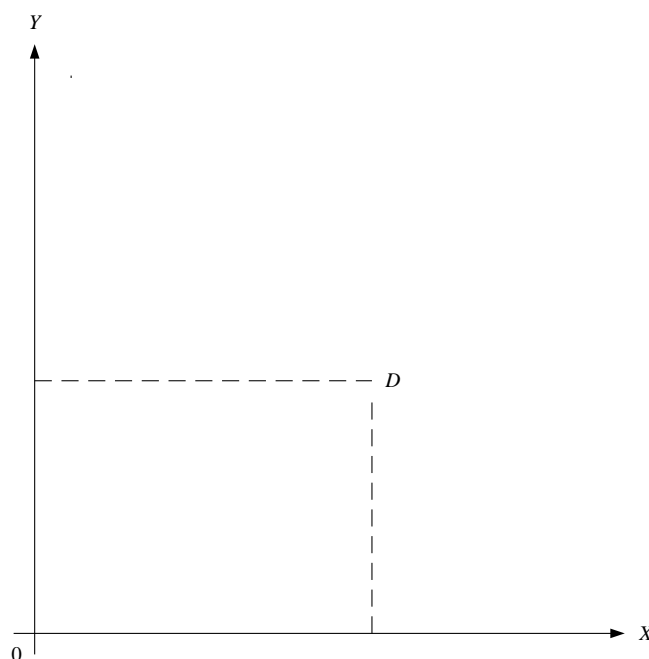
It is important however to recognise the definition of a consumer generally used by economists in the context of consumer choice. The theory of consumer choice is concerned with how ‘individuals’ make choices, where an ‘individual’ is defined as a consumer unit, namely a household. Consumer choice theory, at this level, does not predominantly concern itself with how households are organised or intra-household relations.

1. The Objects of Choice

The objects of choice are *available* goods and services both within one time-period and over time. Without loss of generality let us assume that the consumer faces a choice of two goods, *X* and *Y*, and for the moment that consumption is only allowed in one time-period. We can then represent the choice set by a two-dimensional diagram, i.e., where the axes represent quantities.

Any point in the positive quadrant, e.g., *D*, is within the choice set. But, this choice set is unbounded, i.e., there are no constraints on consumer choice.

Figure 1.1 Choice Set



2. Consumer Tastes & Preferences - the indifference-curve theory

The economic models we will consider start from a set of initial conditions and then analyse how economic systems will react to given stimuli. The initial conditions are typically expressed as series of assumptions that define the domain of the model.

It is assumed that the consumers are seeking to maximise their welfare/utility; hence consumer theory is sometimes called utility theory. As such consumers' value goods and services in terms of how much utility they gain from consuming those goods and services. The concepts are widely adhered to but they are not without controversy.

2.1 Assumptions

The standard assumptions of orthodox, or neoclassical, consumer theory are

A1: Rationality

- the consumer seeks to maximise utility given the choices available, her tastes and preferences and the constraints under which she must operate, e.g., income and prices.

A2: Ordinal Utility

- the consumer can rank the utility provide by different consumption bundles, but not precisely quantify the level of satisfaction.

A3: Completeness

- the consumer can rank all consumption bundles.

A4: Transitivity

If $A > B$ and $B > C$ then $A > C$

this ensures consistent preferences, i.e., if $A > B$, then $B \not> A$.

A5: More is preferred to less

- the utility function is

$$U = u(q_1, q_2, \dots, q_n)$$

$$\frac{\partial U}{\partial q_i} \geq 0 \quad i = 1, \dots, n. \quad (1)$$

often this assumption is extended to nonsatiation, i.e., the marginal utility is always positive.

A6: Diminishing Marginal Rate of Substitution

- this ensures that indifference curves are convex to the origin. This is a critical axiom.

A7: Full Information

- Full knowledge of all relevant information.

These assumptions are adequate to ensure three key properties of indifference curves:

- they are convex to the origin
- they do not intersect
- the further an indifference curve is from the origin, along any ray line, the greater the level of satisfaction.

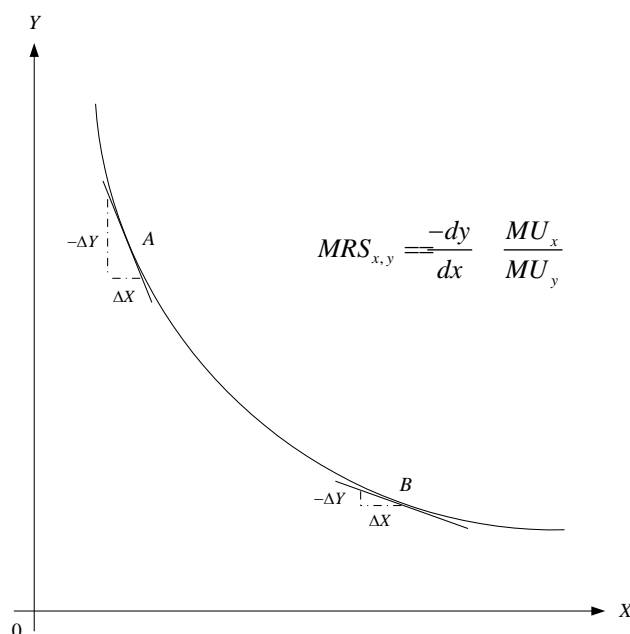
You should be able to demonstrate why all these properties hold; the ability to do so will be considered legitimate examination questions.

2.2 Marginal Rate of Substitution (in consumption)

The Marginal Rate of Substitution (*MRS*) is concerned with the rate at which a consumer is prepared to substitute one good for another and still maintain the same level of utility. To

remain on an indifference curve an increase the consumption of X it is necessary if the consumption of Y is reduced.

Figure 2.1 Marginal Rate of Substitution



What is the relationship between the MRS and the MU s for X and Y ?

The utility function for a two good indifference curve with fixed utility is

$$U = f(X, Y) = k \quad (2)$$

where k is a constant. Totally differentiating the utility function gives

$$dU = \frac{\partial U}{\partial Y} dY + \frac{\partial U}{\partial X} dX = (MU_y) dY + (MU_x) dX \quad (3)$$

which along any given indifference curve is equal to **zero**, i.e.,

$$dU = (MU_y) dY + (MU_x) dX = 0. \quad (4)$$

Rearranging this equation gives

$$-\frac{dY}{dX} = \frac{MU_x}{MU_y} = MRS_{x,y} \quad \text{or} \quad -\frac{dX}{dY} = \frac{MU_y}{MU_x} = MRS_{y,x} \quad (5)$$

3. The Budget Constraint

The budget constraint encompasses two facts:

- i) the consumers income in the period, i.e., I
- ii) the prices of goods X and Y , i.e., p_x and p_y .

Assuming, not unreasonably, that the consumer is a price taker, then the prices, p_x and p_y , are fixed and constant. Thus, the budget constraint can be written as:

$$I \geq p_x X + p_y Y \quad (6)$$

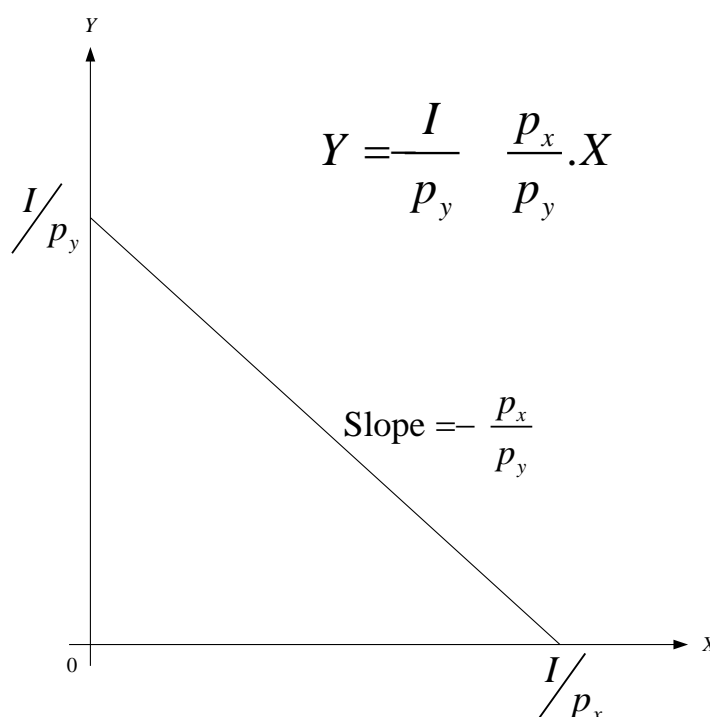
Further if all income is spent then we can rearrange this expression to give

$$Y = \frac{I}{p_y} - \frac{p_x}{p_y} X \quad (7)$$

which is a formula for a straight line, where I/p_y is the intercept and $-p_x/p_y$ the slope.

The budget constraint can be represented on the choice set diagram such that it constrains consumer choice by defining all those combinations of goods that the consumer can choose from given the levels of income and the prices, i.e.,

Figure 3.1 Budget Constraint



If all income is spent then the consumption bundle, in terms of the quantities of X and Y , lies on the budget constraint. Otherwise the consumption bundle lies to the left of the budget constraint, while consumption bundles to the right are not feasible.

A common comment by students at this point is: What about savings? Two points can be made about savings:

- i) the comment is logically inconsistent because we assumed consumption decisions could only be made one period at a time, i.e., saving, or deferred consumption, was excluded
- ii) by modifying the model's assumptions, inter-temporal consumption decisions can be conceptualised. (We will do this in the future.)

These points are illustrations of an important feature of the methodology used by economists. By ruling out savings the *domain* of the model was defined, i.e., the model sought only to examine current consumption decisions and therefore to criticise it for not doing something else would be inappropriate.

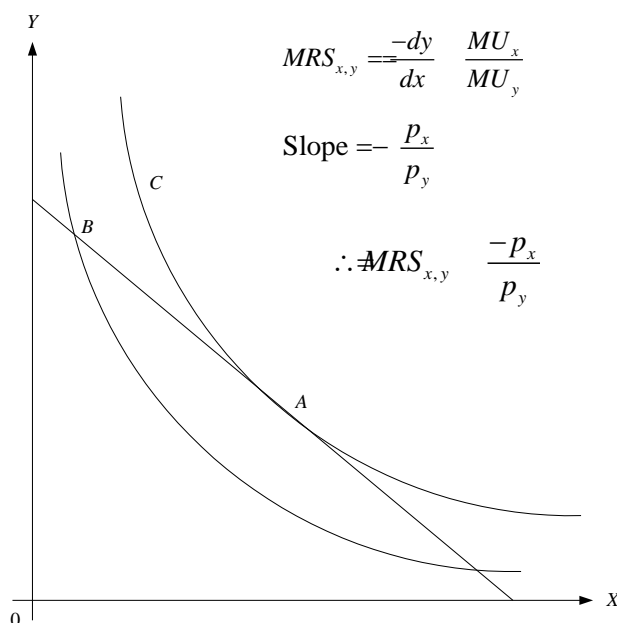
Note also that a decision to consume other than on the budget constraint would not be rational. It would involve foregoing consumption for no possible benefit, given our initial assumption.

Rational individuals will choose to consume somewhere on the budget constraint. Why? But where?

4. Solution to the Choice Problem

By combining the indifference curve representation and the budget constraint we can identify the utility maximising consumption bundle, i.e.,

Figure 4.1 Solution to the Choice Problem



By A4, $C > B$ and, by definition, $C = A$, therefore by A4, $A > B$ (i.e., $A = C$ and $C > B$ therefore $A > B$). Thus, A is the point at which utility is maximised subject to the constraints imposed by income, prices and preferences.

It is useful to examine the story from the perspective of utility. Along any indifference curve

$$dU = (MU_y).dY + (MU_x).dX = 0 \quad (8)$$

and therefore

$$-\frac{dY}{dX} = \frac{MU_x}{MU_y} = MRS_{y,x} \quad (9)$$

at the equilibrium choice set, A, i.e., where utility is maximized. Furthermore, the slopes of the indifference curve and the budget constraint equate at the equilibrium and therefore

$$-\frac{dY}{dX} = \frac{MU_x}{MU_y} = MRS_{y,x} = \frac{p_x}{p_y}. \quad (10)$$

Rearranging this expression gives

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{MU_x}{p_x} = \frac{MU_y}{p_y} \quad (11)$$

such that at equilibrium the ratios of the marginal utility of goods to their own price are constant for all goods.

Two points arise from this.

- i) The units in which utility is measured are irrelevant since only the ratios are important, provided the units used for each good are consistent.
- ii) The ratio of a MU to its own price represents the MU per unit of expenditure. If these do not equate for all goods, then a consumer can increase their welfare by reallocating expenditure from the good with the lower ratio to the good with the higher ratio.

4.1 A Formal Statement of the Choice Problem

Why bother with formalistic rubbish? Several reasons exist for concerning ourselves with a formal mathematical solution:

- i) It sharpens the analysis - it makes transparent the relationship between marginal utilities and prices in this model.
- ii) Diagrams are fine while we must use only 2 dimensions, but are the 2 good models generalisable to n -good situations? In fact, they are, and you can verify this in a few minutes by formulating a 4, or more, good constrained optimisation problem.
- iii) The diagrams in microeconomics have their basis in these algebraic formulations. It is therefore important to bear this in mind when drawing diagrams, and to think formally about the relations implicit to specific diagrams.
- iv) And perhaps most important, this type of constrained maximisation problem occurs throughout microeconomic theory. It must be faced sooner or later so let's do it now. In fact, all of you should be capable already of interpreting the meaning of a tangency between an isoquant and a cost constraint that will appear in producer theory.
- v) It rationalises the use of ordinal utility through the interpretation of the Lagrange multiplier.

Formally we can state the consumer's problem as the maximization of utility subject to a budget constraint, tastes & preferences, and the prices of the goods, i.e.,

$$\text{Max } U = u(X, Y) \quad (12)$$

$$\text{sto} \quad I = p_x X + p_y Y \quad (13)$$

By the method of Lagrangian multipliers we can combine (12) and (13) as

$$\mathfrak{J} = u(X, Y) + \lambda(I - p_x X - p_y Y) \quad (14)$$

where λ is the Lagrange multiplier. Differentiating \mathfrak{J} with respect to the variables and setting each equation equal to zero gives us the first-order conditions for a maximum.

$$\frac{\partial \mathfrak{J}}{\partial X} = \frac{\partial U}{\partial X} - \lambda p_x = 0 \Rightarrow \lambda = \frac{MU_x}{p_x} \quad (15)$$

$$\frac{\partial \mathfrak{J}}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda p_y = 0 \Rightarrow \lambda = \frac{MU_y}{p_y} \quad (16)$$

$$\frac{\partial \mathfrak{J}}{\partial \lambda} = I - p_x X - p_y Y = 0 \Rightarrow I = p_x X + p_y Y \quad (17)$$

Equation (17) simply states that the maximum must occur on the budget constraint because more is preferred to less.

Combining (15) and (16) we get

$$\lambda = \frac{MU_x}{p_x} = \frac{MU_y}{p_y} \quad (18)$$

and therefore, that

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} = MRS_{x,y} \quad (19)$$

At the maximum, the slopes of the indifference curve and the budget constraint are equal, that is the maximum, is a tangency point. Hence the MRS is equal to the ratio of the marginal utilities, i.e., the slope of the indifference curve, and the ratio of the prices, i.e., the slope of the budget constraint. This of course is no more than the conclusion we arrived at by the simple diagrammatic approach.

Interpreting λ

The interpretation of λ is important. Differentiate the utility function with respect to income

$$\frac{dU}{dI} = MU_x \frac{dx}{dI} + MU_y \frac{dy}{dI} \quad (20)$$

but $\lambda p_x = MU_x$ and $\lambda p_y = MU_y$, therefore

$$\frac{dU}{dI} = \lambda p_x \frac{dx}{dI} + \lambda p_y \frac{dy}{dI} = \frac{\lambda(p_x \cdot dx + p_y \cdot dy)}{dI}. \quad (21)$$

The total differential of the budget constraint is

$$dI = p_x \cdot dx + p_y \cdot dy \quad (22)$$

and substituting (22) into (21) give

$$\frac{dU}{dI} = \frac{\lambda(p_x \cdot dx + p_y \cdot dy)}{(p_x \cdot dx + p_y \cdot dy)} = \lambda \quad (23)$$

which simply says that the marginal utility of income is equal to the Lagrange multiplier, i.e., how much utility changes as income changes when the changes are marginal.

Note also from the first-order conditions that the ratios of the MUs are equal to the MU of income, i.e.,

$$\frac{p_x}{MU_x} = \lambda = \frac{p_y}{MU_y}. \quad (24)$$

Alternatively, by concentrating on good X

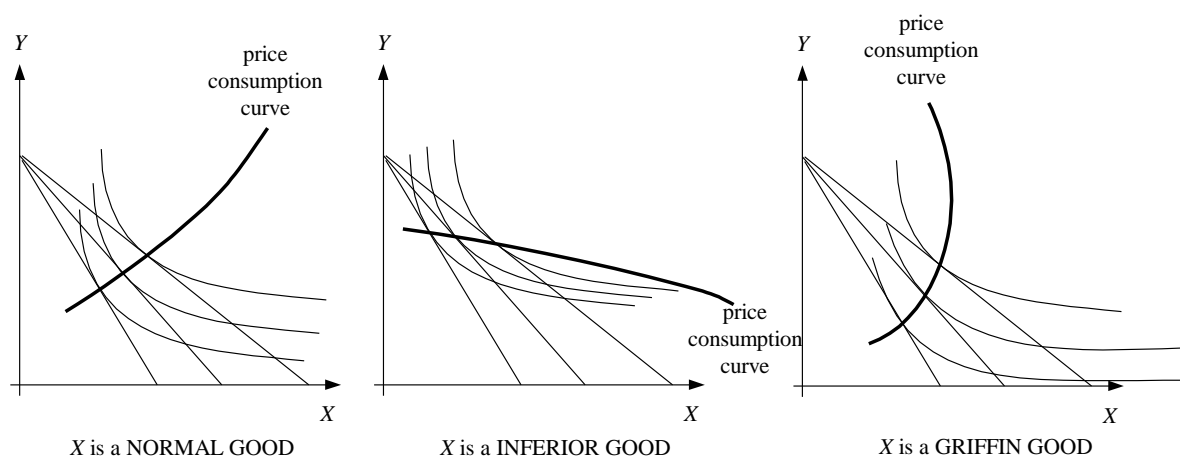
$$\lambda = \frac{\partial U / \partial X}{p_x} = \frac{\partial U}{\partial X \cdot p_x} \quad (25)$$

or, in English, λ identifies the change in utility, ∂U , consequent upon a change in expenditure on X, $(\partial X \cdot p_x)$. It is therefore the marginal utility of money income, which, by definition, is equal for both X and Y at the maximum. Notice also that the magnitude of λ is not relevant to the equilibrium condition and hence that cardinality is not needed in the solution to the choice problem because we do not require a quantitative value of the marginal utility of income.

5. Price Consumption and Demand Curves

For price consumption curves, we hold everything else constant and allow one price to vary, e.g., p_x . Thus, we can draw

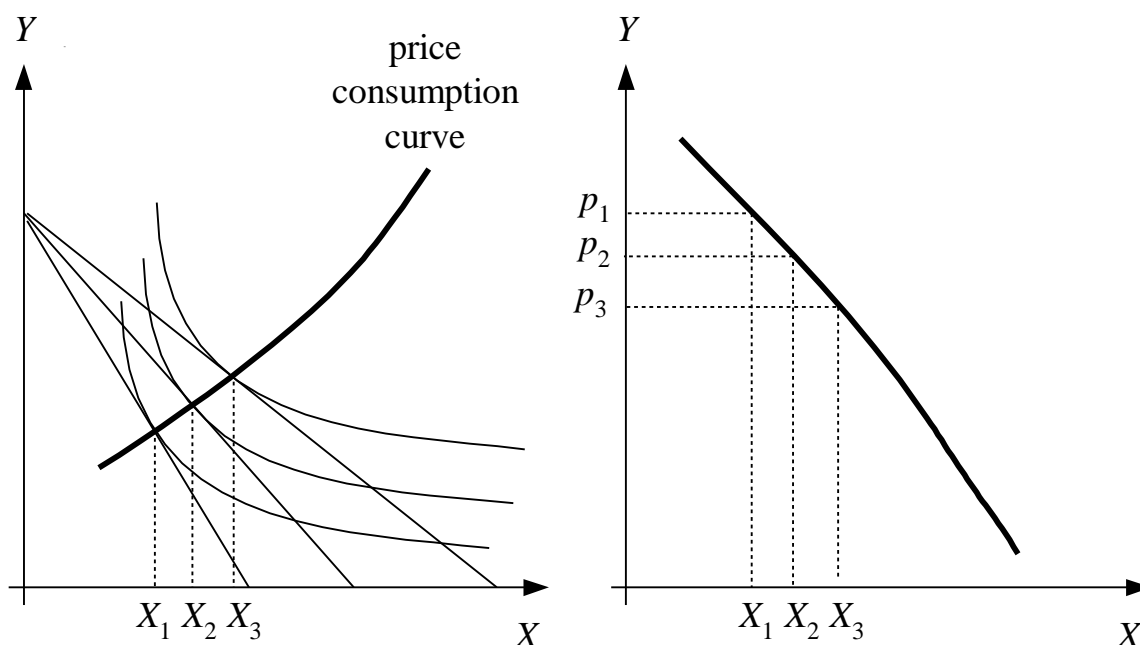
Figure 5.1 Price Consumption Curves



where p_X increases the slope of the budget constraint, p_X/p_Y , increases and hence less and less of X is purchased.

As as before we can derive the price consumption curve, but this time relating demand for X to the price of X , p_X , i.e., the demand curve

Figure 5.2 Price Consumption and Demand Curves



This demand curve is for a normal good. What shape would it have for a Giffin Good and what shape would the price consumption curve have?

But can a demand curve be derived from the formal Lagrangean statement for the optimum choice of a consumer? Rewriting the Lagrangean equation with a specific, Cobb-Douglas, utility function produces

$$\mathfrak{J} = X^{0.75}Y^{0.25} + \lambda(I - p_x X - p_y Y). \quad (26)$$

Differentiating \mathfrak{J} with respect to the variables and setting each equation equal to zero gives us the first-order conditions for a maximum.

$$\frac{\partial \mathfrak{J}}{\partial X} = \frac{3}{4} X^{-0.25} Y^{0.25} - \lambda p_x = 0 \quad (27)$$

$$\frac{\partial \mathfrak{J}}{\partial Y} = \frac{1}{4} X^{0.75} Y^{-0.75} - \lambda p_y = 0 \quad (28)$$

$$\frac{\partial \mathfrak{J}}{\partial \lambda} = I - p_x X - p_y Y = 0 \quad (29)$$

Solving for X and Y gives

$$X = \frac{3}{4} \cdot \frac{I}{p_x} \quad (30)$$

and

$$Y = \frac{1}{4} \cdot \frac{I}{p_y} \quad (31)$$

which are the demand functions for X and Y .

The solutions for X and Y are straightforward if you use (27), (28) and (29) and remember the laws of exponents. From (27) and (28)

$$\lambda p_x = \frac{3}{4} X^{-0.25} Y^{0.25} = MU_x \quad \text{and} \quad \lambda p_y = \frac{1}{4} X^{0.75} Y^{-0.75} = MU_y$$

and in equilibrium

$$\begin{aligned}\frac{MU_x}{P_x} &= \frac{MU_y}{P_y} = \lambda \\ \frac{0.75X^{-0.25}Y^{0.25}}{P_x} &= \frac{0.25X^{0.75}Y^{-0.75}}{P_y} \\ \frac{0.75X^{0.75}Y^{0.25}}{P_x X} &= \frac{0.25X^{0.75}Y^{0.25}}{P_y Y} \\ 0.75P_y Y &= 0.25P_x X\end{aligned}$$

which on substitution into the budget constraint (29) produces

$$\begin{aligned}I &= P_x X + \frac{0.25}{0.75} P_x X \quad \text{and} \quad I = \frac{0.75}{0.25} P_y Y + P_y Y \\ I &= X \left(\frac{4}{3} P_x \right) \quad \text{and} \quad I = 4P_y Y \\ X &= \frac{3}{4} \cdot \frac{I}{P_x} \quad \text{and} \quad Y = \frac{1}{4} \cdot \frac{I}{P_y}\end{aligned}$$

5.1 Price Elasticity of Demand

Define the own price elasticity of demand for good X as

$$\epsilon_{p_x} = \frac{\partial X / \partial p_x}{X / p_x} = \frac{\partial X / X}{\partial p_x / p_x} \quad (32)$$

then the elasticity of demand for good X using the CD function from above is

$$\epsilon_{p_x} = \frac{3}{4} \cdot \frac{I}{P_x^2} \cdot \left(\frac{P_x}{\left(\frac{3}{4} \cdot \frac{I}{P_x} \right)} \right) = -1 \quad (33)$$

5.2 Elasticity and Total Expenditure

The value of the price elasticity is important for many reasons in economic analysis. There is an important relationship between the price elasticity and how total expenditure (*TE*) varies as the price varies. Defining

$$TE_x = P_x \cdot X \quad (34)$$

then

$$\begin{aligned}
 \frac{dTE_x}{dp_x} &= X + p_x \cdot \frac{dX}{dp_x} \\
 &= X + X \cdot \frac{p_x}{X} \cdot \frac{dX}{dp_x} \\
 &= X \left(1 + \frac{p_x}{X} \cdot \frac{dX}{dp_x} \right) \\
 &= X (1 + \varepsilon_x)
 \end{aligned}
 \tag{35}$$

which means that for inelastic demand total expenditure increases/decreases as price rises/falls; for elastic demand, total expenditure increases/decreases as price falls/rises; and for unit elastic demand total expenditure neither increases/decreases as price rises/falls.

5.3 Cross Price Elasticity of Demand

Define the cross-price elasticity of demand between goods X and Y as

$$e_c = \frac{\partial X / \partial p_y}{X / p_y} = \frac{\partial X / X}{\partial p_y / p_y} \tag{36}$$

such that $e_c > 0$ for substitutes and $e_c < 0$ for complements. You should verify this for yourself.

Since both X and p_y are positive it follows that the sign on e_c depends upon the signs on ∂X and ∂p_y . If $e_c > 0$ then $\partial X / \partial p_y > 0$, if $\partial X > 0$ then $\partial p_y > 0$ or if $\partial X < 0$ then $\partial p_y < 0$, i.e., if the price of Y increases then the demand for X increases – the consumer substitutes X for Y , or if the price of Y decreases then the demand for X decreases – the consumer substitutes Y for X . Hence X and Y are substitutes. Furthermore, if $e_c < 0$ then $\partial X / \partial p_y < 0$, if $\partial X > 0$ then $\partial p_y < 0$ or if $\partial X < 0$ then $\partial p_y > 0$, i.e., if the price of Y decreases then the demand for X increases – the consumer demands more of both X and Y , or if the price of Y increases then the demand for X decreases – the consumer demands less of both X and Y . Hence, X and Y are complements.

Own and Cross price elasticities are formally related. From the budget constraint

$$I = p_x X + p_y Y \tag{37}$$

and differentiating

$$dI = X \cdot dp_x + p_x \cdot dX + Y \cdot dp_y + p_y \cdot dY \tag{38}$$

and setting $dI = 0$ (constant money income) and $dp_Y = 0$ (constant price of Good Y), then

$$X.dp_x + p_x.dX - p_Y.dY = 0 \quad (39)$$

Multiplying through by $(p_x/I.dp_x)$ and using $X/X = Y/Y = 1$, gives

$$\begin{aligned} X.dp_x \cdot \frac{p_x}{I.dp_x} + p_x.dX \cdot \frac{p_x}{I.dp_x} \cdot \frac{X}{X} + p_Y.dY \cdot \frac{p_x}{I.dp_x} \cdot \frac{Y}{Y} &= 0 \\ \frac{p_x X}{I} + \left(\frac{dX}{X} \cdot \frac{p_x}{dp_x} \right) \left(\frac{p_x X}{I} \right) + \left(\frac{dY}{Y} \cdot \frac{p_x}{dp_x} \right) \frac{p_Y Y}{I} &= 0 \end{aligned} \quad (40)$$

which simplifies to

$$\alpha_1(1 + e_p) + \alpha_2 e_c = 0 \quad (41)$$

where

$$\alpha_1 = \frac{p_x X}{I}$$

$$\alpha_2 = \frac{p_Y Y}{I}$$

and therefore, define the expenditure shares.

How is this useful? If we know the shares of X and Y in total expenditure, then the own price elasticity of demand can be used to calculate the cross-price elasticity, and vice versa.

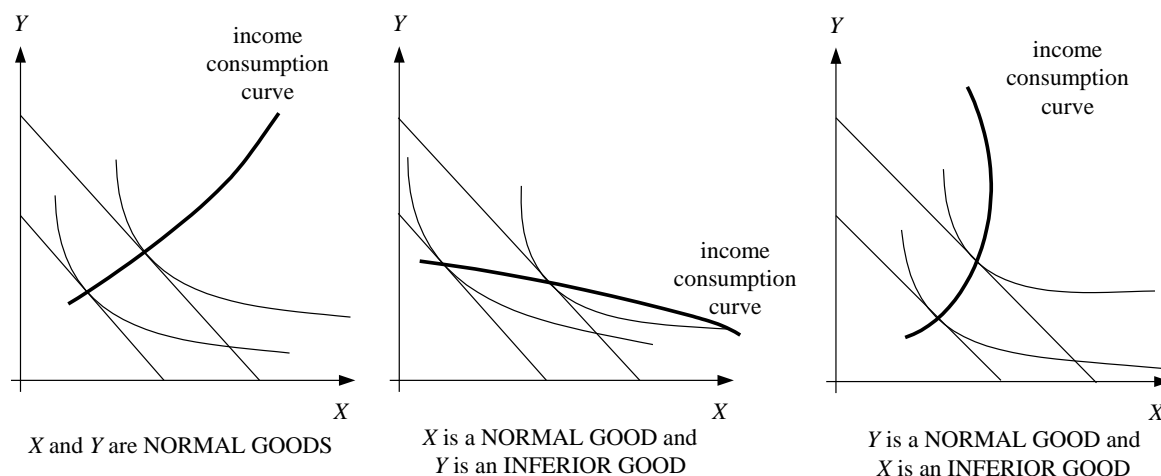
6. Income Consumption and Engel Curves

So far we have addressed only the question of consumption choice at a given time for a fixed income and fixed prices. Let us now relax only the fixed income assumption in which case we are concerned with parallel shifts of the budget constraint, i.e., in

$$Y = \frac{I}{p_y} - \frac{p_x}{p_y} \cdot X \quad (42)$$

only I and therefore only the intercept changes.

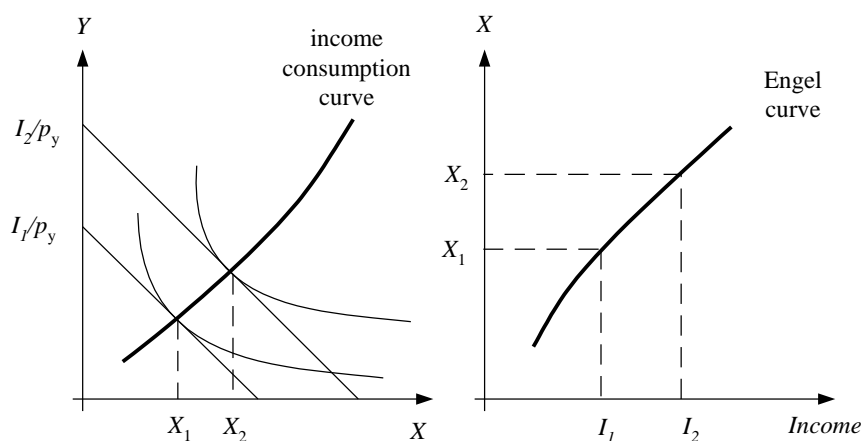
We can illustrate this simply in a diagram

Figure 6.1 Income Consumption Curve

As income increases the budget constraint shifts out, and because indifference curves are everywhere dense and obey the axiom of diminishing MRS , we can find a series of tangency points. These define how consumption changes as income changes - the income consumption curve.

Now clearly the shape of the income consumption curve will depend upon the shape of the indifference curve. In this illustration X and Y are both normal goods, i.e., consumption of both increases as income increases. Either X or Y could be inferior; this will determine the income consumption curves shape. You should draw these out for yourself. Given our assumptions can both X and Y be inferior?

We might be interested in how the demand for a good, say X , increases as income increases. This can easily be derived from the income consumption curve diagram.

Figure 6.2 Income Consumption Curve and the Engel Curve

Why are the slopes of income consumption and Engel curves as represented? Could the Engel curve be upward sloping and increasingly steep?

6.1 Income Elasticity of Demand

With respect to an Engel curve let us consider two concepts that lead to a third. The slope of the Engel curve indicates how the consumption of X changes as income, I , changes and is known as the marginal propensity to consume, i.e.,

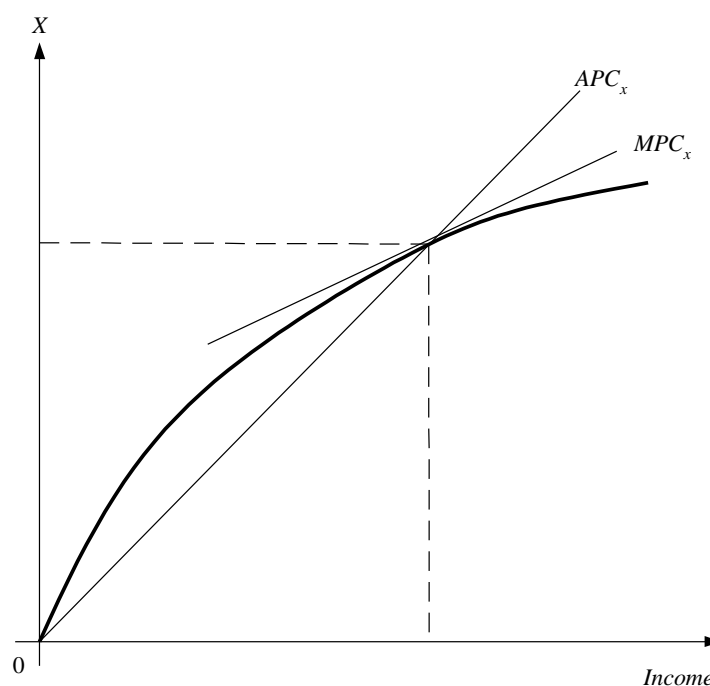
$$MPC_x = \frac{\partial X}{\partial I}$$

Using partial differentiation, ∂ , since all other things are being held constant. And the quantity of X bought for any given level of I is the average propensity to consume, i.e.,

$$APC_x = \frac{X}{I}$$

These can both be illustrated on an Engel Curve diagram in way that will be useful elsewhere, i.e.,

Figure 6.3 Engels Curve and Income Elasticity



The income elasticity of demand is defined as the percentage change in demand for X divided by the percentage change in income, i.e.,

$$e_M = \frac{\partial X/X}{\partial I/I} = \frac{\partial X/\partial I}{X/I} = \frac{MPC_X}{APC_X}$$

You should now be able to work out the relationships between the magnitudes of income elasticities of demand and the shapes of Engel Curves.

7. Revealed Preference

A major difficulty with indifference curve analysis is the unobservability of indifference curves. However, while indifference curves appear to be crucial to the foregoing analysis it is possible to replicate the previous conclusions without them, by means of observations about prices, incomes and consumption patterns. This is known as revealed preference analysis and was introduced by Samuelson in 1938.

The key assumptions of the revealed preferences hypothesis are

A1: Rationality

- the consumer seeks to maximise utility given the choices available, her tastes and preferences and the constraints under which she must operate, e.g., income and prices.

A2: Consistency

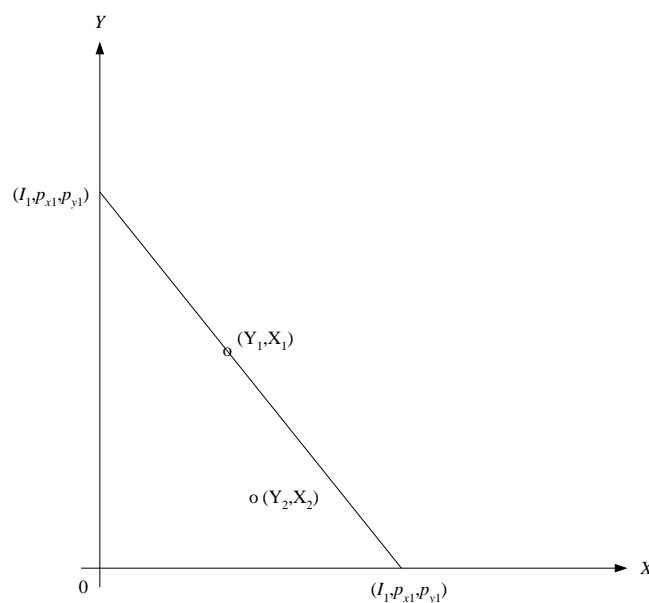
If $A > B$, then $B \not> A$.

A3: Transitivity

If $A > B$ and $B > C$ then $A > C$

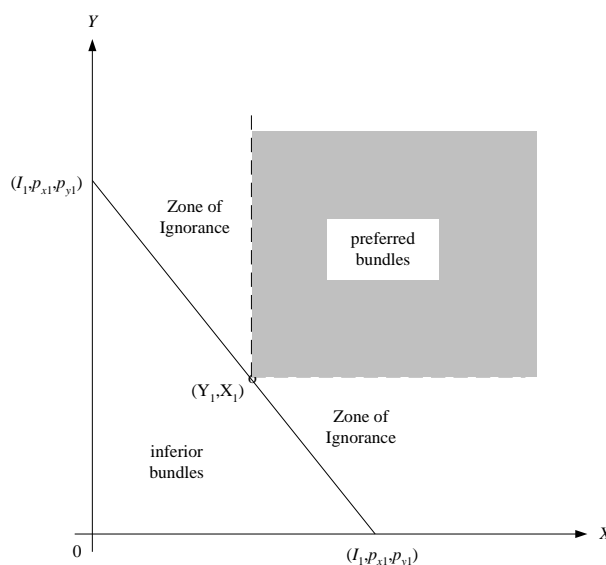
A4: Revealed Preference Axiom

By choosing a bundle of goods in one budget situation the consumer reveals her preference for that bundle over all others that were available.

Figure 7.1 Idea of Revealed Preference

By virtue of the assumptions, (Y_1, X_1) is revealed to be preferred to any bundle inside the budget constraint, e.g., (Y_2, X_2) , because all those bundles could have been afforded given the budget, and since any bundle on the constraint could also have been afforded but was not chosen, so (Y_1, X_1) is the revealed preferred choice.

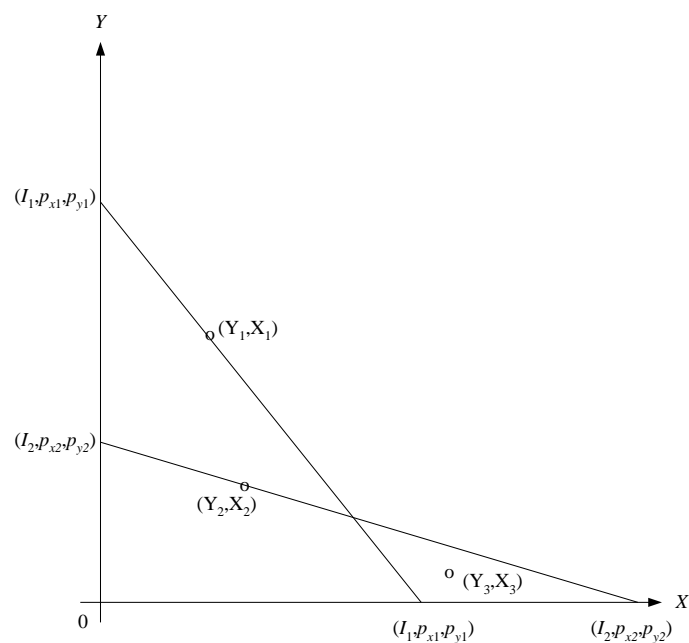
Does this provide any information on the shape of the indifference curves? Consider the diagram below. Any indifference curve through (Y_1, X_1) must lie above the budget constraint and outside the area that identifies the ‘preferred bundles’, i.e., it must pass through the ‘zone of ignorance’. If we have more observations representing higher levels of income, then we can further define the area within which an indifference curve must lie.

Figure 7.2 Revealed Preferences and Indifference Curves

Let us extend this argument to a situation in which the prices and or budget have changed, i.e., so-called indirectly revealed preferences. In period 1 we observe that (Y_1, X_1) is chosen on the budget constraint defined by (I_1, p_{X1}, p_{Y1}) , in which case we can conclude that (Y_1, X_1) is directly revealed preferred to (Y_2, X_2) . But if we observe that (Y_2, X_2) is chosen in period 2 when the budget constraint is defined by (I_2, p_{X2}, p_{Y2}) what can we infer about the changes in the budget constraint. Namely that (Y_1, X_1) was not affordable at (I_2, p_{X2}, p_{Y2}) .

This is the **Weak Axiom of Revealed Preference**

If (Y_1, X_1) is directly revealed preferred to (Y_2, X_2) , and the two bundles are not the same, then (Y_2, X_2) cannot be directly revealed preferred to (Y_1, X_1) .

Figure 7.3 Indirect Revealed Preferences

What does WARP allow you to say about the bundle (Y_3, X_3) relative to bundles (Y_2, X_2) and (Y_1, X_1) ?

Consider the situation in the diagram below. Why does a consumer who chooses both (Y_1, X_1) and (Y_2, X_2) violate WARP?

Figure 7.4 Violating WARP