Household as Supplier

The concern in this set of notes is with how households obtain their money income. This is done in terms of the household/individual's endowments – by reference to goods and services including factors, they are currently endowed with. To date we have assumed, implicitly, that the relative quantities and prices of the endowment are fixed and known, and hence that the budget constraint through the point of the initial endowment is fixed and known. This might not always be the case.

1. Endowments and Prices

We can represent endowments and money income by using a variant of the budget constraint diagram. Assume the initial endowments are $\bar{X}_0$ and $\bar{Y}_0$, the e_0 represents the point in choice space consistent with those endowments. But the individual can sell and buy the goods on product markets at the fixed prices p_x and p_y. Hence, the individual can convert her existing endowments into money income,

$$I_0 = p_x.\bar{X}_0 + p_y.\bar{Y}_0$$

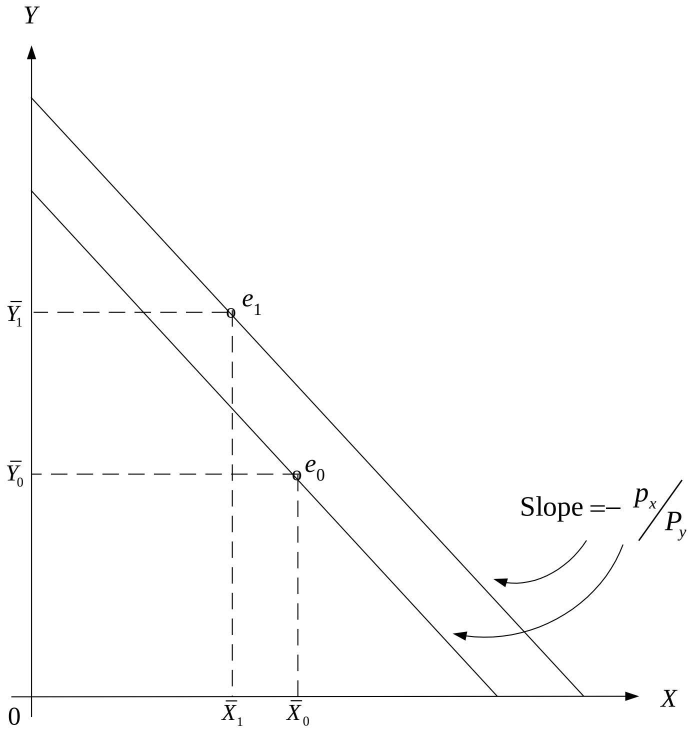
and then choose to spend the money income on those quantities of X and Y that best satisfy her preferences. If her endowments increase to, say, $\bar{X}_1$ and $\bar{Y}_1$, then e_1 represents the point in choice space consistent with the new endowments. Again, the individual can sell and buy the goods on product markets at the fixed prices p_x and p_y, and hence the individual can convert her existing endowments into money income,

$$I_1 = p_x.\bar{X}_1 + p_y.\bar{Y}_1$$

and then choose the consumption set.

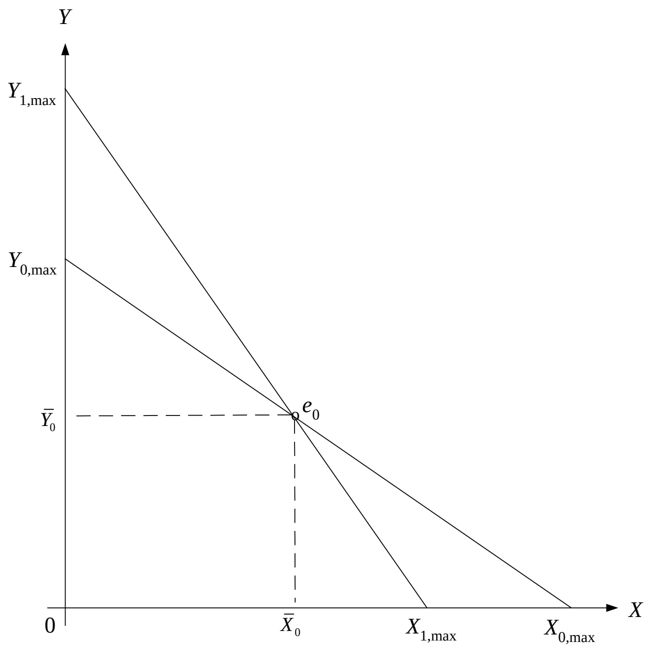
The increase in endowments causes the budget constraint to shift outwards. This represents an unambiguous increase in welfare, which can be verified by a simple application of the revealed preference hypothesis with respect to points e_0 and e_1.

Figure 1.1 Endowments and Wealth Effects



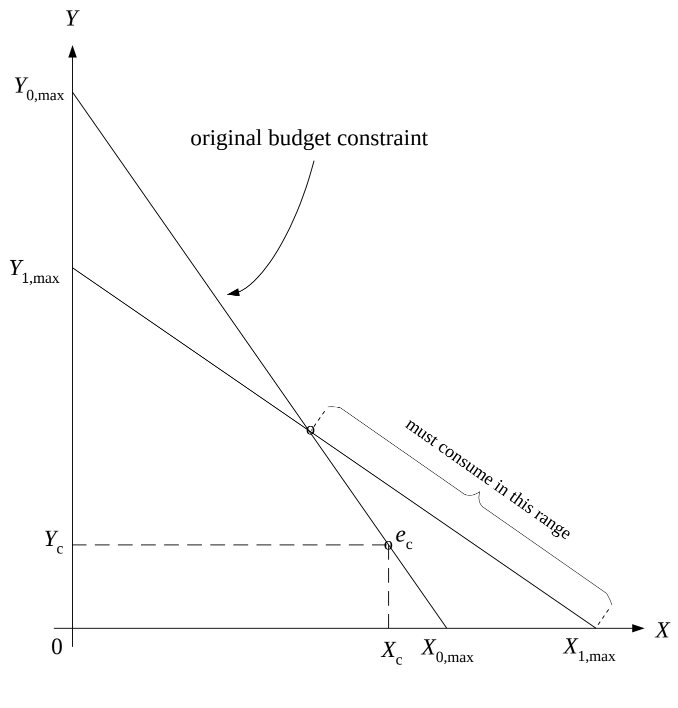
What may be more difficult is to disentangle are the consequences of changes in the prices of the two goods. If this happens the impact is to change the slope of the budget constraint, but whereas previously the budget constraint was defined by reference to money income, now the budget constraint is defined by reference to the endowments.

Figure 1.2 Endowments and Prices



Now the critical question is whether or not the individual is better or worse off. This can be determined by applying the revealed preference hypothesis to both $X > X_0$ and $X < X_0$. If the original consumption bundle is greater than X_0, i.e., we sell some X and buy some Y, and the price of X_1 increases then the following diagram holds.

Figure 1.3 Endowments, Prices and Revealed Preference



What would be the case if X_0 was less than the endowment level?

1.1 Slutsky Equation Revisited

When we considered the Slutsky equation earlier it was assumed that money income was exogenously set. Now we need a specification that is more general. Consider our simple diagram of the Slutsky decomposition; in this case the budget line pivots around an endowment point that is not on an axis. This requires a more general specification of the income effect. Naming the original income effect as the 'ordinary income effect' we can now identify an 'endowment income effect' associated with the impact of price changes on the value of the endowment, i.e.,

Total Effect = Substitution Effect + Ordinary Income Effect + Endowment Income Effect

Or more formally

$$\frac{\Delta X}{\Delta p_x} = \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(-X_0.\frac{\Delta X}{\Delta I}\right) + \textit{endowment income effect} \tag{1}$$

where ΔX is the quantity change associated with a given price change, Δp_x, and 'comp' is the 'compensated response', or substitution effects associated with the price change.

The endowment income effect is more complex. We need to identify how money income changes as the price changes and how demand changes when income changes. The former is straightforward. Endowment income is defined

$$I_0 = p_{x,0}.\overline{X} + p_{y,0}.\overline{Y}$$

where $\overline{X}$ and $\overline{Y}$ are the fixed endowments. Hence, by partial differentiation,

$$\frac{\Delta I_0}{\Delta p_x} = \overline{X}\ .$$

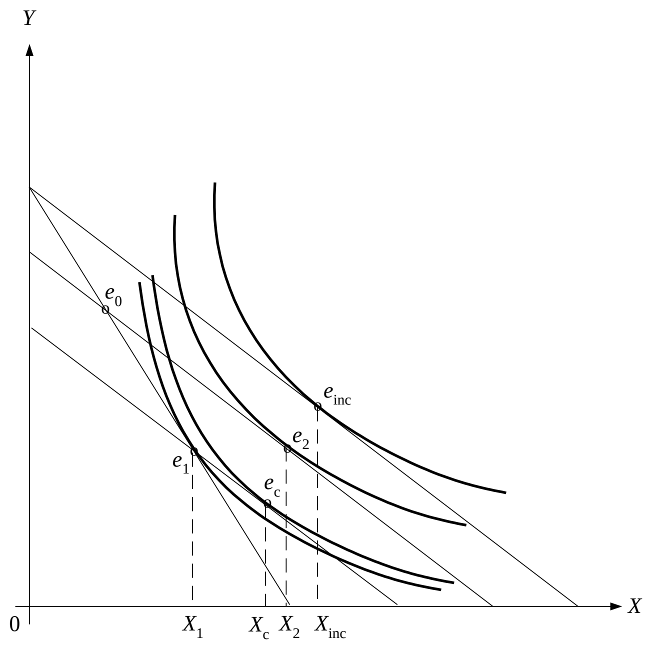
The change in demand when income changes is part of the ordinary income effect, $(\Delta X/\Delta I)$, and therefore the endowment income effect is

$$\frac{\Delta X}{\Delta I}.\frac{\Delta I_0}{\Delta p_x} = \frac{\Delta X}{\Delta I}.\overline{X}$$

and the total effect is

$$\frac{\Delta X}{\Delta p_x} = \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(-X_0.\frac{\Delta X}{\Delta I}\right) + \left(\bar{X}.\frac{\Delta X}{\Delta I}\right)$$
$$= \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(\bar{X} - X_0\right)\left(\frac{\Delta X}{\Delta I}\right)$$

Figure 1.4 Total Effect and an Endowment Income Effect



The initial endowment point is e_0 and the initial consumption point is e_1. Given the change in price of *X*, i.e., a price fall, the budget constraint pivots around the endowment point. The new consumption bundle is e_2. The substitution effect is the move from e_1 to e_c. The ordinary income effect is the move from e_c to e_{inc}, i.e., holding the money income constant via intercept on the vertical axis. But the fall in the price of X has reduced the value of the endowment; hence the movement from e_{inc} to e_2 is the endowment income effect.

2. Labour Supply

We start with arguably the most basic endowment that an individual may have, namely time. In this case the consumer's choice set is over consumption (*c*) and leisure (*n*). However, we assume that the individual can only consume if they have access to income, but her only endowment is her labour time. Therefore, consumption requires that the individual forgoes leisure to acquire income to fund consumption, i.e., the opportunity cost of consumption is leisure.

Let us assume

A1: constant wage rate (w);

A2: free choice over number of hours to work (l);

A3: the individual's only asset is time;

A4: constant price of consumption good (p_c);

A5: individual's tastes and preferences satisfy standard assumptions;

A6: perfect knowledge.

The individual is therefore free to choose how many hours to work, i.e., has full freedom to trade off leisure for consumption. This is a definition for a budget constraint, which we can write as

$$p_c * C = w * l = w * (T - n) \tag{1}$$

i.e., expenditure = income (from work) – income (from leisure foregone), which can be rearranged as

$$\left(p_c * C\right) + \left(w * n\right) = w * T\,. \tag{2}$$

The RHS defines the maximum possible income: the 'value of time endowment' or 'full income'. The LHS defines the cost of 'consumption' in terms of the cost of consuming goods plus the cost of 'consuming' leisure, which is the income forgone via the decision to devote time to leisure rather than work.

Solving (2) for C gives the maximum level of consumption

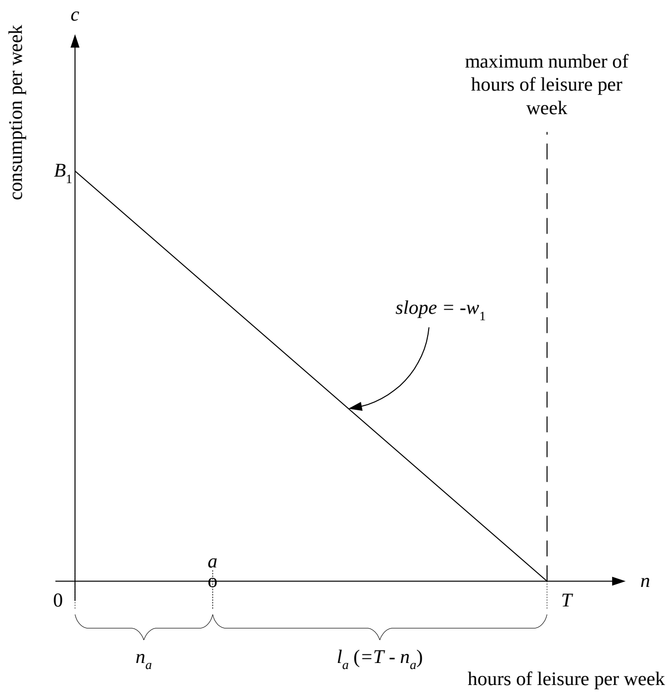
$$C = \frac{w * T}{p_c} - \left(\frac{w}{p_c}\right) * n \tag{3}$$

i.e., a straight-line budget constraint, where the slope, $-w/p_c$, is the relative price of work to the consumption good. A common practice is to normalize on p_c, i.e., to set p_c equal to 1. If we do this, we can write (3) as

$$C = w * T - w * n \tag{4}$$

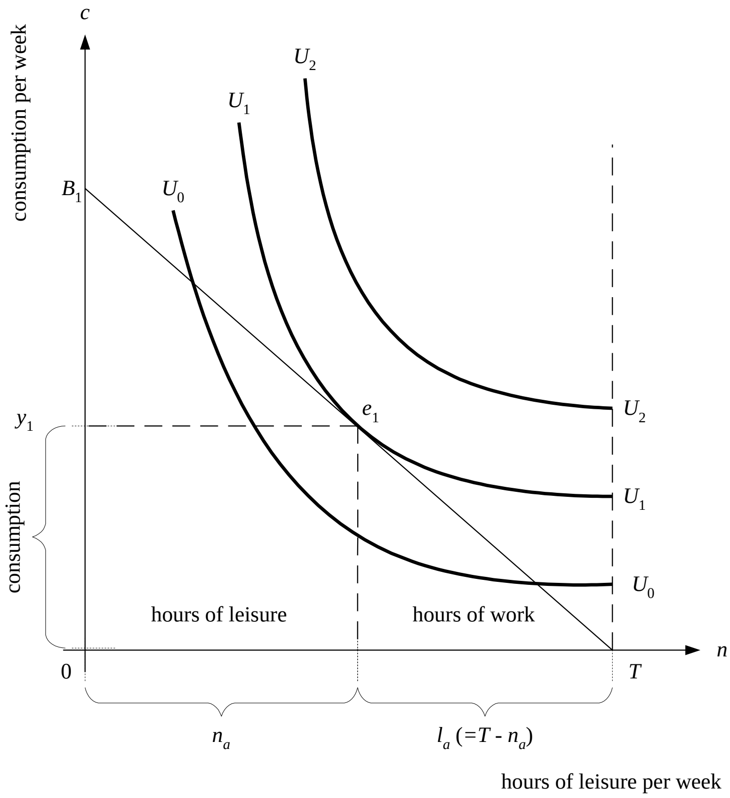
in which case the slope is $-w$.

Figure 2.1 Consumption, Leisure and the Budget Constraint



Adding the budget constraint to the choice set defines the range of feasible choices available to the individual. The decision about the allocation of time between leisure and work, and hence consumption, depends upon the preferences of the individual. These are represented by the utility function of the individual, which defines her indifference map, and determines the individual's supply of labour. Notice how the vertical line at *T* defines the range of feasible choices and therefore how the indifference curves do not exist to the right of *T*.

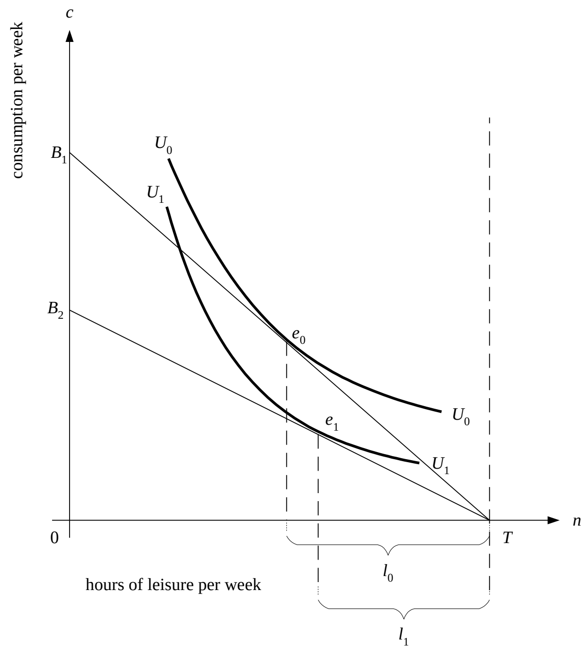
Figure 2.2 Equilibrium Labour Supply



This simple labour supply model can be used to examine how individuals might respond to changes in the relative price of labour to consumption goods. Note that the process of normalisation means that the representation is couched in terms of the price of labour, where the price of labour is defined as the REAL price of labour, i.e., relative to the price of the consumption good.

If the price of labour changes then the budget constraint will pivot around the intercept on the horizontal axis, and thereby alter the feasible region. How this impacts upon the choice over the amount of labour to supply will depend upon consumer preferences. In the case below we have assumed that the individual's preferences are such that at a lower wage rate she will substitute leisure for consumption. But as drawn the diagram does not rule out the opposite effect; and NOTHING in our model allows us to preclude that possibility. This reflects the fact that we have not assumed that an individual has a minimum consumption requirement.

Figure 2.3 Labour Supply and a Wage Reduction



The indifference map we have drawn could be produced for an individual who has a C-D utility function, since such a function assumes that substitution possibilities exist over all ranges of the arguments. A simple solution to this problem is to assume the individual has a Stone-Geary utility function. This is a utility function identical to a C-D function except for the possibility of minimum consumption requirements for any of the arguments, e.g., a minimum level of consumption, or minimum amount of leisure (this could be viewed as implicit in the definition of *T*).

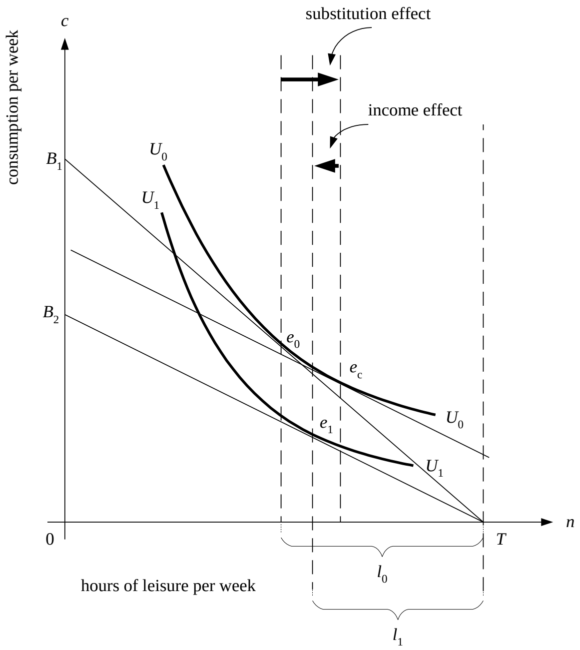
The reduction in the supply of labour resulting from the reduction in the real wage rate can be broken down into two effects: a substitution effect and an income effect. As with the standard consumer choice representation the substitution effect is necessarily negative, but the income effect can either be positive of negative. The critical issue becomes that of whether the absolute size of the income effect is greater than or less than the substitution effect. In this case the income effect on hours worked is positive but less than the substitution effect and therefore the individual offers up less labour time.

We can formalize this presentation by reference to the Slutsky equation. Adapting the Slutsky equation to the present context produces

$$\frac{\Delta n}{\Delta w}=\underset{(-)}{\left(\frac{\Delta n}{\Delta w}\right)_{comp}}+\underset{(+)}{\left(\bar{n}-n\right)}\underset{(+)}{\frac{\Delta n}{\Delta I}} \tag{5}$$

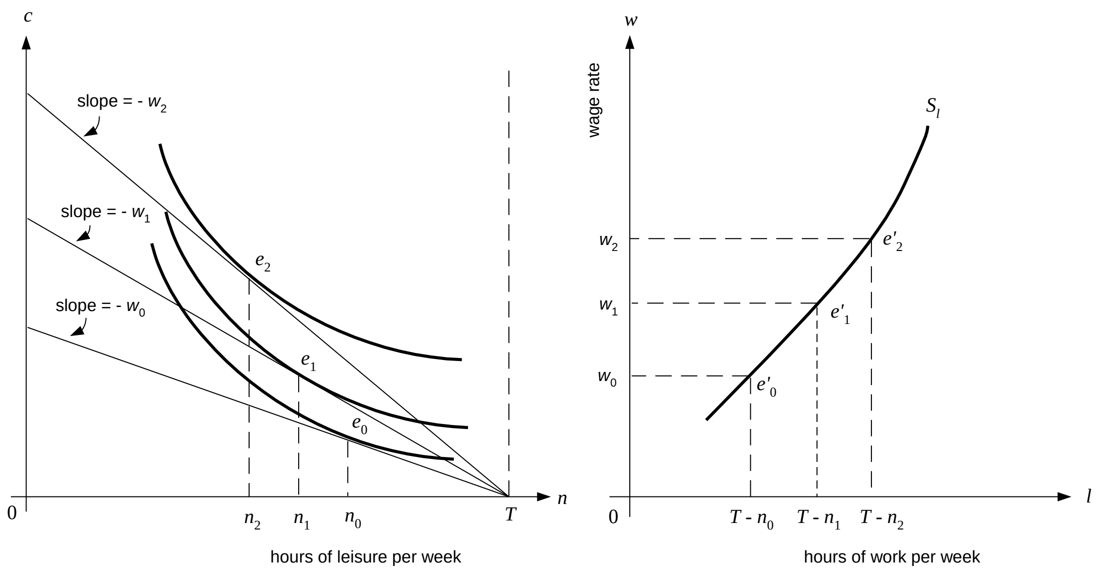
which formalises the relationship between the income and substitution and total effects derived from the diagrams.

Figure 2.4 Labour Supply and Income and Substitution Effects



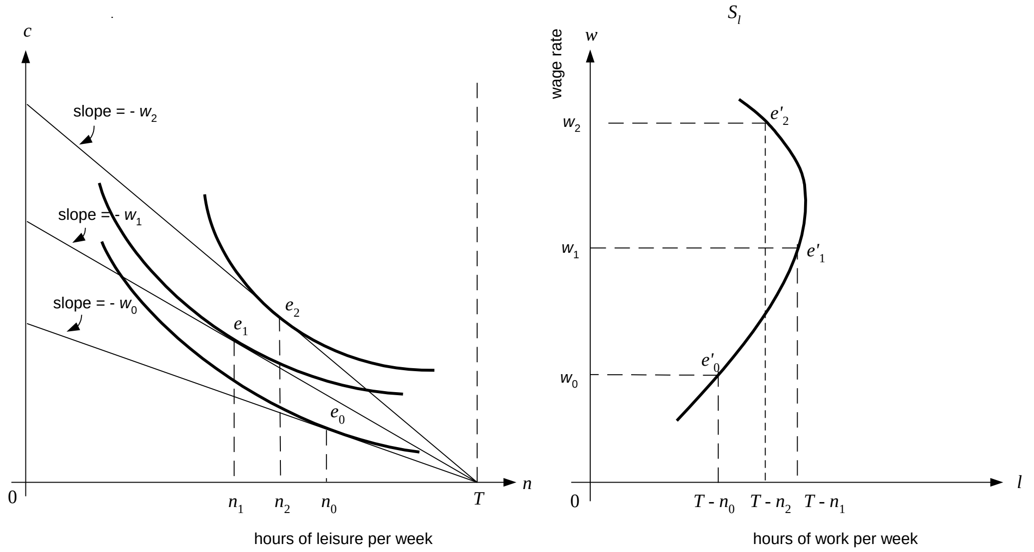
If we plot out the optimal choices for an individual given different real wage rates, then we can derive the labour supply curve for an individual.

Figure 2.5 Labour Supply Curve – Upward Sloping



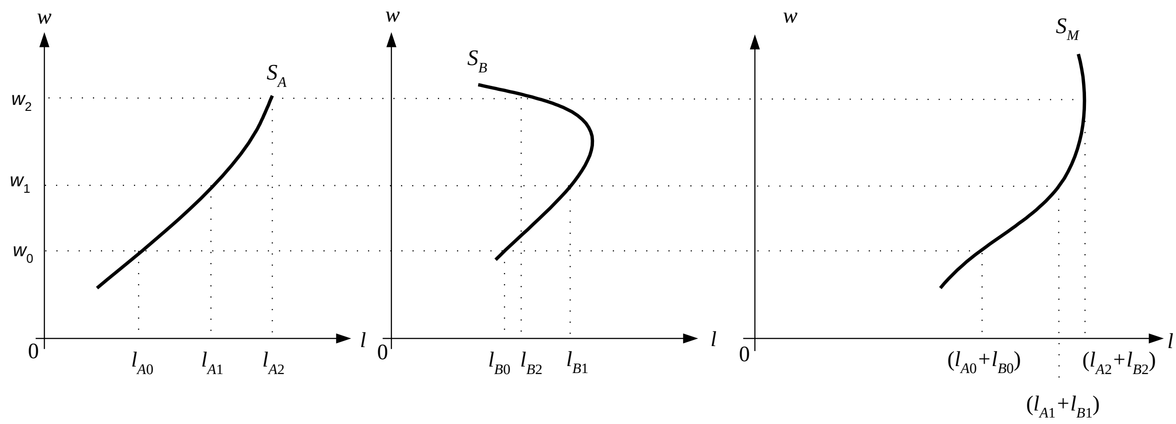
We might assume that as wage rates increase so will the amount of labour supplied. But this implicitly ignores the income effect. Indeed, it might be reasonable to argue that as income increases so might the absolute magnitude of the income effect; to such an extent that the labour supply curve becomes backward sloping. Note how this is not a case of perverse supply response because the substitution effect remains negative, rather it is a consequence of the relative magnitude of the income effect.

Figure 2.6 Labour Supply Curve – Backward Bending



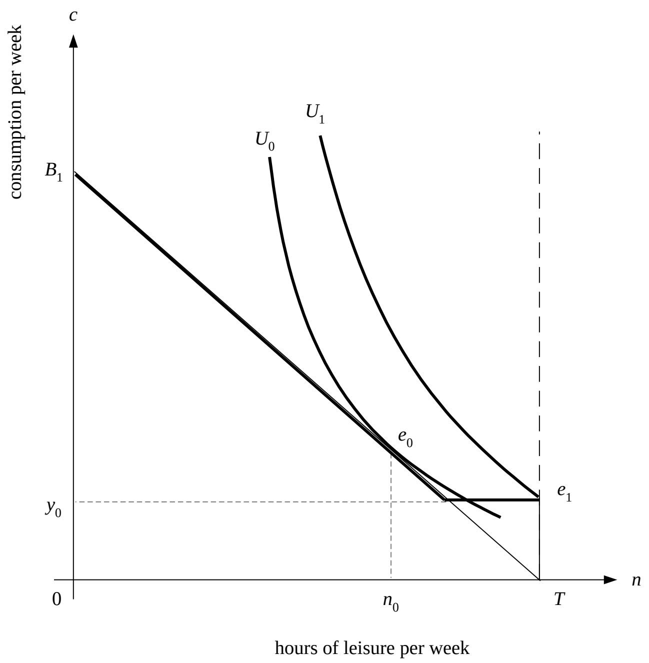
At the root of this lies the issue of individual preferences. Assume we have an economy with two groups of individuals each of which has identical preferences. For one group the substitution effect always dominates whereas for the other the income effect increases as the wage rate increases. In such a case the market supply of labour is given by the horizontal summation of the groups labour supply curves, and its shape will depend upon the combined preferences of the groups and may or may not become backward bending at some point.

Figure 2.7 Market Supply of Labour



An extension of the labour supply decision to allow for autonomous income is straightforward. If the household has some autonomous income, then the budget constraint is kinked and hence it is possible to argue that in some circumstances households may choose to offer no labour to the market.

Figure 2.8 Labour Supply and Autonomous Income



3. Allocation of Consumption Over Time

Note: these notes are not concerned with factor markets or income distribution theory.

In the initial analysis of consumer choice, it was assumed that all income must be consumed within a single time-period, i.e., deferred consumption/saving, was excluded by assumption. We will now relax is restrictive assumption. This is a standard analytical method used repeatedly by economists: restrictive assumptions are made to develop a simple model and then, these assumptions are relaxed progressively. It is done to facilitate the tractability of the model <u>not</u> because we believe the assumptions to be realistic.

Let us assume the following:

A1: The planning horizon is limited to two periods;

A2: All income must be consumed during the two periods;

A3: The capital market is perfect;

A4: The single rate of interest is i.

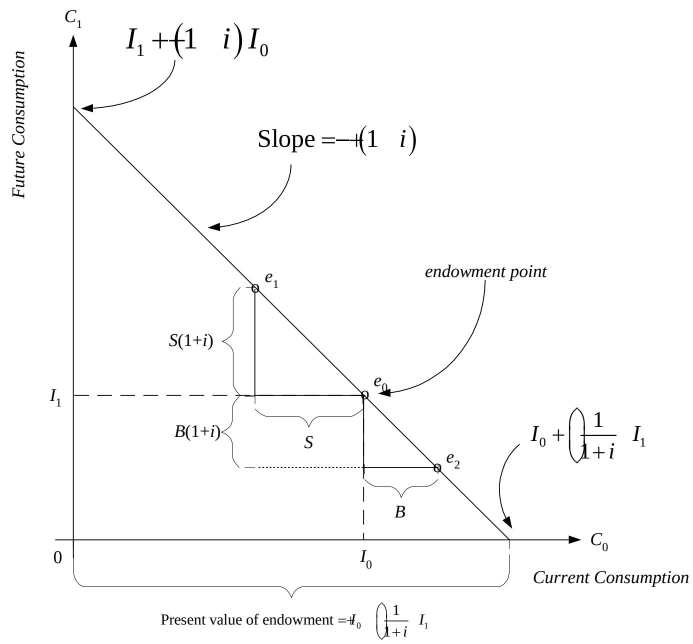
A5: No transaction costs;

A6: individual's tastes and preferences satisfy standard assumptions;

A7: The income in period 0 and I_0 and in period 1 is I_1, and is known and certain.

From these assumptions, we can deduce the existence of a 'budget constraint', analogous to that in our previous analysis. The axes are consumption in period 0, C_0, and consumption in period 1, C_1

Figure 3.1 Intertemporal Budget Constraint



If the individual does not enter the capital market, then

$$C_0 = I_0 \quad \underline{\underline{\text{and}}} \quad C_1 = I_1 \tag{6}$$

But what happens if the individual enters the capital market? Assuming C_0 = 0, then I_0 can be lent earning the interest rate i to produce $(1 + i)I_0$ in period 1. Thus

$$C_{1_{(C_0=0)}} = I_1 + (1+i)I_0 \tag{7}$$

and similarly, if C_1= 0, then

$$C_{0_{(C_1=0)}} = I_0 + \left(\frac{1}{1+i}\right)I_1 \tag{8}$$

Hence we can draw a 'budget constraint' with intercepts of $\left[I_1 + (1+i)I_0\right]$ and $\left[I_0 + \left(\frac{1}{1+i}\right)I_1\right]$ and the slope is - (1 + i), since for each unit of current consumption, C_0, (1+i) units of future consumption must be given up, i.e., the opportunity cost of current consumption in (1+i).

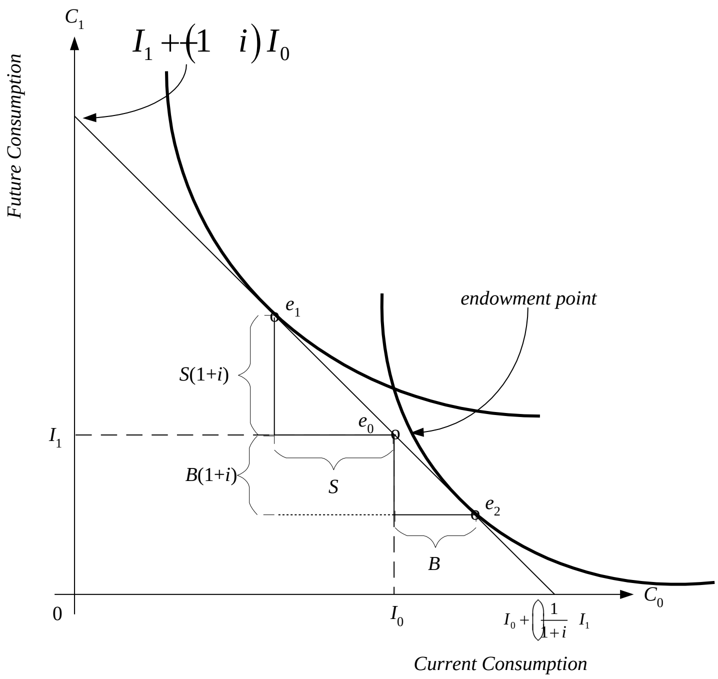
Such a 'budget constraint' is often termed a market opportunity curve.

Just as we could conceive of combinations of 2 goods in the same period yielding identical levels of well-being, so can we conceive of bundles of goods consumed in two time periods giving identical utility. Thus, we can conceive of a utility function, and hence indifference map, relating present and future consumption. But will they also be convex to the origin?

Economists have adopted the axiom that present consumption is preferred to future consumption, and therefore that consumers must be induced to give up present consumption in return for greater future consumption: that the marginal rate of substitution is less than -1. This is known as a positive rate of time preference. Furthermore, it is presumed that as the quantity of present consumption deferred increases, so must the quantity of future consumption increase. These presumptions ensure that the intertemporal indifference curves are convex to the origin.

Hence, as before, we can represent the consumers' equilibrium as the tangency between an intertemporal indifference curve and the market opportunity curve. Notice how this allows the individual to separate the consumption decision from the endowment constraint, i.e., consumption in each period is related to but partially independent from endowments.

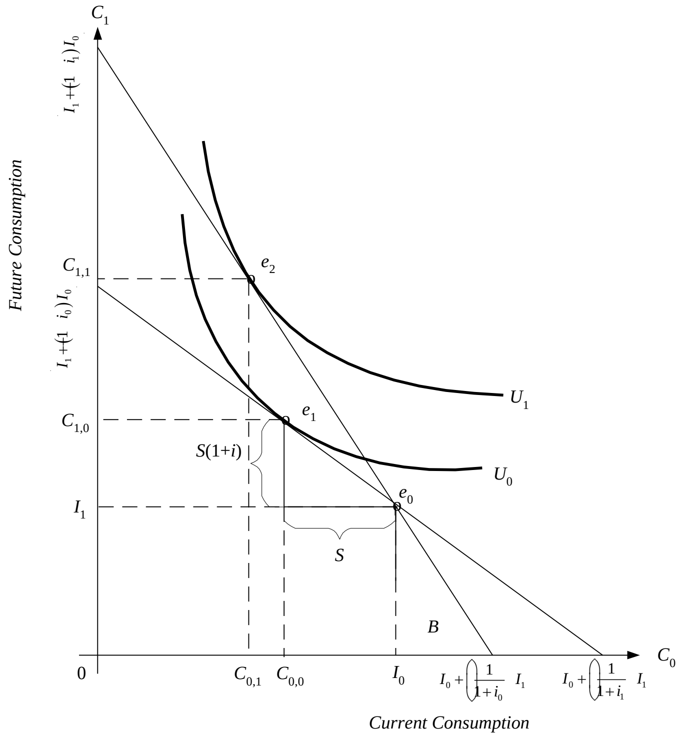
Figure 3.2 Intertemporal Consumption Equilibrium



3.1 Consumption and the Level of Interest Rates

It is perhaps worth asking whether deferred consumption, or savings, will necessarily increase or decrease with an interest rate rise. In fact, both outcomes are possible.

Figure 3.3 Intertemporal Consumption & Interest Rate Changes



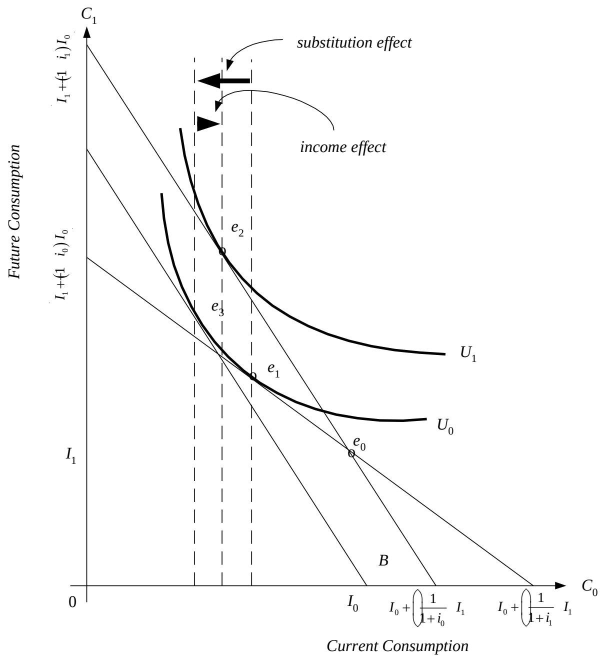
The increase in interest rate pivots the market opportunity curve from 1 to 2. Note: that if $C_1 = I_1$ then $C_0 = I_0$. And the equilibrium combination of C_1 moves from e_1 to e_2, but e_2 represents a decrease in C_0 and therefore more saving.

In fact, the explanation for this is simply another example a substitution effect and an ‘income’ effect or more appropriately as wealth effect. Why wealth?

Consider the meaning of the intercept with the horizontal axis. This tells us the present value of current income, I_0, plus the present value of future income, $\frac{1}{(1+i)}.I_1$. Thus, we can deduce that current consumption depends upon ‘wealth’ and the rate of interest, and not solely on current income.

Now we can conduct a Slutsky compensating variation to decompose the impact of different interest rates.

Figure 3.4 Intertemporal Consumption, Interest Rates and Income & Substitution Effects



In this case, the substitution effect increases savings by the move from e_1 to e_3, which reduces C_0. But the wealth effect works in the opposite direction to reduce savings by the move from e_3 to e_2. Since the substitution effect will unambiguously increase savings, just as we found when analysing price change substitution effects, the important question is the relative sizes of the substitution and wealth effects, provided current consumption is a normal good.

As you can easily verify the critical factor is the, unknown, shape of the consumer's indifference map.

We can also formalise this presentation by reference to the Slutsky equation. Adapting the Slutsky equation to the present context produces

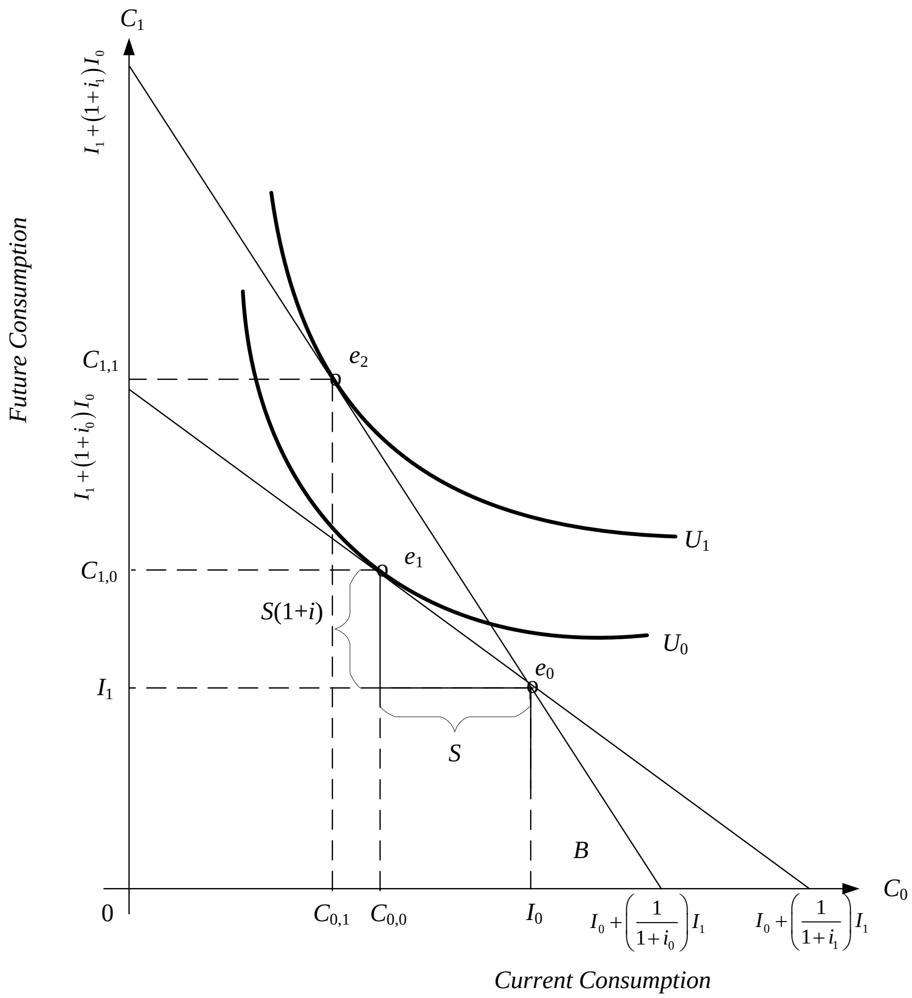
$$\underset{}{\frac{\Delta C_0}{\Delta p_0}} = \underset{(-)}{\left(\frac{\Delta C_0}{\Delta p_0}\right)_{comp}} + \underset{(?)}{\left(I_0 - C_0\right)} \underset{(+)}{\frac{\Delta C_0}{\Delta I}} \tag{9}$$

which simply formalises the relationship between the income and substitution and total effects derived from the diagrams.

3.2 Consumption and Changes in the Rate of Interest

So far we considered the impact of indifference interest rates upon inter-temporal consumption patterns where a once-and-for-all decision was to be made. This analysis applies wherever the method of saving does not alter the capital value of the savings to the lender, e.g., savings in a building society. But if lending takes place through a bond purchase - a promise to pay a sum of money in the next period - the anlaysis requires modification.

Figure 3.5 Intertemporal Consumption & Interest Rates 3



The preferred position is at A given the initial interest rate: a bond is therefore purchased for (Y_t - C_t) for which (C_{t+1} - Y_{t+1}) will be received in period t+1. Changes in the interest rate will now not affect the individual, nor can she move from A because the payment (C_{t+1} - Y_{t+1}) is fixed. But if the interest rate changes she faces the option of re-entering the bond market. A rise in the interest rate will now pivot the market opportunity curve clockwise about A, in which case she may purchase a bond to move to point D.

Thus, in this case there is clearly an unambiguous relationship between interest rates and deferred consumption. You should work out what would happen if interest rates fall. *Hint:* You can buy or sell bonds.

3.3 Time Preference and Present Value

Why is the interest rate positive? Economists justify this by calling upon the axiom of a positive rate of time preference, i.e., we prefer to consume now to later and therefore if we are to defer consumption we require an incentive to do so, that incentive is a positive rate of interest. This axiom of time preference underlies the concept of present value, which economist make repeated recourse to when analysing decisions over time.

Assuming perfect knowledge, we can identify the future values for a stream of benefits that might arise from some action or investment, e.g., investing in education. A typical problem we might face is a choice between different actions that imply different future streams of benefits. How can we choose between the actions?

If we accept the axiom of a positive rate of time preference, then we can conclude that the present value of a future benefit is less than the value of the same benefit now. Moreover, the rate of time preference allows us to quantify the difference since it defines the amount we would require to forgo current consumption by one period, i.e., the future value of present consumption. Taken over a period of years we can recognize this as compound interest.

The reverse is known as discounting, i.e.,

$$PV = \frac{I_0}{(1+i)^0} + \frac{I_1}{(1+i)^1} + \frac{I_2}{(1+i)^2} + ... + \frac{I_T}{(1+i)^T} . \qquad (10)$$

This is the concept that we used over 2 periods in our simple diagrams above. What it does is allow us to extend the concept beyond the confines of simple 2 dimensional diagrams.

4. Human Capital

Investing in human capital is an action from which we hope to gain an enhanced future income stream. We can use a variant of the simple intertemporal optimisation diagram to illustrate how the decision might be made and to encompass several variables.

Let us assume the following:

A1: The planning horizon is limited to two periods;

A2: All income must be consumed during the two periods;

A3: The capital market is perfect;

A4: The single rate of interest is i.

A5: No transaction costs;

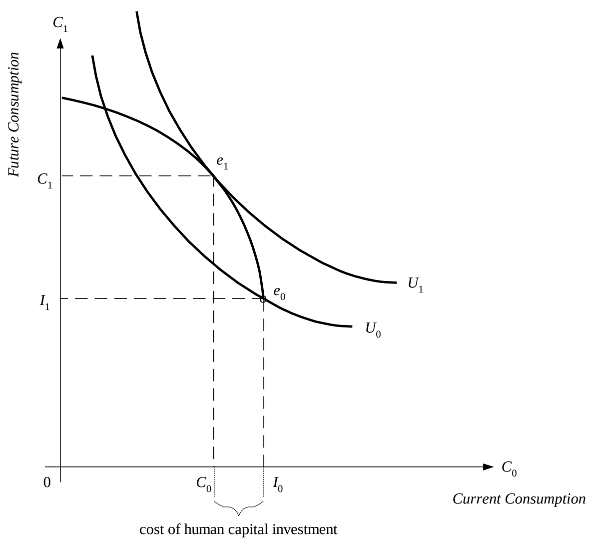
A6: individual's tastes and preferences satisfy standard assumptions;

A7: The income in period 0, I_0, is known and certain;

A8: The income in period 1, I_1, consequent upon different levels of human capital is known and certain.

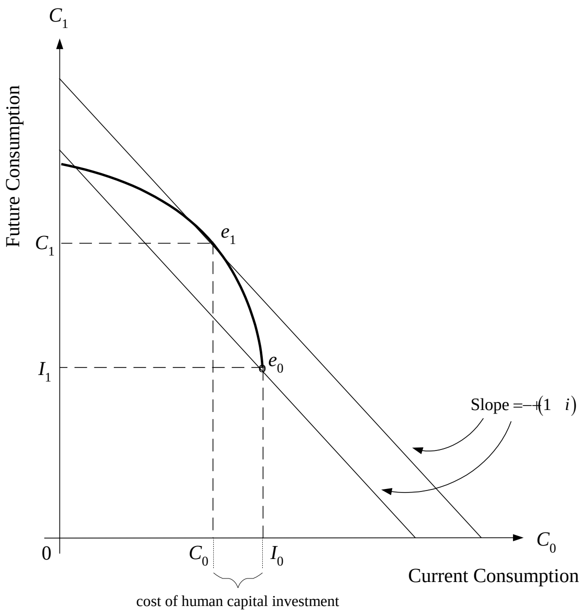
A9: The relationship between investment in 'education' and human capital is known and certain.

Figure 4.1 Human Capital Decision with No Capital Market



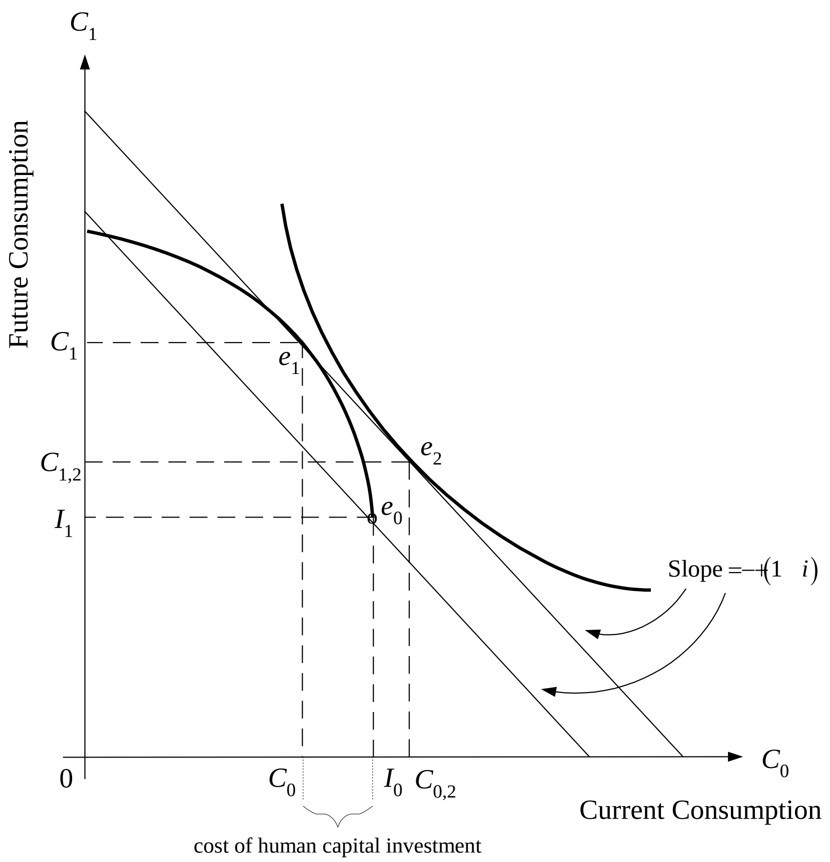
If we make no investment in human capital then our income in periods one and two are known and are equal to I_0 and I_1, and consumption and income in each period are equal at e_0. Since we know the relationship between investment in 'education' and human capital and the returns in period 1 to any level of human capital, we can define a transformation function between income forgone in period 0 and income received in period 1. Without an option to invest in 'education' the individual has utility U_0 at e_0. But if there is an option to invest in 'education' the individual can achieve a higher level of utility, U_1 at e_1, for the cost of $I_0 - C_0$ consumption forgone in period 0.

Figure 4.2 Human Capital Decision with a Capital Market



This, of course, presumes that C_0 is an adequate level of consumption in period 0. What happens if we introduce a perfect capital market? Now the individual can separate the current consumption decision from the current income level by borrowing against future incomes, which are assumed to be known. Now given the known rate of interest, e_1 defines a point on the intertemporal budget constraint whose slope is –(1 + i), and the individual is free to choose any point on that budget constraint to determine consumption in periods 0 and 1.

Figure 4.3 Human Capital, Capital Market and Consumption Decisions



5. Physical Capital

It is relatively straightforward to derive an analysis for physical capital that combines the arguments for labour supply and for human capital.