Stone-Geary Function
Linear Expenditure System

1

Outline

- Stone-Geary Utility Function
- Linear Expenditure System
- Properties
  - Complete demand system
  - Marginal 'budget' shares
  - Allows for minimum/subsistence consumption
- Calibration
- Welfare
  - Hicksian & Slutsky welfare measures

2

cgemod

Properties

3

cgemod

Stone-Geary Function & the LES

Stone-Geary Utility Function

$$U_h\left(q_{c,h}\right)=\prod_c\left(q_{c,h}-\gamma_{c,h}\right)^{\beta_{c,h}}$$

Linear Expenditure System

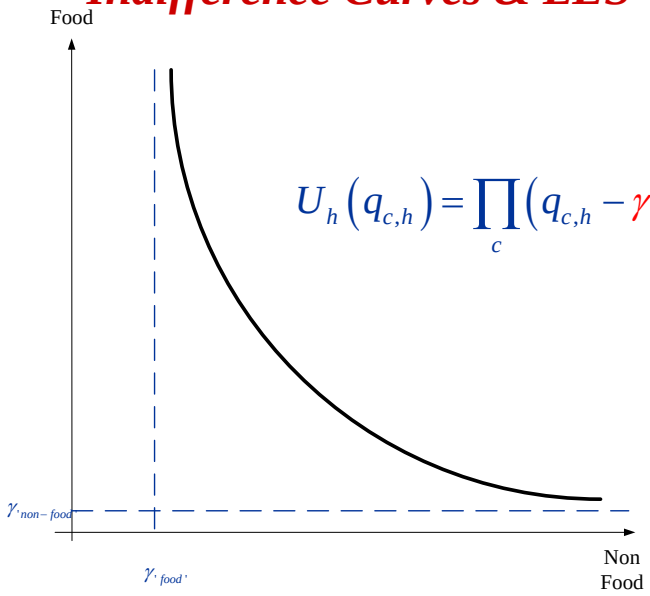
$$q_{c,h}=\gamma_{c,h}+\frac{\beta_{c,h}}{p_c}\left(y-\sum_c\left(\gamma_{c,h}*p_c\right)\right)$$

4

cgemod

cgemod

Indifference Curves & LES



$$U_h\left(q_{c,h}\right)=\prod_c\left(q_{c,h}-\gamma_{c,h}\right)^{\beta_{c,h}}$$

5

cgemod

Linear Expenditure System

Commodity demand

$$q_{c,h}=\gamma_{c,h}+\frac{\beta_{c,h}}{p_c}\left(y-\sum_c\left(\gamma_{c,h}*p_c\right)\right)$$

Commodity expenditure (linear)

$$q_{c,h}*p_c=\left(\gamma_{c,h}*p_c\right)+\beta_{c,h}\left(y-\sum_c\left(\gamma_{c,h}*p_c\right)\right)$$

$\therefore \beta_{c,h}$ **= marginal expenditure share**

NO Subsistence demand

$$q_{c,h}*p_c=\beta_{c,h}\left(y\right)$$

$\therefore \beta_{c,h}$ **= average expenditure share**

6

cgemod

cgemod

Calibration

7

cgemod

Calibration Method

- Follow Dervis, de Melo and Robinson (pp 482-3, 1984)
- Obtain estimates of
  - Frisch parameter (μ)
    - elasticity of the marginal utility of income with respect to income
  - Income elasticities of demand (ε)

$$\mu_h = -\frac{y_h}{y_h - \sum_c \left(\gamma_{c,h} * p_c\right)}$$

$$\gamma_{c,h} = \left(\frac{y_h}{p_c}\right) * \left[\alpha_{c,h} * \left(\frac{\beta_{c,h}}{\mu_h}\right)\right]$$

$$\beta_{c,h} = \varepsilon_{c,h} * \alpha_{c,h}$$

8

cgemod

cgemod

Household Demand: Equations

Subsistence consumption

$$q_{c,h} * p_c = (\gamma_{c,h} * p_c) + \beta_{c,h}\left(y - \sum_c (\gamma_{c,h} * p_c)\right)$$

Marginal 'budget' shares

Consumption expenditure

Subsistence expenditure

Requirements

1. Subsistence consumption volumes
2. Marginal budget share

9

cgemod

Household Consumption Expenditure

e.g., Household income less other expenditure

SAM	
	h_poor
cagr	43,663
cnres	344
cmanu	109,228
cserv	105,472
dirtax	16,331
i_s	28,595

$$tyh_h = \frac{SAM("dirtax",h)}{SAM("total",h)} = \frac{VTYH_h}{VYH_h}$$

$$= \frac{16,331}{303,633} = 0.0537853$$

```
tyh0(h) = SAM("dirtax",h)/SAM("total",h) ;
```

$$shh_h = \frac{SAM("i\_s",h)}{SAM("total",h) - SAM("dirtax",h)} = \frac{VSH_h}{(VYH_h - VTYH_h)}$$

$$= \frac{28,595}{(303,633 - 16,331)} = 0.0995294$$

```
shh0(h)= SAM("i_s",h)/[SAM("total",h)-SAM("dirtax",h)] ;
```


10

cgemod

Household Demand: Calibration

SAM	
	h_poor
cagr	43,663
cnres	344
cmanu	109,228
cserv	105,472

ELASTY	
	h_poor
cagr	0.4
cnres	0.6
cmanu	1.2
cserv	2

ELASTMU	
	frisc
h_poor	-3.50

$$\alpha_{c,h} = \frac{SAM_{c,h}}{\sum_c SAM_{c,h}}$$

$$\varepsilon_{c,h} = \frac{ELASTY_{c,h}}{\sum_c \alpha_{c,h} * ELASTY_{c,h}}$$

$$\gamma_{c,h} = \left(\frac{y_h}{p_c}\right) * \left[\alpha_{c,h} * \left(\frac{\beta_{c,h}}{\mu_h}\right)\right]$$

$$\beta_{c,h} = \varepsilon_{c,h} * \alpha_{c,h}$$

Practical CGE, 2024 © cgemod 11

11

Household Demand: Elasticities

ELASTMU	
	frisc
h_rich	-1.20
h_poor	-3.50

Frisch is negative. Subsistence increases as Frisch becomes MORE negative

IF Frisch = - 1 AND ALL ELASTY = 1 ∀c Utility function is CD WHY?

ELASTY		
	h_rich	h_poor
cagr	0.8	0.4
cnres	0.8	0.6
cmanu	1.5	1.2
cserv	2.2	2

Practical CGE, 2024 © cgemod 12

12

cgemod

Household Demand: Calibration

SAM

	h_poor
cagr	43,663
cnres	344
cmanu	109,228
cserv	105,472

ELASTY

	h_poor
cagr	0.4
cnres	0.6
cmanu	1.2
cserv	2

ELASTMU

	frisc
h_poor	-3.50

```
sumelast(h)= SUM(c, alphah(c,h)*ELASTY(c,h)) ;

yhelast(c,h)$sumelast(h)
              = ELASTY(c,h)/sumelast(h) ;

yhelast(c,h)$(alphah(c,h) eq 0)  = 0 ;
```

```
beta(c,h) = yhelast(c,h)*alphah(c,h) ;
```

```
frisch(h) = ELASTMU(h) ;

qcdconst(c,h)$(PQCD0(c) and frisch(h))
   = (HEXP0(h)/PQCD0(c))
        *(alphah(c,h)+(beta(c,h)/frisch(h))) ;
```


13

cgemod cgemod

Linear Expenditure System

The End

14

cgemod