STAGE_CC: A Standard Applied General Equilibrium Model: Technical Documentation

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Abstract

This paper provides a technical description of a version of a single country computable general equilibrium (CGE) model that is calibrated using data presented as a Social Accounting Matrix (SAM) with satellite accounts. This model was used in a study for UNFCCC; its use is not endorsed.

Keywords: Computable General Equilibrium; Social Accounting Matrix.

[1] The STAGE model is subject to ongoing developments; this version of the technical document contains details of developments up to the given date. See Appendix 1 on the model's genealogy for details.

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1. Introduction

This document provides a description of the comparative static version of STAGE single-country computable general equilibrium (CGE) model, that is a variant/development of the STAGE 3 single country CGE model. Recursive dynamic applications of the STAGE family of CGE models all start from the respective comparative static models, by exploiting the LOOP facility provided by GAMS (General Algebraic Modelling System[2]), so that the recursive dynamic applications operate as series of comparative static simulations where the 'dynamic' updates are implemented between each comparative static simulation. Thus, an in-depth understanding of the comparative static versions of the model is essential before progressing to the recursive dynamic versions. This technical description encompasses both the comparative static version of STAGE and the core model components required for the recursive dynamic variant, i.e., it only relates to the 'solution' phase. It does not include any details about the behavioural relationships included in the 'update' phase.[3]

This model is characterised by several distinctive features.

1. The model allows for a generalised treatment of trade relationships by incorporating provisions for non-traded exports and imports, i.e., commodities that are neither imported nor exported, competitive imports, i.e., commodities that are imported and domestically produced, non-competitive imports, i.e., commodities that are imported but not domestically produced, commodities that are exported and consumed domestically and commodities that are exported but not consumed domestically.
2. The model allows the relaxation of the small country assumption for exported commodities that do not face perfectly elastic demand on the world market.
3. the model allows for modeling of multi-product activities using various assumptions; fixed proportions of commodity outputs by activities with commodities differentiated by the activities that produce them, varying output mixes by activities in response to changes in the (basic) prices of commodities, and domestically produced commodities that are differentiated by source activity or are

2 See www.gams.com for details about GAMS.

3 Recursive dynamic CGE models can be thought of as being a series of comparative static 'solution' phases with 'update' phases that are used to calibrate successive 'solution' phase.

homogeneous, i.e., undifferentiated by source activity. Hence the numbers of commodity and activity accounts are not necessarily the same.

4. The (value added) production technologies can be specified as systems of n-level generalised nested Constant Elasticity of Substitution (CES) using factors and selected intermediate inputs.
5. Trade and transport margins between factory and dock gate and the consumer are levied on domestic consumption.
6. Consumption expenditure by each representative household group (RHG) is represented by nested CES and Stone-Geary utility functions.
7. Household consumption commodities include ‘leisure’ where the ‘leisure’ consumed by each RHG can only be produced by labour factors owned by the respective RHG; this introduces a labour-leisure trade-off into the model.
8. The model includes behavioral relationships that track emissions of, and taxes on, greenhouse gases based on environmental satellite accounts.
9. The model is calibrated using two matrices (satellite accounts) detailing the factor use by activity and the factor ownership by institution.
10. The functional distribution of income is endogenously determined through the specification of the ownership (domestic and foreign) of factors used within the economy being defined as a series of variables.
11. Investment functions can be specified for multiple types of new capital goods.

A guiding principle underpinning the STAGE family of models, inherited from the work by Sherman Robinson and various colleagues, is that the models should be agnostic with respect to macroeconomic closure and market clearing conditions, i.e., the models should be coded to allow users substantial degrees of freedom to impose their own views on how an economy operates. The models also include a substantial number of variables included to simplify the coding of adjustments needed for experiments; by and large these adjustment instruments use a standard formulation.

The model is designed for calibration using a reduced form of a Social Accounting Matrix (SAM) that conforms to the UN System of National Accounts (SNA). Table 1 contains a macro-SAM in which the active sub matrices are identified by *X* and the inactive sub matrices are identified by 0. In general, the model will run for any SAM that does not contain information in the inactive sub matrices and conforms to the rules of a SAM.[4] In some cases a SAM might contain payments from and to both transacting parties, in which case recording the transactions as net payments between the parties will render the SAM consistent with the structure laid out in Table 1.

Table 1 Macro SAM for the Standard Model

	Commodities	**Activities**	**Factors**	**Households**	**Enterprises**	**Government**	**Capital Accounts**	**RoW**
Commodities	0	X	0	X	X	X	X	X
Activities	X	0	0	0	0	0	0	0
Factors	0	X	0	0	0	0	0	X
Households	0	0	X	0	X	X	0	X
Enterprises	0	0	X	0	0	X	0	X
Government	X	X	X	X	X	0	0	X
Capital Accounts	0	0	X	X	X	X	0	X
RoW	X	0	X	X	X	X	X	0
Total	X	X	X	X	X	X	X	X

The most notable differences between this SAM and one consistent with the SNA are:

1) The SAM is assumed to contain only a single ‘stage’ of income distribution. However, if the SAM has a multi-stage treatment of income distribution, transforming the functional distribution of income using apportionment (see Pyatt, 1989) will render the SAM consistent.
2) A series of tax accounts are identified (see below for details), each of which relates to specific tax instruments. Thereafter a consolidated government account is used to bring together the different forms of tax revenue and to record government expenditures. These adjustments do not change the information content of the SAM, but they do simplify the modeling process. However, they do

4 If users have a SAM that does not run with no information in inactive sub matrices the authors would appreciate a copy of the SAM to further generalise the model.

have the consequence of creating a series of reserved names that are required for the operation of the model.[5]

The model contains a section of code, immediately after the data have been read in, that resolves several common ‘issues’ encountered with SAM databases by transforming the SAM so that it is consistent with the model structure without any marked loss of information. Specifically, all transactions between an account with itself are eliminated by setting the appropriate cells in the SAM equal to zero. Second, some transfers from domestic institutions to the Rest of the World and between the Rest of the World and domestic institutions are treated net as transfers to the Rest of the World and domestic institutions, by transposing and changing the sign of the payments. And third, some transfers between domestic institutions and the government are treated as net and as payments from government to the respective institution. Since these adjustments change the account totals, which are used in calibration, the account totals are recalculated within the model.

In addition to the SAM, which records transactions in value terms, two additional databases are used by the model. The first are a series satellite accounts that record quantity data that link to various sub-matrices of transaction values in the SAM: these include ‘quantities’ of primary inputs used by each activity, the quantities of factors owned by each institution, emissions associated with input use by activities, etc. If such quantity data are not available, the code allows for the use of value quantities: for instance, if entries in the factor use and ownership matrices are zero then value quantities are derived from the transaction values in the corresponding sub matrices of the SAM. The second series of additional data are the elasticities of substitution for imports and exports relative to domestic commodities, the elasticities of substitution for the CES production functions, the income elasticities of demand for the linear expenditure system and the Frisch (marginal utility of income) parameters for each household, and factor mobility and household migration elasticities.

All the data are accessed by the model from data recorded in Excel and GDX (GAMS data exchange) files. All the data recorded in Excel are converted into GDX format as part of the model.

5 These and other reserved names are specified below as part of the description of the model.

A key design principle of the model is that this is a ‘template’ model, where the term ‘template’ means that the model has been compiled with the expectation that users of the model are likely to, and/or should, make changes to the model to customise the model to the specific circumstances of the economy being studied and/or the policy issues be simulated.

2. The Computable General Equilibrium Model

The model is a member of the class of single country CGE models that are descendants of the approach to CGE modeling described by Dervis *et al*., (1982). More specifically, the implementation of this model, using the GAMS (General Algebraic Modeling System) software, is a direct descendant and development of models devised in the late 1980s and early 1990s, particularly those models reported by Robinson *et al*., (1990), Kilkenny (1991) and Devarajan *et al*., (1994). The model is a SAM based CGE model, wherein the SAM serves to identify the agents in the economy and provides the transactions value database with which the model is calibrated. Since the model is SAM based it contains the important assumption of the law of one price, i.e., prices are common across the rows of the SAM.[6] The SAM also serves an important organisational role since the groups of agents identified by the SAM structure are also used to define sub-matrices of the SAM for which behavioural relationships need to be defined. As such the modeling approach has been influenced by the 'SAM Approach to Modeling' (Pyatt, 1989; Drud *et al*., 1986).

The description of the model proceeds in five stages. The first stage is the identification of the behavioural relationships; these are defined by reference to the sub matrices of the SAM within which the associated transactions are recorded. The second stage is definitional and involves the identification of the components of the transactions recorded in the SAM, while giving more substance to the behavioural relationships, especially with those governing inter-institutional transactions, and in the process defining the notation. The third stage uses figures to explain the nature of the price and quantity systems for commodity and activity accounts that are embodied within the model. In the fourth stage an algebraic statement of the model is provided. A full listing of the parameters and variables contained within the model are in Appendix 1.[7] Finally, in the fifth stage, there is a discussion of the default and optional macroeconomic closure and market clearing rules available within the model.

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6 The one apparent exception to this is for exports. However, the model implicitly creates a separate set of export commodity accounts and thereby preserves the 'law of one price', hence the SAM representation in the text is a somewhat condensed version of the SAM used in the model (see McDonald, 2007).

7 The model includes specifications for transaction valuess that were zero in the SAM. This is an important component of the model. It permits the implementation of policy experiments with exogenously imposed changes that impact upon transactions that were zero in the base period.

2.1 Behavioural Relationships

While the accounts of the SAM determine the agents that can be included within the model, and the transactions recorded in the SAM identify the transactions that took place, the model is defined by behavioural relationships. The behavioural relationships in this model are a mix of non-linear and linear relationships that govern how the model's agents will respond to exogenously determined changes in the model's parameters and/or variables. Table 2 summarises these behavioural relationships by reference to the sub matrices of the SAM.

Households are assumed to choose the bundles of commodities they consume to maximise utility subject to utility functions that are nested CES and Stone-Geary, where the arguments in the Stone-Geary functions are, typically, aggregates. For a developing country a Stone-Geary function may be generally preferable because it allows for subsistence consumption expenditures, which is an arguably realistic assumption when there are substantial numbers of poor and very poor consumers.[8] But the assumption that households define subsistence consumption requirements at the level of the individual commodity, however disaggregated the commodity accounts are in the data/model, is both highly restrictive and unrealistic. It is realistic however to assume that households (of all sorts) will have subsistence consumption requirements across 'broad' commodity groups, e.g., food, while within those commodity groups households may elect to substitute between commodities (natural commodities) of the 'broad' groups, e.g., between different grains (wheat, rice, etc.,) and between vegetable and meat commodities. Consequently, this model includes nested CES and LES utility functions that at the top (LES) level involve substitution between 'broad' commodity groups, subject to subsistence consumption constraints on these 'broad' groups, while at the lower-level households are willing, and able, to substitute between the component commodities that make up the 'broad' commodity groups. The model includes facilities for the user to define the 'broad' commodity groups and the components of each group, thus the user could define 'food' as a broad commodity group that is a composite formed from different natural food commodities, e.g., meat, grains, fruits, etc. Another alternative would be to distinguish between commodities that are purchased on the market and those produced at home to produce a composite commodity consumed by households, e.g.,

8 A Stone-Geary function reduces to a Cobb-Douglas function given appropriate specification of the parameters.

composite wheat made up of market and home-produced wheat where the types of wheat can be distinguished by time and/or place and/or variety.

The households choose their consumption bundles of natural commodities from a set of ‘composite’ commodities that are aggregates of domestically produced and imported commodities. These ‘composite’ commodities are formed as Constant Elasticity of Substitution (CES) aggregates that embody the presumption that domestically produced and imported commodities are imperfect substitutes. The optimal ratios of imported and domestic commodities are determined by the relative prices of the imported and domestic commodities. This is the so-called Armington ‘insight’ (Armington, 1969), which allows for product differentiation via the assumption of imperfect substitution (see Devarajan *et al*., 1994). The assumption has the advantage of rendering the model practical by avoiding the extreme specialisation and price fluctuations associated with other trade assumptions, e.g., the Salter/Swan or Australian model. In this model the country is assumed to be a price taker for all imported commodities.

Domestic production uses an *n*-stage production process. In the first stage aggregate ‘value added’, defined as an aggregate of ‘factor’ inputs, composed of primary inputs and selected commodity (here called factor commodities) inputs and an aggregate of non-factor inputs (commodities) are combined using either CES or Leontief technologies to generate the outputs of activities. If a CES specification is chosen then the proportion of aggregate factor and non-factor inputs vary with the (composite) prices of the aggregates, while if a Leontief specification is chosen then aggregate factor and non-factor inputs are in fixed proportions. At the second level aggregate non-factor inputs are generated using Leontief technology so that non-factor input demands are in fixed proportions relative to aggregate non-factor inputs of each activity. For ‘factor’ inputs the user can define a substantial number of additional levels for the production system depending on the configuration of sets set by the user: the number of levels can be activity specific. In a simple configuration the second level may combine capital with aggregate labour and land, where at the third level aggregate labour is an aggregation of different types of labour and aggregate land is an aggregation of land and fertilizer. It should be noted that for each extra level added to the production system it is necessary to define additional substitution elasticities for which data are unlikely to be available; consequently, there is a trade-off between making the production system 'more realistic' but reducing the extent of necessary assumptions.

This system of generalised nesting of CES functions produces flexible functional forms that are also tractable and well-behaved. It is also 'flexible' in the sense that it subsumes the production systems that characterised previous variants of the STAGE model, e.g., STAGE_EN (an energy model), STAGE_W (a water model), etc., that can now be included simultaneously in a single model variant.

The activities are defined as multi-product activities that produce combinations of commodity outputs. The model allows for a range of different assumptions governing the output mix produced by each activity. The first is a pure by-product assumption whereby the proportionate combinations of commodity outputs produced by each activity/industry remain constant; hence for any given vector of commodities demanded there is a unique vector of activity outputs that must be produced.[9] Alternatively, activities can adjust their output mixes in the response to changes in the relative (basic) prices of domestically produced commodities using CET technologies. The user can assign some activities to each of these two alternatives. The total supply of domestically produced commodities across activities can be defined in two ways: first the commodities can be differentiated by domestic activity and then aggregated using CES technologies or the commodities can be assumed to be homogenous – this latter assumption requires that the users configure the model so as to define the scale of output of the homogenous commodities, e.g., electricity in an energy model, from different activities.

The vector of commodities demanded is determined by the domestic demand for domestically produced commodities and export demand for domestically produced commodities. Using the assumption of imperfect transformation between domestic demand and export demand, in the form of a Constant Elasticity of Transformation (CET) function, the optimal distribution of domestically produced commodities between the domestic and export markets is determined by the relative prices on the alternative markets. The model can be specified as a small country, i.e., price taker, on all export markets, or selected export commodities can be deemed to face downward sloping export demand functions, i.e., a large country assumption.

9 This specification is found in the IFPRI standard model (Lofgren *et al*., 2001).

Table 2 **Behavioural Relationships for the Standard Model**

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital	RoW	Total	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Nested CES and Stone-Geary Utility Functions	Fixed in Real Terms	Fixed in Real Terms and Export Taxes	Fixed Shares of Savings	Commodity Exports	Commodity Demand	Consumer Commodity Price Prices for Exports
Activities	Domestic Production	0	0	0	0	0	0	0	Constant Elasticity of Substitution Production Functions	
Factors	0	Factor Demands (CES)	0	0	0	0	0	Factor Income from RoW	Factor Income	
Households	0	0	Variable Shares of Factor Income	Fixed shares of income	Fixed Shares of Dividends	Fixed (Real) Transfers	0	Remittances	Household Income	
Enterprises	0	0	Variable Shares of Factor Income	0	0	Fixed (Real) Transfers	0	Transfers	Enterprise Iincome	
Government	Tariff Revenue Domestic Product Taxes	Indirect Taxes on Activities	Variable Shares of Factor Income Direct Taxes on Factor Income	Direct Taxes on Household Income	Fixed Shares of Dividends Direct Taxes on Enterprise Income	0	0	Transfers	Government Income	
Capital	0	0	Depreciation	Household Savings	Enterprise Savings	Government Savings (Residual)	0	Current Account 'Deficit'	Total Savings	
Rest of World	Commodity Imports	0	Variable Shares of Factor Income	0	0	0	0	0	Total 'Expenditure' Abroad	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Enterprise Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad		
	Producer Commodity Prices Domestic and World Prices for Imports	Value Added Prices								

The model includes code for the endogenous determination of the functional distribution of income. Specifically factor supplies are defined by reference to their ownership by different domestic (households, enterprises, and government) and foreign institutions. In its simplest form this formulation defines the quantities of factors supplied by each institution as fixed and equal to the quantities owned by each institution: this requires that factors and institutions cannot transition between categories and that there is full employment and hence the functional distribution of income is in fixed proportions. However, in this model variant the quantities of factors supplied and owned by each institution can change and hence the functional distribution of income must be defined by variables. The most common application of this, in comparative static applications, is in the context surplus labour whereby some institutions/households may be able to supply more labour; if this happens then the share of labour supplied by each institution/household may change and hence the functional distribution of the income from that factor should change. Other applications include circumstances where there is a labour-leisure trade-off at the level of the utility functions of households and in dynamic applications where patterns of capital accumulation, and hence ownership, vary across institutions.

The other behavioural relationships in the model are generally linear. A few features do however justify mention. First, all the tax rates are declared as variables with various adjustment and/or scaling factors that are declared as variables or parameters according to how the user wishes to vary tax rates. If a fiscal policy constraint is imposed, then one or more of the sets of tax rates can be allowed to vary equiproportionately and/or additively to define a new vector of tax rates that is consistent with the fiscal constraint. Relative tax rates can also be adjusted by the settings chosen by the user. Similar adjustment and/or scaling factors are available for several key parameters, e.g., household and enterprise savings rates and inter-institutional transfers. Second, technology changes can be introduced through changes in the activity specific efficiency variables – adjustment and/or scaling factors are also available for the efficiency parameters. Third, the proportions of current expenditure on commodities defined to constitute subsistence consumption can be varied. Fourth, although a substantial proportion of the sub matrices relating to transfers, especially with the rest of the world, contain zero entries, the model allows changes in such transfers, e.g., aid transfers to the government from the rest of the world may be defined equal to zero in the database but they can be made positive, or even negative, for model simulations. And fifth, the model is set up with a range of flexible macroeconomic closure rules and market clearing conditions. While the base model has a standard neoclassical model closure, e.g., full employment, savings driven investment and a floating exchange rate, these closure conditions can all be readily altered.

2.2 Transaction Relationships

The transactions relationships are laid out in Table 3, which is in two parts. The prices of domestically consumed (composite) commodities are defined as PQD_c, and they are the same irrespective of which agent purchases the commodity, except for final demand by households on which VAT can be levied. The quantities of commodities demanded domestically are divided between intermediate demand, $QINTD_c$, and final demand, with final demand further subdivided between demands by households, QCD_c (natural commodities) and $QCD2_{cag}$ (aggregate commodities from the nested utility functions), enterprises, $QENTD_c$, government, QGD_c, investment, $QINVD_{c,i}$, and stock changes, $dstocconst_c$. The value of total domestic demand, at purchaser prices, is therefore $PQD_c * QQ_c$. Consequently, the decision to represent export demand, QE_c, as an entry in the commodity row is slightly misleading, since the domestic prices of exported commodities, $PE_c = PWE_c * ER$, do not accord with the law of one price. The representation is a space saving device that removes the need to include separate rows and columns for domestic and exported commodities.[10] The price wedges between domestic and exported commodities are represented by export duties, TE_c, that are entered into the commodity columns. Commodity supplies come from domestic producers who receive the common prices, PXC_c, for outputs irrespective of which activity produces the commodity, with the total domestic production of commodities being denoted as QXC_c. The inclusion of the possibility of multi-product activities, using CET functions, means that activity specific prices ($PXAC_{a,c}$) and quantities ($QXAC_{a,c}$) must be allowed for commodities. Commodity imports, QM_c, are valued carriage insurance and freight (cif) paid, such that the domestic price of imports, PM_c, is defined as the world price, PWM_c, times the exchange rate, ER, plus an adjustment for *ad valorem* import duties, TM_c. All domestically consumed

10 In this model the allocation by domestic producers of commodities between domestic and export markets is made on the supply side; implicitly there are two supply matrices – supplies to the domestic market and supplies to the export market.

commodities are subject to a variety of product taxes, sales taxes, TS_c, excise taxes, TEX_c. and value added taxes, TV_c. Other taxes can be readily added.

Domestic production activities receive average prices for their output, PX_a, that are determined by the commodity composition of their outputs. Since activities produce multiple outputs their outputs can be represented as an index, QX_a, formed from the commodity composition of their outputs. In addition to intermediate inputs, activities also purchase primary inputs, $FD_{f,a}$, for which they pay average prices, WF_f. To create greater flexibility the model allows the price of each factor to vary according to the activity that employs the factor. Finally, each activity pays production taxes, the rates, TX_a, for which are proportionate to the value of activity outputs.

The model allows for the domestic use of both domestic and foreign owned factors of production, and for payments by foreign activities for the use of domestically owned factors. Factor incomes therefore accrue from payments by domestic activities and foreign activities, $factwor_f$, where payments by foreign activities are assumed exogenously determined and are denominated in foreign currencies. After allowing for depreciation, $deprec_f$, and the payment of factor taxes, TF_f, the residual factor incomes, $YFDIST_f$, are divided between domestic institutions (households, enterprises and government) and the rest of the world in fixed proportions.

Households receive incomes from factor rentals and/or sales ($INSVA_{h,f}$), inter household transfers, $HOHO_{,h}$, transfers from enterprises, $HOENT_{h,e}$, and government, $hogovconst_h$, and remittances from the rest of the world, $howor_h$, where remittances are defined in terms of the foreign currency. Household expenditures consist of payments of direct/income taxes, TY_h, after which savings are deducted, where the savings rates, SHH_h, are fixed exogenously in the base configuration of the model. The residual household income is then divided between inter household transfers and consumption expenditures, with the pattern of consumption expenditures determined by the household utility functions.

Table 3 Transactions Relationships for the Standard Model

	Commodities	Activities	Factors	Households
Commodities	0	$(PQD_c * QINTD_c)$	0	$(PQCD_{cles} * QCD_{cles})$ $(PQCD_{cag} * QCD2_{cag})$
Activities	$(PXC_c * QXC_c);(PX_a * QX_a)$ $(PXAC_{a,c} * QXAC_{a,c})$	0	0	0
Factors	0	$(WF_f * FD_{f,a})$	0	0
Households	0	0	$\sum_f INSVASH_{h,f}$	$HOHO_{h,hh}$
Enterprises	0	0	$\sum_f INSVASH_{e,f}$	0
Government	$(TM_c * PWM_c * QM_c * ER)$ $(TE_c * PWE_c * QE_c * ER)$ $(TV_c * PQD_c * QCD_c)$ $(TS_c * PQS_c * QQ_c)$ $(TEC_c * PQS_c * QQ_c)$	$(TX_a * PX_a * QX_a)$	$\sum_f INSVASH_{gt,f}$ $TF_f * YFDISP_f$	$(TY_h * YH_h)$
Capital	0	0	$\sum_f deprec_f$	$HSAV_h$
Rest of World	$(PWM_c * QM_c * ER)$	0	$\sum_f INSVASH_{w,f}$	0
Total	$(PQD_c * QQ_c)$	$(PX_a * QX_a)$	YF_f	YH_h

Table 3 (cont) Transactions Relationships for the Standard Model

	Enterprises	Government	Capital	RoW	Total
Commodities	$\left(PQD_c * QENTD_c\right)$	$\left(PQD_c * QGD_c\right)$	$\left(PQD_c * QINVD_{c,i}\right)$ $\left(PQD_c * dstocconst_c\right)$	$\left(PWE_c * QE_c * ER\right)$	$\left(PQD_c * QQ_c\right)$
Activities	0	0	0	0	$\left(PX_a * QX_a\right)$
Factors	0	0	0	$\left(factwor_f * ER\right)$	YF_f
Households	$HOENT_{h,e}$	$\left(hogovconst_h * HGADJ\right)$	0	$\left(howor_h * ER\right)$	YH_h
Enterprises	0	$\left(entgovconst * EGADJ\right)$	0	$\left(entwor * ER\right)$	$EENT$
Government	$GOVENT_e$ $\left(TYE * YE\right)$	0	0	$\left(govwor * ER\right)$	EG
Capital	$ENTSAV_e$	$\left(YG - EG\right)$	0	$\left(CAPWOR * ER\right)$	$TOTSAV$
Rest of World	0	0	0	0	Total 'Expenditure' Abroad
Total	YE	YG	$INVEST$	Total 'Income' from Abroad	

The enterprise accounts receive income from factor sales (*INSVA*), primarily in the form of retained profits,[11] transfers from government, *entgovconst*, and foreign currency denominated transfers from the rest of the world, *entwor*. Expenditures then consist of the payment of direct/income taxes, *TYE*, consumption, which is assumed fixed in real terms,[12] and savings, which are defined as a residual, i.e., the difference between income, *YE*, and committed expenditure, *EENT*. There is an analogous treatment of government savings, i.e., the internal balance, which is defined as the difference (residual) between government income, *YG*, and committed government expenditure, *EG*. The enterprise accounts are defined to include privately and publicly owned (parastatals) incorporated business enterprises and other institutions, e.g., charities, that may be economically relevant.[13]

In the absence of a clearly definable set of behavioural relationships for the determination of government consumption expenditure, the quantities of commodities consumed by the government are fixed in real terms, and hence government consumption expenditure will vary with commodity prices.[14] Transfers by the government to other domestic institutions are fixed in nominal terms, although there is a facility to allow them to vary, e.g., with consumer prices. On the other hand, government incomes can vary widely. Incomes accrue from the various tax instruments (import and export duties, sales, production and factor taxes, and direct taxes), that can all vary due to changes in the values of production, trade, and consumption, and from factors (*INSVA*). The government also receives foreign currency denominated transfers from the rest of the world, *govwor*, e.g., aid transfers.

Domestic investment demand consists of fixed capital formation, $QINVD_{c,i}$, and stock changes, $dstocconst_c$. The comparative static nature of the model and the absence of a capital composition matrix underpin the assumption that the commodity composition of fixed capital formation is fixed by the type of capital good, while a lack of information means that stock changes are assumed invariant. However, the value of fixed capital formation will vary with

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11 Hence the model contains the implicit presumption that the proportions of profits retained by incorporated enterprises are constant.

12 Hence consumption expenditure is defined as the fixed volume of consumption, $QENTD_c$, times the variable prices. It requires only a simple adjustment to the closure rules to fix consumption expenditures. Without a utility function, or equivalent, for enterprises it is not possible to define the quantities consumed as the result of an optimisation problem.

13 Typically incorporated business enterprises are not reported to engage in consumption expenditures, however other domestic institutions may do so, hence the inclusion of final demand by enterprises in the model.

14 The closure rules allow for the fixing of government consumption expenditure rather than real consumption.

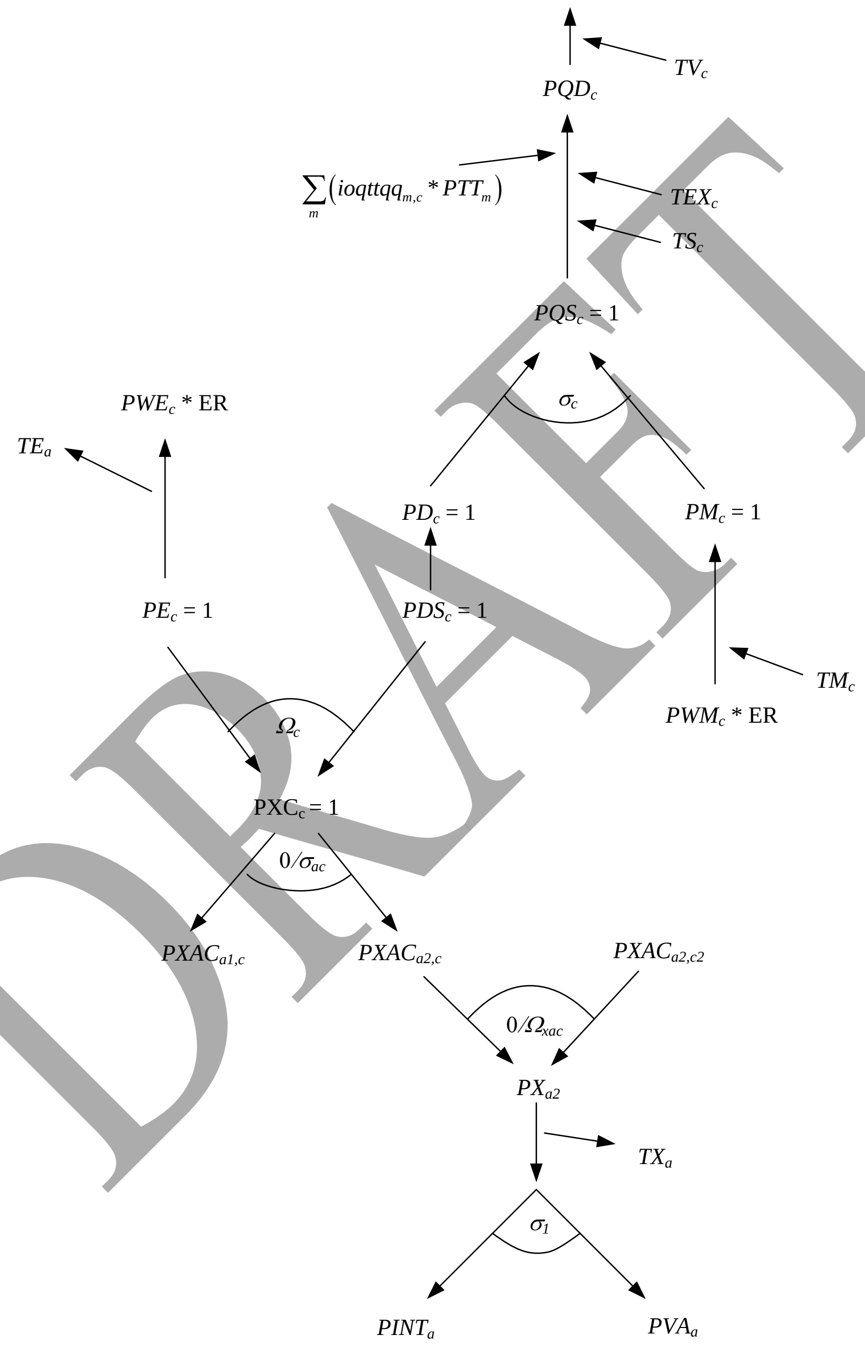
commodity prices while the volume of fixed capital formation can vary both because of the volume of savings changing or changes in exogenously determined parameters. In the base version of the model domestic savings are made up of savings by households, enterprises, the government (internal balance) and foreign savings, i.e., the balance on the capital account or external balance, *CAPWOR*. The various closure rules available within the model allow for different assumptions about the determination of domestic savings, e.g., flexible versus fixed savings rates for households, and value of ‘foreign’ savings, e.g., a flexible or fixed exchange rate.

Incomes to the rest of the world account, i.e., expenditures by the domestic economy in the rest of the world, consist of the values of imported commodities and factor services. On the other hand, expenditures by the rest of the world account, i.e., incomes to the domestic economy from the rest of the world, consist of the values of exported commodities and NET transfers by institutional accounts. All these transactions are subject to transformation by the exchange rate. In the base model the balance on the capital account is fixed at some target value, denominated in foreign currency terms, e.g., at a level deemed equal and opposite to a sustainable deficit on the current account, and the exchange rate is variable. This assumption can be reversed, where appropriate, in the model closure.

2.3 Core Price and Quantity Relationships

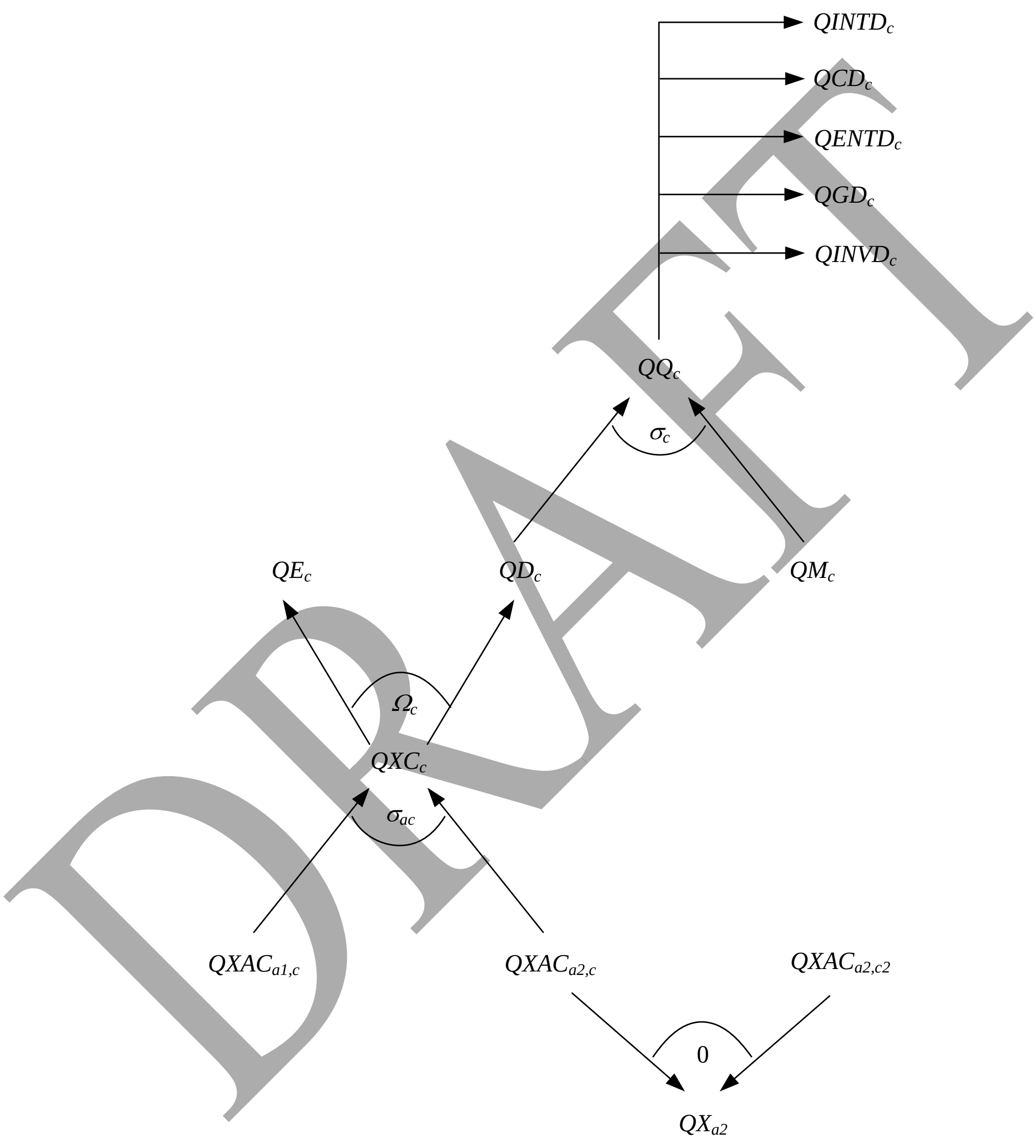
Figures 1 and 2 provide further detail on the interrelationships between the prices and quantities for commodities and activities. The supply prices of the composite commodities (PQS_c) are defined as the weighted averages of the domestically produced commodities that are consumed domestically (PD_c) and the domestic prices of imported commodities (PM_c), which are defined as the products of the world prices of commodities (PWM_c) and the exchange rate (ER) uplifted by *ad valorem* import duties (TM_c). These weights are updated in the model through first order conditions for optima. The average prices exclude sales taxes, and hence must be uplifted by (*ad valorem*) sales and excise taxes (TS_c , TEX_c), and possibly other tax instruments, and by trade and transport margins ($ioqttqq_{m,a} * PTT_m$) to reflect the composite consumer price (PQD_c) and the VAT inclusive prices ($PQCD_c$). The producer prices of commodities (PXC_c) are similarly defined as the weighted averages of the prices received for domestically produced commodities sold on domestic and export (PE_c) markets. These weights are updated in the model through first order conditions for optima. The prices received on the export market are defined as the products of the world price of exports (*PWE_c*) and the exchange rate (*ER*) less any exports duties due, which are defined by *ad valorem* export duty rates (*TE_c*).

Figure 1 Price Relationships for the STAGE Model



The average price per unit of output received by an activity (*PX_a*) is defined as the weighted average of the domestic producer prices (*$PXAC_{a,c}$*), where the weights are constant or variables according to the model configuration. After paying indirect/production/output taxes (TX_a), this is divided between payments to aggregate value added (PVA_a), i.e., the amount available to pay primary inputs, and aggregate intermediate inputs ($PINT_a$). Total payments for intermediate inputs per unit of aggregate intermediate input are defined as the weighted sums of the prices of the inputs (PQD_c).

Figure 3 Quantity Relationships for the STAGE



Total demands for the composite commodities, QQ_c, consist of demands for intermediate inputs, $QINTD_c$, consumption by households, QCD_c, enterprises, $QENTD_c$, and government, QGD_c, gross fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. Supplies from domestic producers, QD_c, plus imports, QM_c, meet these demands; equilibrium conditions ensure that the total supplies and demands for all composite commodities equate. Commodities are delivered to both the domestic and export, QE_c, markets subject to equilibrium conditions that require all domestic commodity production, QXC_c, to be either domestically consumed or exported.

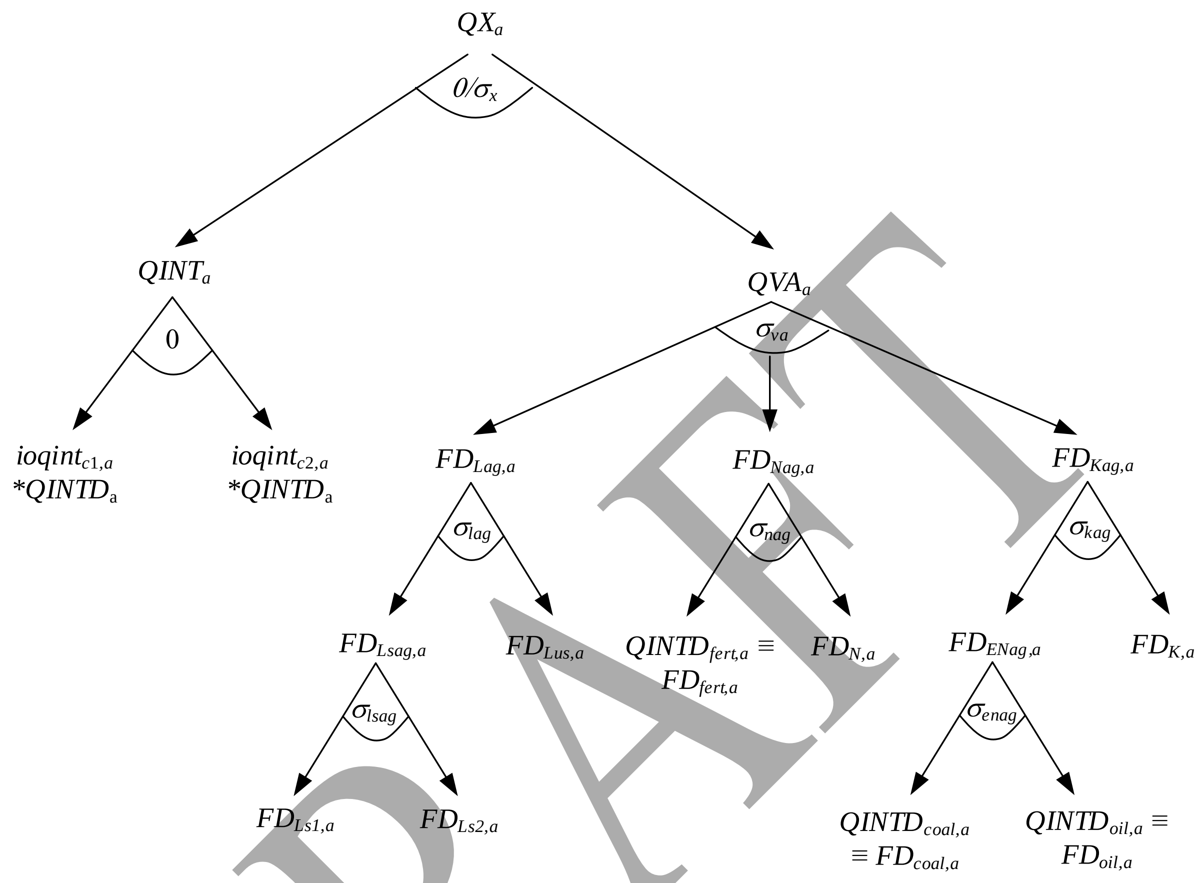
Production Relationships

The production system is set up as a flexible *n*-level nest of CES production functions, with the option of Leontief functions at various levels: the system can be different for each activity. The system is controlled by assigning the memberships of several sets when calibrating the model. This flexibility in the specification of the production system has advantages and disadvantages. On the plus side, it allows the use of one core set of model codes in various contexts, e.g., land fertiliser substitution in agriculture, modelling of water as a produced product and substitution between energy inputs in an energy model. There are negative considerations: first, the model code is more complex and less transparent; second, the set assignments are potentially error prone and ensuring the planned system is the implemented system is less easy; and third, a wider range of options may encourage overelaboration and hence less clarity in the results.

It is important to note that in any nested system, with aggregates being created at the various levels of the system, that the aggregates have no ‘real’ world counterparts. The aggregates are constructs that exist solely to facilitate different degrees of substitutability between the various natural factors and, if in the specification, factor commodities in the system. No one owns these aggregates, and the associated prices are only required to provide solutions to the first-order conditions. Moreover, since these aggregates are not ‘real’ and are not owned they cannot pay taxes, and hence their prices cannot be tax ridden, except to the extent that they encompass taxes paid on natural factors and factor commodities.

Moreover, the plethora of different production systems means that the expositional simplicity associated with a fix production system is lost. Hence, the description here is generic. The system described is for one activity: the system may or may not be same for all activities in a region. The system assumes that the activity uses 5 commodity inputs and five natural/primary factors; three of the commodity inputs will be included in the value-added nest and therefore will be known as factor commodities (*fc*), so the system will effectively have 8 factors (*ff*) and 2 intermediate (commodity) inputs.

Figure 3 Production Quantity System for a Typical Region



Starting with the quantity system illustrated in Figure 3. At the top level aggregate intermediate inputs (*QINT*) are combined with aggregate factor inputs (*QVA*) to produce the output of an activity (*QX*). This top level production function can take either CES or Leontief form, with CES being the default and the elasticities being activity specific.[15] Aggregate intermediate inputs are a Leontief aggregation of the individual commodities used as intermediate inputs, i.e., those commodities that are not factor commodities, where the input-output coefficients (*ioqint*) are defined in terms of input quantities relative to the aggregate intermediate input.[16] The value added production function is a CES function over **aggregate** capital (*Kag*), **aggregate** land (*Nag*) and **aggregate** labour (*Lag*), with the elasticities being activity specific: these aggregates are created at the third level. Aggregate land is defined as a CES aggregate of the natural factor land (*N*)[17] and the commodity factor fertiliser (*fert*), which

15 The model allows the user to specify the share of intermediate input cost in total cost below which the Leontief alternative is automatically selected. The user also has the option to make activity and region-specific decisions about the selection of CES or Leontief forms.

16 A more flexible specification, with substitution possibilities between intermediate inputs, exists but is not included in ANARRES.

17 'Natural' land may be made up of various 'types' of land, e.g., Agro-ecological zones.

is assumed to be a land saving input. The nests for aggregate labour and capital have two levels of nest. Aggregate labour (*Lag*) is defined as a CES aggregate of the natural factor unskilled labour (*Lus*) and aggregate skilled labour (*LSag*), and aggregate skilled labour is defined as a CES aggregate of two types of skilled labour (*Ls1* and *Ls2*); the elasticities are activity and level specific. Finally, aggregate capital (*Kag*) is defined as a CES aggregate of the natural factor capital (*K*) and aggregate energy (*ENag*), and aggregate energy is defined as a CES aggregate of two types of fossil fuels (*coal* and *oil*); the elasticities are activity and level specific.[18]

In the price system for production, the price of aggregate intermediate commodities is defined as the input (coefficient) weighted average of the intermediate commodities prices (*PQD**) where the * indicates that the price is the common purchaser (*PQD*) plus any agent commodity specific taxes. The price of value-added (*PVA*) is determined by the share weighted average of the prices of the three (labour, capital and land) aggregate inputs (WFA); note that these aggregate inputs are not subject to any taxes (hence no *). The price of aggregate land is determined by the by the share weighted average of the prices of the factor commodity (*fert*) and the natural factor land, where *PQD** is the price of the factor commodity, which is composed of the common purchaser price (*PQD*) plus any agent commodity specific taxes, as indicated by the *. The price of aggregate capital is determined by the share weighted average of the prices of the natural factor capital (*WFA*) and the price of aggregate energy input (WFA_{ENag}), which is a share weighted average of the prices of the factor commodities coal and oil capital. Again, and asterisks indicate that a price includes any agent specific taxes. In a similar manner, the price of aggregate labour is determined by the share weighted average of the prices of the natural factor unskilled labour (*WFA*) and the price of aggregate skilled labour (*WFA*), which is a share weighted average of the prices of the two different types of skilled labour (*Ls1* and *Ls2*). actor commodities coal and oil capital.

A critical feature of the price system is the distinction between the prices paid by activities and prices received by the natural factors and commodity factors. The price paid by the activities, indicated by an *, are the relevant prices for the selection of commodity input mixes, i.e., the prices that enter the respective first-order conditions. Whereas the prices received are those that are important to decision making by the agents that own the factors.

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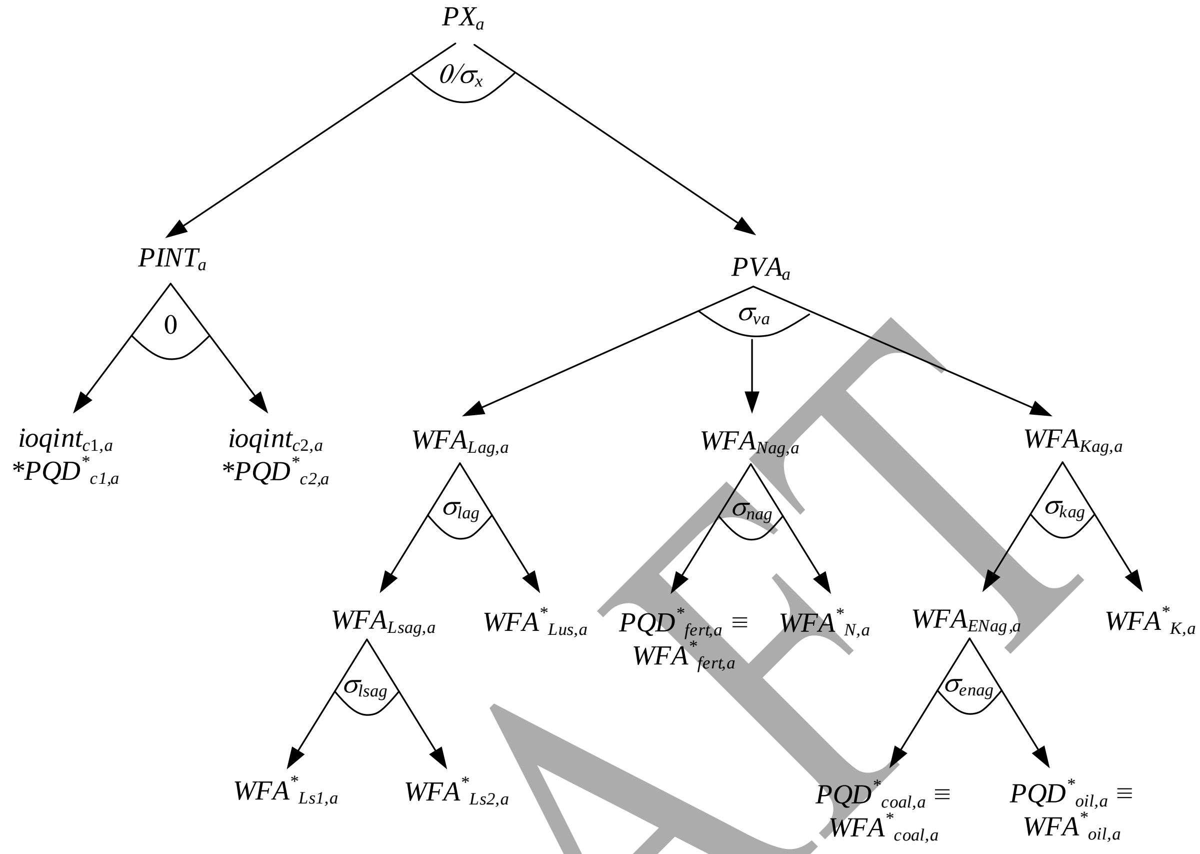
[18] In practical applications it would be expected that the aggregate energy nest is more complex, e.g., aggregate energy is formed from electricity and non-electricity inputs with non-electricity inputs being formed from fossil fuels.

Figure 4 A Production Price System for a Typical Region



The price of activity output (*PX*) a share weighted average of the prices of aggregate intermediates (*PINT*) and aggregated natural factors and commodity factors (*PVA*), uplifted by any the production tax rates (*TX*). The activity prices are a one-to-one mapping of the commodity prices received by activities (*PXC*); this is a consequence of the supply matrix being a square diagonal matrix.

Production relationships by activities are defined by a series of nested Constant Elasticity of Substitution (CES) production functions. In the base version there is a two level production nest, which, in quantity terms, is illustrated in Figure 4. For illustration purposes only, two intermediate inputs and five primary inputs ($FD_{k,a}$, $FD_{l1,a}$, $FD_{l2,a}$, $FD_{l3,a}$ and $FD_{n,a}$) together with one aggregate primary input ($FD_{l,a}$) are identified. Activity output is a CES aggregate of the quantities of aggregate intermediate inputs (*QINT*) and value added (*QVA*), while aggregate intermediate inputs are a Leontief aggregate of the (individual) intermediate inputs and aggregate value added is a CES aggregate of the quantities of two primary and one aggregate inputs demanded by each activity (*FD*). The aggregate primary input is then a CES aggregate of the different primary factors at the third level. The allocation of the finite supplies of factors (*FSI*) between competing activities depends upon relative factor prices via first order conditions for optima.

Figure 4 Production Relationships for the STAGE Model: Quantities

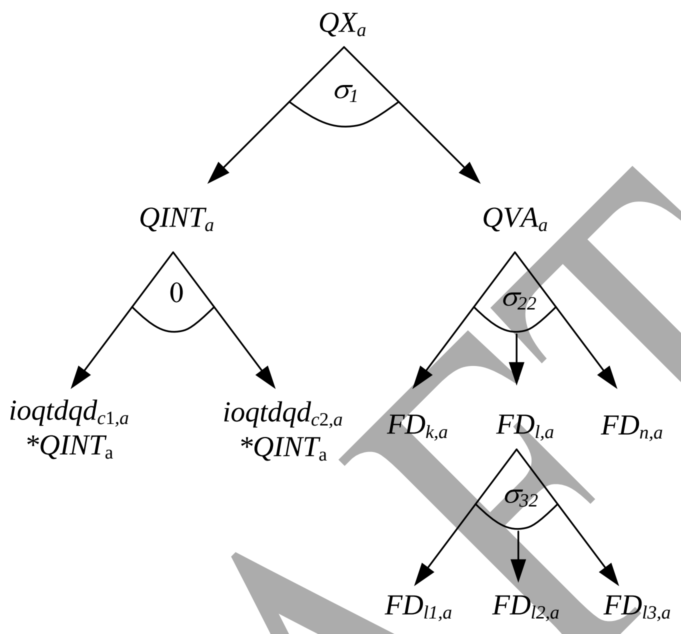
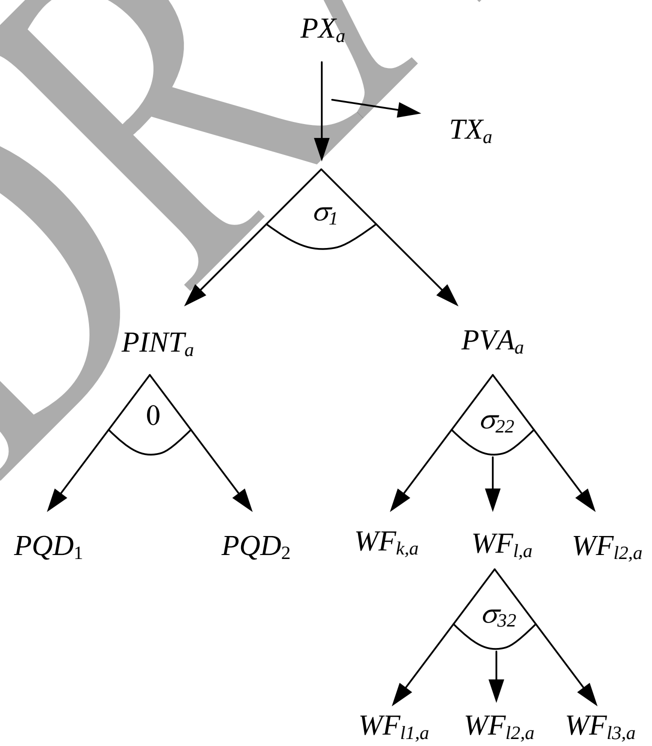


Figure 5 Production Relationships for the STAGE Model: Prices

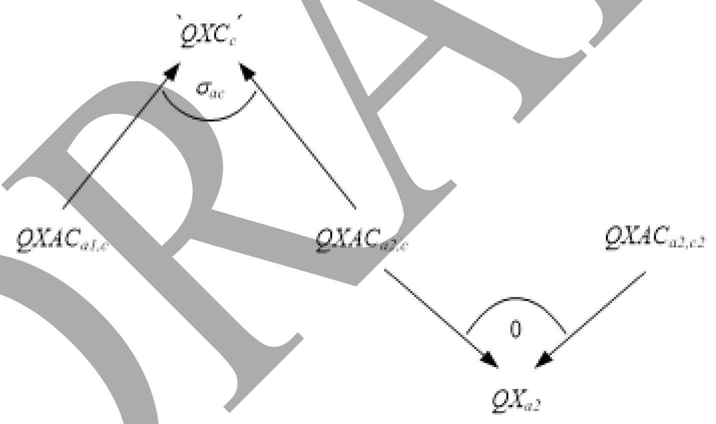


The price relations for the production system are illustrated in Figure 5. Note how the prices paid for intermediate inputs (*PQD*) are the same as paid for final demands, i.e., a ‘law’ of one price relationship holds across all domestic demand. Note also that factor prices are factor and activity specific ($WF_{f,a}$).

<u>Commodity and Activity Output System</u>

The option for multiple product activities means that domestically produced commodities can come from multiple activities, i.e., the total production of a commodity is defined as the sum of the amount of that commodity produced by each activity. The system is flexible with the actual treatment of multi-product activity being activity and commodity specific with the choices controlled by sets. The code allows for commodities produced by different activities to be homogenous or heterogenous and for the output mix by activities being fixed, which is equivalent to a form of by-product assumption, or flexible, which means to mix can change in response to changes in commodity prices.

Figure 6 Quantities for Commodity and Activity Outputs



The domestic production of a commodity (*QXC*) is a CES aggregate of the quantities of that commodity produced by several different activities (*QXAC*), i.e., it is presumed that each activity produces a different (heterogenous) variety of the same commodity, or the simple aggregate of the quantities produced by each activity, i.e., it is presumed that each activity produces an identical (homogeneous) commodity. The output by each activity (*QX*) can be produced in activity specific fixed proportions, i.e., the output of *QXAC* is a Leontief (fixed proportions) aggregate of the output of each activity (*QX*), or a CET aggregate whereby the activity adjusts the output mix according to changes in the relative prices of each commodity it can produce (*QXAC*).

The presence of multiple production activities means that domestically produced commodities can come from multiple activities, i.e., the total production of a commodity is defined as the sum of the amount of that commodity produced by each activity. Hence the domestic production of a commodity (*QXC*) is a CES aggregate of the quantities of that commodity produced by several different activities (*QXAC*), which are produced by each activity in activity specific fixed proportions, i.e., the output of *QXAC* is a Leontief (fixed proportions) aggregate of the output of each activity (*QX*).

<u>Labour Mobility Options</u>

The international standard, i.e., System of National Accounts, for the classification of labour adopts the International Labour Organisation's (ILO) International Standard Classification of Occupations (ISCO-08) that was last endorsed in 2008[19] (https://www.ilo.org/public/english/bureau/stat/isco/docs/groupdefn08.pdf). Consequently, labour force surveys and national accounts report quantities of and payments to labour by occupation categories. The system is well understood and widely adopted. However, the ISCO is essentially a descriptive tool, and this creates some difficulties for the analysis and modeling of labour markets.

Whole economy models typically assume that labour types are rigidly segmented such that, for instance, 'agricultural' workers (ISCO Major Group 6) are precluded from working as 'plant and machine operators' (ISCO Major Group 8) despite there being substantive overlaps between skill sets.[20] This is a restrictive assumption that arguably runs counter to the evidence that, to some extent, workers are mobile across occupations.

The labour mobility options allow the model user to relax this assumption. Specifically, the model assigns ownership of types and quantities of labour to institutions (households and the rest of the world), i.e., factor supplies by institutions (*FSI*), such that each representative household group (RHG) may own a mix of different types and quantities of labour. The model user then defines, pair wise, which labour types by institution can transition between categories and how responsive the transitions are to changes in relative economic incentives, i.e., factor prices.

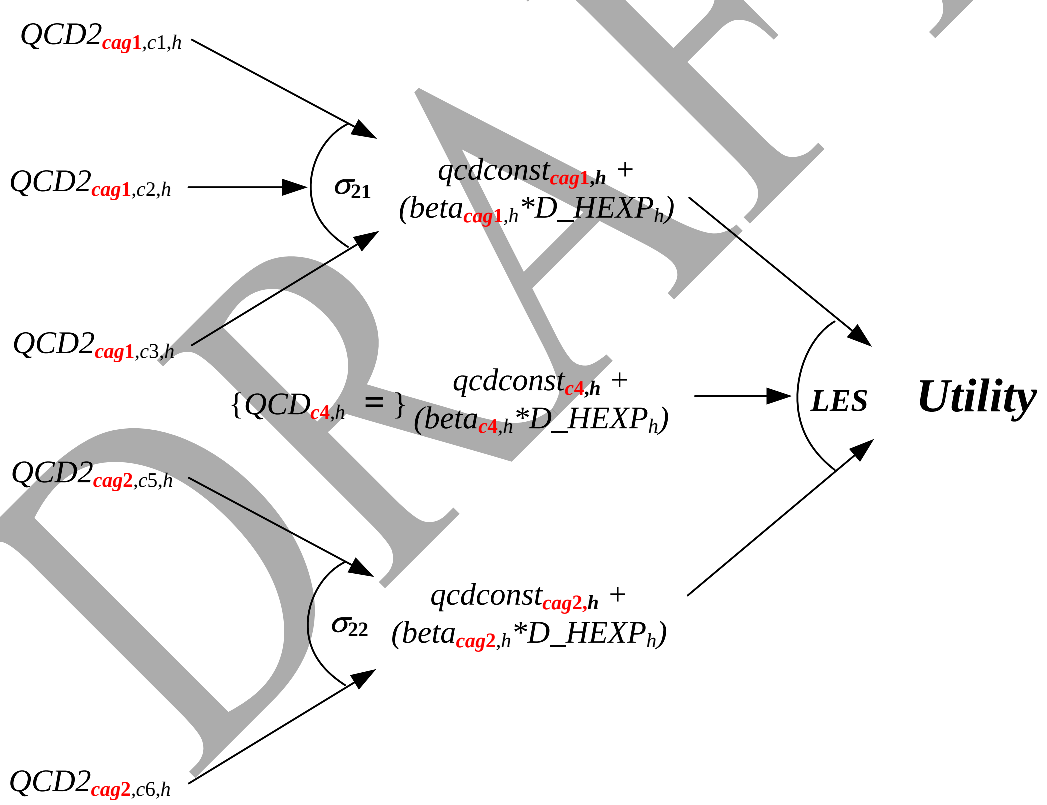
19 The ISCO was first endorsed in 1957 and has been periodically revised.

20 This does not preclude composition of labour forces changing over time due to the occupational characteristics of new entrants to the workforce differing from those of retirees.

Household Utility Relationships

The nested (CES-LES) utility functions have a linear expenditure system (LES) defined over a mix of natural and aggregate commodities. This is illustrated in quantity terms in Figure 6 where the subscript '*cag#*' indicates an aggregate commodity and the subscript '*c#*' a natural commodity. The underlying logic is that each household demands subsistence quantities of certain aggregate commodities, e.g., food, energy, etc., but not necessarily of all-natural commodities, e.g., meat, gas, etc. Thus, the LES utility functions for each household are defined over a mix of aggregate and natural commodities demands for which there are subsistence quantities (*qcdconst*) and marginal budget shares (*beta*) of discretionary household consumption expenditures (*D_HEXP*).

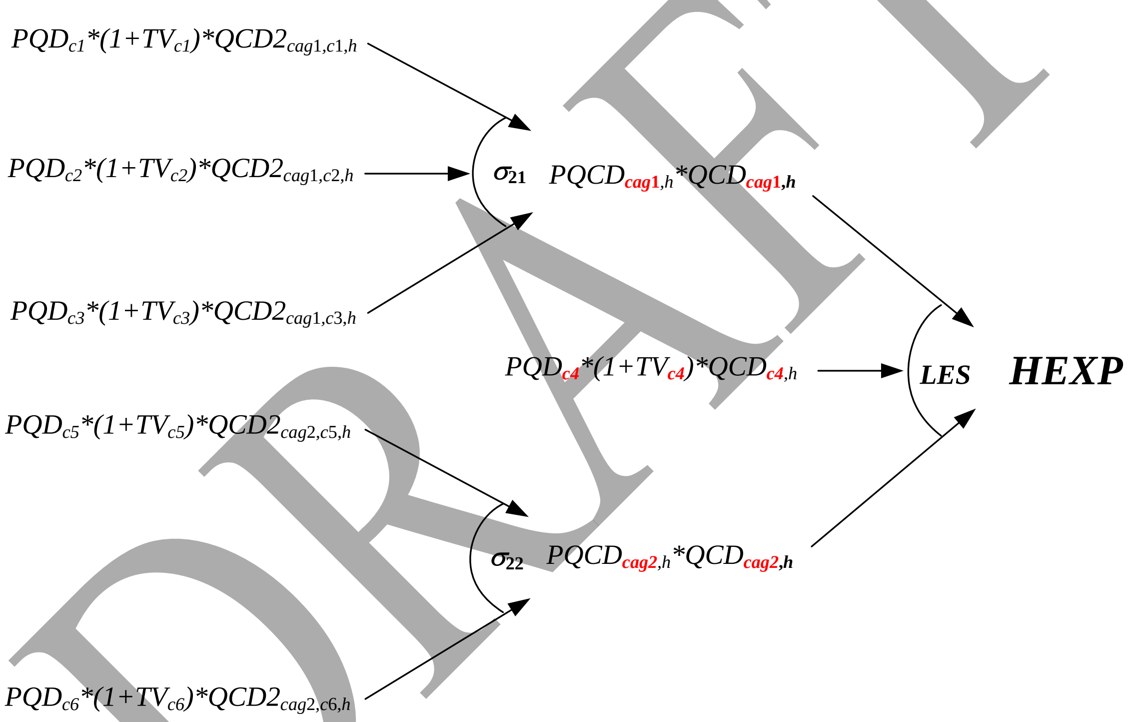
Figure 6 Utility Functions in Quantities



The aggregate commodities are CES aggregates of various natural commodities that are demanded to generate the aggregate. Since each household, *h*, has different preferences, as disclosed by the data, the quantities of each commodity, *c*, used to generate an aggregate, *cag*, the demand for each commodity (*QCD*2) has three arguments. As illustrated in Figure 6 the system is general in the sense that any number of commodities can be used to generate each aggregate and there can be any mix of aggregates and natural commodities in the LES.

The Law of One Price (LOOP) must however be retained. Thus, despite the demand for commodities by each household depending on *c* and *h* the prices paid are only determined by the commodity *c*. However, since the mix of commodities in each aggregate commodity varies by household because the quantities of each natural commodity, the weights, are different for each household. Consequently, the aggregate prices (*PQCD*) are indexed on both the aggregate commodity, *cag*, and the household, *h*. This is illustrated in Figure 7 where the components of the transaction values are identified.

Figure 7 Utility Functions in Transaction Values



Two points deserve emphasis. The prices for aggregate commodities entering into the LES function cannot, by definition, be charged VAT (*TV*) since they have no real equivalent.[21] But the value of the aggregate commodities must be defined so that they include any VAT paid on the natural commodities that make up the aggregate commodities. And second, the requirement for elasticity estimates is both increased and made empirically more difficult. The increase in empirical difficulty is that substitution elasticities are required for the components of the aggregate commodities; the income elasticities of demand are for

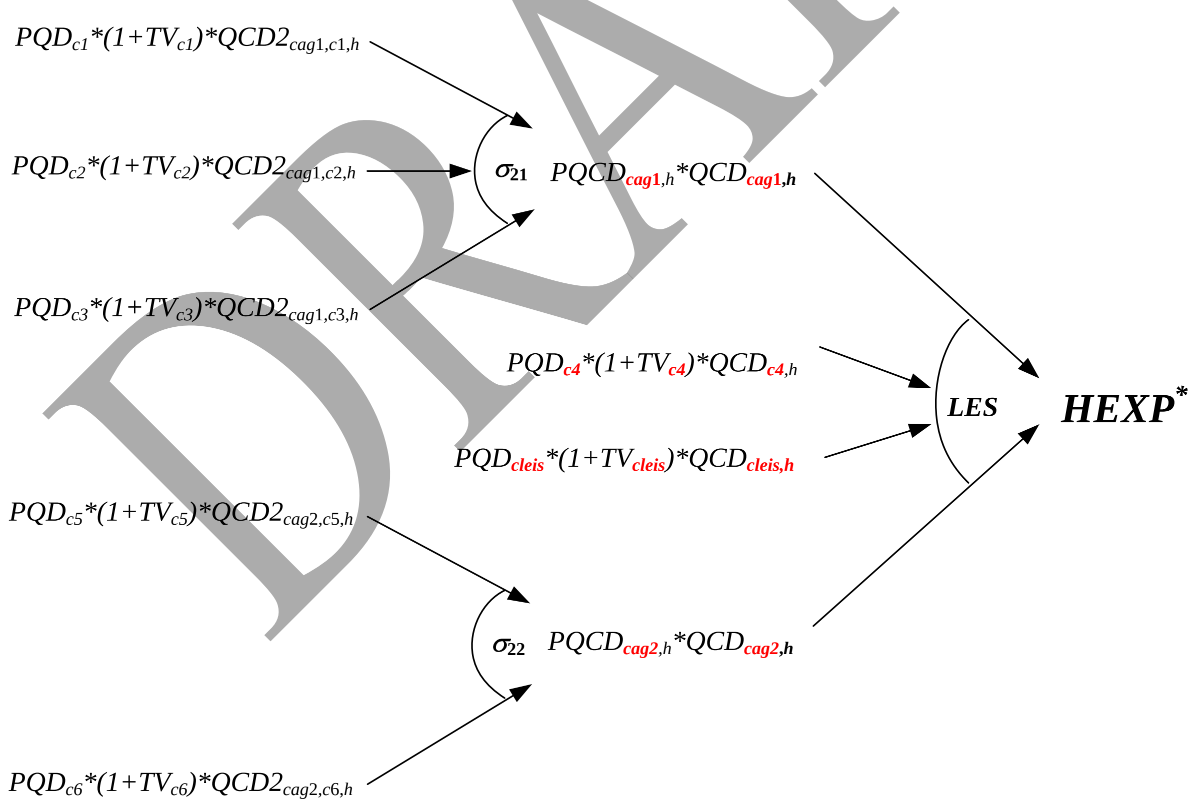
21 The model code appears to include VAT on aggregate commodities in the LES equations in the nest. This is because of how the equations are written; elsewhere in the code the VAT rates on aggregate commodities are fixed at zero. However, it is important to guard against creating a VAT rate on aggregates in policy simulations; to do so the user would need to override controls included to avoid such a possibility.

aggregates, which is the situation encountered with standard implementations of LES functions.

Labour-Leisure Trade Off

Leisure is introduced into the model as 'special' commodities that are RHG specific and can only be produced using labour supplied by the RHG that consumes that type of leisure. Since the compositions and quantities of the labour inputs used to produce leisure are specific to each RHG, each type of leisure has a unique input mix and hence cost of production and price. Thus, the production system is extended to include one leisure activity for each RHG, with each leisure activity producing one RHG specific leisure commodity that can only be produced by that activity. The RHG specific leisure commodities then enter into model utility functions at the LES level, which simply requires an extension to the nested utility functions to include leisure (see Figure 8).

Figure 8 Utility Functions with Leisure



This formulation implies that leisure accounts for a positive marginal budget share (*beta*) so that as RHG consumption expenditures increase so does the demand for leisure, the income effect. The substitution effect depends on the prices of the 'commodities' in the LES nest. Since the prices of the leisure commodities depends solely on the wage rates and labour input shares the behavioural relationships underpinning the substitution effects are the marginal costs of producing leisure by the leisure activities that are linked directly to individual RHG.

Note however that in household consumption variable in Figure 8 (*HEXP**) is different to the household consumption variable in Figure 7 (*HEXP*). This highlights the requirement that the household consumption variable with labour-leisure trade-offs must include the value of leisure. This raises the empirical problem of how to value leisure.

The opportunity cost of labour used in the production of leisure is the marginal wage income foregone; hence the transaction values in the SAM database for labour used in leisure activities are the wage rates for each labour type times the quantities of labour used to produce leisure. Since in this context leisure time is time foregone from the labour market, within the production boundary, by members of the RHGs its valuation avoids the complication associated with defining the production boundary. Specifically, leisure time can only be provided by those persons that enjoy the leisure and the opportunity cost of the time and its market price are identical, and therefore leisure can be given an unambiguous price and hence valuation. Thus, the problem of valuing leisure reduces to deriving estimates of the time 'devoted' to leisure EXCLUDING time taken up providing services outwith the SNA production boundary.[22]

<u>Endogenous Functional Distribution of Income</u>

Any formulation of a CGE model that allows endogenous changes in factor supply requires some method, implicit or explicit, for assigning the changes in factors supply by each institution to the existing structure/pattern of factor supply. In most known CGE models where the code can be verified, e.g., IFPRI, STAGE 1 and PEP, the allocation is implicit: the structure/pattern of factor supply by institution is fixed, i.e., model parameters, which amounts to an assumption that changes in factor supplies are drawn from institutions proportionate to

[22] This should not be interpreted as an endorsement of the SNA's definition of the production boundary. There is a long literature on the issue of the valuation of services provided outside of the SNA's production boundary. In essence the contrasting arguments can be reduced to whether labour used to produce such services should be valued on an opportunity cost basis, i.e., childcare provided by high earners should have a higher valuation that that provided by low earners, or a market prices, i.e., child care should be valued at the cost of hiring nannies.

their supplies in the base period. It is arguable that this assumption, a fixed and exogenously determined functional distribution of income, is generally inappropriate.

A standard assumption is a perfectly elastic supply of labour at the existing wage rate/marginal cost. The assumption that the functional distribution is fixed requires that any labour added/subtracted from the market is drawn/withdrawn equiproportionately from all institutions. Consider the simple case of 2 households, one rich and one poor, both supplying skilled and unskilled labour. It is arguably likely that the rich household will have more skilled labour relative to the poor household, and hence that there is the possibility that any increase in supply of unskilled labour may be biased towards the poor household. However, it may arise, ANY changes in labour supplies are likely to generate some changes in the pattern of factor supply by institution, and that given any systematic behavioural determinants of changes in labour supply changes in the functional distribution of income is highly probable

This model introduces behavioural relationships that ensure the functional distribution of income changes as the patterns of factor supply by institution change.

3. Algebraic Statement of the Model

The model uses a series of sets, each of which is required to be declared and have members assigned. For the majority of the sets the declaration and assignment takes place simultaneously in a single block of code.[23] However, the assignment for a number of the sets, specifically those used to control the modeling of trade relationships is carried out dynamically by reference to the data used to calibrate the model. The following are the basic sets for this model

$$
\begin{aligned}
cc &= \{\text{natural and aggregate commodities}\} \\
c &= \{\text{natural commodities}\} \\
a &= \{\text{activities}\} \\
f &= \{\text{factors}\} \\
ins &= \{\text{domestic institutions}\} \\
insw &= \{\text{domestic institutions and rest of the world}\} \\
h &= \{\text{households}\} \\
g &= \{\text{government}\} \\
e &= \{\text{enterprises}\} \\
i &= \{\text{investment}\} \\
w &= \{\text{rest of the world}\}
\end{aligned}
$$

and for each set there is an alias declared that has the same membership as the corresponding basic set. The notation used involves the addition of a '*p*' suffix to the set label, e.g., the alias for *c* is *cp*.

However, for practical/programming purposes these basic sets are declared and assigned as subsets of a global set, *sac*,

$$sac = \{c, a, f, h, g, e, i, w, total\}.$$

All the dynamic sets relate to the modeling of the commodity and activity accounts and therefore are subsets of the sets *c* and *a*. The subsets are

23 For practical purposes it is often easiest if this block of code is contained in a separate file that is then called up from within the *.gms file. This is how the process is implemented in the worked example.

$$ce(c) = \{\text{export commodities}\}$$
$$cen(c) = \{\text{non-export commodities}\}$$
$$ced(c) = \{\text{export commodities with export demand functions}\}$$
$$cedn(c) = \{\text{export commodities without export demand functions}\}$$
$$cm(c) = \{\text{imported commodities}\}$$
$$cmn(c) = \{\text{non-imported commodities}\}$$
$$cx(c) = \{\text{commodities produced domestically}\}$$
$$cxn(c) = \{\text{commodities NOT produced domestically AND imported}\}$$
$$cd(c) = \{\text{commodities produced AND demanded domestically}\}$$
$$cdn(c) = \{\text{commodities NOT produced AND demanded domestically}\}$$

and members are assigned using the data used for calibration. Additionally, there are some sets, referring to commodities and activities, which are used to control the behavioural equations implemented in specific cases. These are

$$cxac(c) = \{\text{differentiated commodities produced domestically}\}$$
$$cxacn(c) = \{\text{UNdifferentiated commodities produced domestically}\}$$
$$cles(cc) = \{\text{natural and aggregate commodities in top level LES utility functions}\}$$
$$cces(c) = \{\text{natural commodities in second level CES utility functions}\}$$
$$cag(cc) = \{\text{aggregate commodities from second level CES utility functions}\}$$
$$aqx(a) = \{\text{activities with CES aggregation at Level 1}\}$$
$$aqxn(a) = \{\text{activities with Leontief aggregation at Level 1}\}$$
$$acet(a) = \{\text{activities with CET aggregation of commodity outputs}\}$$
$$acetn(a) = \{\text{activities with by-product - fixed proportion - aggregation of commodity outputs}\}$$

and their memberships are set during the model calibration phase.

Finally, a set is declared and assigned for a macro SAM that is used to check model calibration. This set and its members are

$$ss = \{commdty, activity, valuad, hholds, entp, govtn, kapital, world, totals\}.$$

Reserved Names

The model also uses a number of names that are reserved, in addition to those specified in the set statements detailed above. The majority of these reserved names are components of the government set; they are reserved to ease the modeling of tax instruments. The required members of the government set, with their descriptions, are

$$g = \left\{ \begin{array}{lcc} & IMPTAX & \text{Import Taxes} \\ & EXPTAX & \text{Export Taxes} \\ & SALTAX & \text{Sales Taxes} \\ & ECTAX & \text{Excise Taxes} \\ tf, & VATTAX & \text{VAT Taxes} \\ & INDTAX & \text{Indirect Taxes} \\ & FACTTAX & \text{Factor Taxes} \\ & DIRTAX & \text{Direct Taxes} \\ & GOVT & \text{Government} \end{array} \right\}$$

where *tf* is the set of factor use taxes, with one member of *tf* for each member of the set of factors, *f*.

The other reserved names are for the factor account and for the capital accounts. For simplicity the factor account relating to residual payments to factors has the reserved name of *GOS* (gross operating surplus); in many SAMs this account would include payments to the factors of production land and physical capital, payments labeled mixed income and payments for entrepreneurial services. Where the factor accounts are fully articulated *GOS* would refer to payments to the residual factor, typically physical capital and entrepreneurial services.

The capital account includes provision for multiple expenditure accounts relating to investment. All expenditures on stock changes are registered in the account *dstoc*, while all investment expenditures are registered to investment accounts *k***. All incomes to the capital account accrue to the *i_s* account and stock changes are funded by an expenditure levied on the *i_s* account to the *dstoc* account.

Conventions

The equations for the model are set out in eleven ‘blocks’; which group the equations under the following headings ‘trade’, ‘commodity price’, ‘numéraire’, ‘production’, ‘factor’, ‘household’, ‘enterprise’, ‘government’, ‘kapital’, ‘foreign institutions’ and ‘market clearing’. This grouping of equations is intended to ease the reading of the model rather than being a requirement of the model; it also reflects the modular structure that underlies the programme and which is designed to simplify model extensions/developments.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modeling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- The standard prefixes for variable names are: *P* for price variables, *Q* for quantity variables, *E* for expenditure variables, *Y* for income variables, and *V* for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.
- A series of variables are declared that allow for the equiproportionate adjustment of groups of parameters. These variables are named using the convention ***ADJ*, where ** is the parameter series they adjust.
- All parameters are in lower case, except those used to initialise variables.
- Names for parameters are derived using account abbreviations with the row account first and the column account second, e.g., *actcom*** is a parameter referring to the activity:commodity (supply or make) sub-matrix;
- Parameter names have a two or five character suffix which distinguishes their definition, e.g., ***sh* is a share parameter, ***av* is an average and ***const* is a constant parameter;
- The names for all parameters and variables are kept short.

Trade Block Equations

Trade relationships are modeled using the Armington assumption of imperfect substitutability between domestic and foreign commodities. The set of eleven equations are split across two sub-blocks – exports and imports - and provide a general structure that accommodates most eventualities found with single country CGE models. In particular these equations allow for traded and non-traded commodities while simultaneously accommodating commodities that are produced or not produced domestically and are consumed or not consumed domestically and allowing a relaxation of the small country assumption of price taking for exports.

<u>Exports Block</u>

The domestic price of exports (E1) is defined as the product of the world price of exports (*PWE*), the exchange rate (*ER*) and one minus the export tax rate[24]and are only implemented for members of the set *c* that are exported, i.e., for members of the subset *ce*. The cost of transporting commodities in the form of prices of per unit margin services are also included in determining PE_c. The world price of imports and exports are declared as variables to allow relaxation of the small country assumption, and are then fixed as appropriate in the model closure block.

Export Block Equations

$$PE_c = PWE_c * ER * \left(1 - TE_c\right) - \sum_m \left(ioqttqe_{m,c} * PTT_m\right) \qquad \forall ce \tag{E1}$$

$$PE.FX_c = 0.0 \qquad \not\forall ce \tag{E1B}$$

$$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + \left(1 - \gamma_c\right) * QD_c^{rhot_c}\right)^{\frac{1}{rhot_c}} \qquad \forall ce \ \textbf{AND} \ cd \tag{E2}$$

$$QXC.FX_c = 0.0 \qquad \not\forall cx \tag{E2b}$$

$$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{\left(1 - \gamma_c\right)}{\gamma_c}\right]^{\frac{1}{\left(rhot_c - 1\right)}} \qquad \forall ce \ \textbf{AND} \ cd \tag{E3}$$

$$QE.FX_c = 0.0 \qquad \not\forall ce \tag{E3b}$$

$$QXC_c = QD_c + QE_c \qquad \forall \left(cen \ \textbf{AND} \ cd\right) \ \textbf{OR} \ \left(ce \ \textbf{AND} \ cdn\right) \tag{E4}$$

$$QD.FX_c = 0.0 \qquad \not\forall cd \tag{E4b}$$

$$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c}\right)^{-eta_c} \qquad \forall ced \tag{E5}$$

[24] ALL tax rates are expressed as variables. How the tax rate variables are modeled is explained below.

The output transformation functions (E2), and the associated first-order conditions (E3), establish the optimum allocation of domestic commodity output (*QXC*) between domestic demand (*QD*) and exports (*QE*), by way of CET functions, with commodity specific share parameters (γ), elasticity parameters (*rhot*) and shift/efficiency parameters (*at*). The first order conditions define the optimum ratios of exports to domestic demand in relation to the relative prices of exported (*PE*) and domestically supplied (*PD*) commodities. But (E2) is only defined for commodities that are both produced and demanded domestically (*cd*) **and** exported (*ce*). Thus, although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced **and** demanded domestically but **not** exported, and those cases where commodities are produced domestically **and** exported but **not** demanded domestically.

If commodities are produced domestically but **not** exported, then domestic demand for domestically produced commodities (*QD*) is, by definition (E5), equal to domestic commodity production (*QXC*), where the sets *cen* (commodities not exported) and *cd* (commodities produced and demanded domestically) control implementation. On the other hand if commodities are produced domestically but **not** demanded by the domestic output, then domestic commodity production (*QXC*) is, by definition (E4), equal to commodity exports (*QE*), where the sets *ce* (commodities exported) and *cdn* (commodities produced but not demanded domestically) control implementation.

The equations E1 to E4 are sufficient for a general model of export relationships when combined with the small country assumption of price taking on all export markets. However, it may be appropriate to relax this assumption in some instances, most typically in cases where a country is a major supplier of a commodity to the world market, in which case it may be reasonable to expect that as exports of that commodity increase so the export price (*PE*) of that commodity might be expected to decline, i.e., the country faces a downward sloping export demand curve. The inclusion of export demand equations (E5) accommodates this feature, where export demands are defined by constant elasticity export demand functions, with constants (*econ*), elasticities of demand (*eta*) and prices for substitutes on the world market (*pwse*).

<u>Imports Block</u>

The domestic price of competitive imports (M1) is the product of the world price of imports (*PWM*), the exchange rate (*ER*) and one plus the import tariff rate (TM_c). These equations are only implemented for members of the set *c* that are imported, i.e., for members of the subset *cm*.

Import Block Equations

$$PM_c = \left(PWM_c * \left(1 + TM_c\right)\right) * ER \qquad \forall cm \quad \text{(M1)}$$

$$PM.FX_c = 0.0 \qquad \not\forall cm \quad \text{(M1b)}$$

$$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + \left(1 - \delta_c\right) QD_c^{-rhoc_c}\right)^{-\frac{1}{rhoc_c}} \qquad \forall cm \textbf{ AND } cx \quad \text{(M2)}$$

$$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{\left(1 - \delta_c\right)}\right]^{\frac{1}{\left(1 + rhoc_c\right)}} \qquad \forall cm \textbf{ AND } cx \quad \text{(M3)}$$

$$QM.FX_c = 0.0 \qquad \not\forall cm \quad \text{(M3b)}$$

$$PD.FX_c = 0.0 \qquad \not\forall cd \quad \text{(M3c)}$$

$$QQ_c = QD_c + QM_c \qquad \forall \left(cmn \textbf{ AND } cx\right) \textbf{ OR } \left(cm \textbf{ AND } cxn\right) \quad \text{(M4)}$$

The domestic supply equations are modeled using Constant Elasticity of Substitution (CES) functions and associated first order conditions to determine the optimum combination of supplies from domestic and foreign (import) producers. The domestic supplies of the composite commodities (*QQ*) are defined as CES aggregates (M2) of domestic production supplied to the domestic market (*QD*) and imports (*QM*), where aggregation is controlled by the share parameters (δ), the elasticity of substitution parameters (*rhoc*) and the shift/efficiency parameters (*ac*). The first order conditions (M3) define the optimum ratios of imports to domestic demand in relation to the relative prices of imported (*PM*) and domestically supplied (*PD*) commodities. But (M2) is only defined for commodities that are both produced domestically (*cx*) **and** imported (*cm*). Although this condition might be satisfied for most commodities, it is also necessary to cover those cases where commodities are produced but **not** imported, and those cases where commodities are **not** produced domestically **and** are imported.

If commodities are produced domestically but **not** imported, then domestic supply of domestically produced commodities (*QD*) is, by definition (M4), equal to domestic commodity demand (*QQ*), where the sets *cmn* (commodities not imported) and *cx* (commodities produced domestically) control implementation. On the other hand if commodities are **not** produced domestically but are demanded on the domestic market, then commodity supply (*QQ*) is, by definition (M4), equal to commodity imports (*QM*), where the sets *cm* (commodities imported) and *cxn* (commodities not produced domestically) control implementation.

Trade and Transport Margins Block

Trade and transport margins – margin services - record the costs of transferring commodities from their source (factory gate and port of entry) to consumer (domestic or foreign). At source commodities are valued at basic prices while at the point of consumption they are valued at purchaser prices, i.e., inclusive of indirect taxes and trade and transport margins.

Trade and Transport Margins Block Equations

$$PTT_m = \sum_c ioqtdqtt_{c,m} * PQD_c \quad . \qquad \text{(M1)}$$

$$QTT_m = \sum_c \left(ioqttqq_{m,c} * QQ_c \right) + \sum_c \left(ioqttqe_{m,c} * QE_c \right) \quad . \qquad \text{(M2)}$$

$$QTTD_c = \sum_m ioqtdqtt_{c,m} * QTT_m \quad . \qquad \text{(M3)}$$

The key assumption is that trade and transport margins are represented by the quantity of trade and transport services required to deliver a unit of the commodity to the consumer (*ioqttqq* and *ioqttqe* – for supplies to the domestic and foreign consumers respectively). Thus, the quantity of trade and transport services required by the economy (QTT) is defined by the quantity of commodities demand times the quantity of margin services per unit of delivered commodity (M2).

The quantities of the commodities required (*QTTD*) to produce a unit of margins services are defined by Leontief technologies where the input coefficient ($ioqtdtt_{c,m}$) define the quantities of commodity c required to produce a unit of the margin services m (M3). Given the Leontief technologies the unit cost of the margin services (*PTT*) is a simple weighted average of the costs of the commodities used in its production (M1).

Commodity Price Block

The supply prices for commodities (P1) are defined as the volume share weighted sums of expenditure on domestically produced (*QD*) and imported (*QM*) commodities. These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QQ*) above.[25] This equation is implemented for all commodities that are imported (*cm*) and for all commodities that are produced and consumed domestically (*cd*).

Domestic agents consume composite consumption commodities (*QQ*) that are aggregates of domestically produced and imported commodities. The prices of these composite commodities (*PQD*) are defined (P2) as the supply prices of the composite commodities plus *ad valorem* sales taxes (*TS*) and excise taxes (*TEX*) and the per unit cost of the margin services used in its delivery to consumers. It is relatively straightforward to include additional commodity taxes.

In addition to commodity taxes paid by all domestic agents, households also pay value added taxes (*TV*). The prices paid by households ($PQCD_c$) are therefore defined (P3) as *ad valorem* increases of the composite commodity prices (*PQD*); note that these prices are only defined for natural commodities, *c*. Because of the nested CES-LES utility functions (see below) require 'intermediate' prices for aggregate commodities it is also necessary to define the prices (P4) for the aggregate commodities ($PQCD_{cagg}$) are defined as quantity weighted

25 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

shares of the components of each aggregate (P5): the mapping set *map_cagg_cc* identifies the natural commodities aggregate to form each aggregate commodity.

Commodity Price Block Equations

$$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \qquad \forall cd_c \textbf{ OR } cm_c \quad . \tag{P1}$$

$$PQD_c = PQS_c * \left(1 + TS_c + TSS_c\right) + TEX_c + \sum_m \left(ioqttqq_{m,c} * PTT_m\right) \qquad \forall cd_c \textbf{ OR } cm_c \quad . \tag{P2}$$

$$PQD.FX_{cc} = 0.0 \qquad \not\forall PQD0_{cc} \quad . \tag{P2b}$$

$$PQCD_c = PQD_c * \left(1 + TV_c\right) \qquad \forall cd_c \textbf{ OR } cm_c \tag{P3}$$

$$PQCD.FX_c = 0.0 \qquad \not\forall PQCD_c \quad . \tag{P3b}$$

$$\sum_h PQCD_{cc} * QCD_{cc,h} = \sum_{cc,h\$map\_ccag\_cc_{cc,cc}} PQCD_{cc} * QCD_{cc,h} \qquad \forall ccag_{cc} \tag{P4}$$

$$PQCD.FX_{cc} = 0.0 \qquad \forall PQCD0_{cc} = 0.0 \tag{P4b}$$

$$PXC_c = \frac{PD_c * QD_c + \left(PE_c * QE_c\right)\$ce_c}{QXC_c} \qquad \forall cx \quad . \tag{P5}$$

$$PXC.FX_c = 0.0 \qquad \not\forall cx \quad . \tag{P5b}$$

There are four points to note in this relationship that derives from the application of Euler's theorem to linear homogenous functions. First, the prices of the natural commodities (*QCD*) are only indexed on the natural commodity, *c*. Second, VAT taxes are levied on the natural commodities. Third, no VAT is levied on the aggregate commodity. And fourth, the weights are quantities, and the quantities are variables and therefore change as the solution is determined.

Finally, domestically produced commodities (*QXC*) are supplied to either or both the domestic and foreign markets (exported). The supply prices of domestically produced commodities (*PXC*) are defined as the volume share weighted sums of expenditure on domestically produced and exported (*QE*) commodities (P2). These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QXC*) below.[26] This equation is implemented for all commodities that are produced domestically (*cx*), with a control to only include terms for exported commodities when there are exports (*ce*).

<u>Numéraire Price Block</u>

The price block is completed by two price indices that can be used for price normalisation. Equation (N1) is for the consumer price index (*CPI*), which is defined as a weighted sum of composite commodity prices (*PQD*) in the current period, where the weights are the shares of each commodity in total demand (*comtotsh*). The domestic producer price index (*PPI*) is defined (N2) by reference to the supply prices for domestically produced commodities (*PD*) with weights defined as shares of the value of domestic output for the domestic market (*vddtotsh*).

Numéraire Block Equations

$$CPI = \sum_{c} comtotsh_c * PQCD_c \quad . \qquad \text{(N1)}$$

$$PPI = \sum_{c} vddtotsh_c * PD_c \quad . \qquad \text{(N2)}$$

Production Block

The supply prices of domestically produced commodities are determined by purchaser prices of those commodities on the domestic and international markets. Allowing for the possibility that the optimal output mix produced by an activity can vary according to the relative prices

[26] Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

paid for the commodities produced by each activity means that the (weighted) average activity prices (*PX*) where the weights are quantities of each commodity produced by each activity (*IOQXACQX*).[27] The determination of the optimal mixes of commodities produced by each activity are detailed below (X19).

In this model a three-stage production process is adopted, with the top level as a CES or Leontief function. If a CES is imposed for an activity the value of activity output can be expressed as the volume share weighted sums of the expenditures on inputs after allowing for the production taxes (*TX*), which are assumed to be applied *ad valorem* (X1). This requires the definition of aggregate prices for intermediates (*PINT*); these are defined as the intermediate input-output coefficient weighted sum of the prices of intermediate inputs (X3), where $ioqtdqd_{c,a}$ are the intermediate input-output coefficients where the output is the aggregate intermediate input (*QINT*).

Production Block Equations: Top Level

$$PX_a = \sum_c IOQXACQXV_{a,c} * PXC_c \quad . \tag{X1}$$

$$PX_a * \left(1 - TX_a\right) * QX_a = \left(PVA_a * QVA_a\right) + \left(PINT_a * QINT_a\right) \quad . \tag{X2}$$

$$PINT_a = \sum_c \left(ioqtdqd_{c,a} * PQD\right)_c \tag{X3}$$

$$ADX_a = \left[\left(adxb_a + dabadx_a\right) * ADXADJ\right] + \left(DADX * adx01_a\right) \tag{X4}$$

$$QX_a = AD_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + \left(1 - \delta_a^x\right) QINT_a^{-rhoc_a^x}\right)^{-\frac{1}{rhoc_a^x}} \qquad \forall aqx_a \quad . \tag{X5}$$

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{\left(1 - \delta_a^x\right)}\right]^{\frac{1}{\left(1 + rhoc_a^x\right)}} \qquad \forall aqx_a \quad . \tag{X6}$$

[27] In the special case of each activity producing only one commodity **and** each commodity only being produced by a single activity, which is the case in the reduced form model reported in Dervis *et al.*, (1982), then the aggregation weights *ioqxacqx* correspond to an identity matrix.

$$QVA_a = ioqvaqx_a * QX_a \qquad \forall aqxn_a \tag{X7a}$$

$$QINT_a = ioqintqx_a * QX_a \qquad \forall aqxn_a \tag{X7b}$$

With CES technology the output by an activity, (*QX*) is determined by the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and aggregate intermediates used (*QINT*), where δ_a^x is the share parameter, $rhoc_a^x$ is the substitution parameter and AD_a^x is the efficiency variable (X5). Note how the efficiency/shift factor is defined as a variable and an adjustment mechanism is provided (X4), where *adxb* is the base values, *dabadx* is an absolute change in the base value, *ADXADJ* is an equiproportionate (multiplicative) adjustment factor, *DADX* is an additive adjustment factor and *adx*01 is a vector of zeros and non zeros used to scale the additive adjustment factor. The operation of this type of adjustment equation is explained below for the case of the import duty case. The associated the first order conditions defining the optimum ratios of value added to intermediate inputs can be expressed in terms of the relative prices of value added (*PVA*) and intermediate inputs (*PINT*), see (X6).

With Leontief technology at the top level the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and intermediates used (*QINT*), are determined by simple aggregation functions, (X7a) and (X7b), where *ioqvaqx* and *ioqintqx* are the (fixed) volume shares of *QVA* and *QINT* (respectively) in *QX*. The choice of top-level aggregation function is controlled by the membership of the set *aqx*, with the membership of *aqxn* being the complement of *aqx*.

Factor prices can vary by both factor type and by the activity that employs each factor ($WFA_{ff,a}$) where *ff* identities the natural factors (*f*), aggregate factors (*fag*) and intermediate inputs that are included in the value added arm of the production nest (*fc*), and *a* the activity. To allow for variations in factor prices that originate from the factor and/or activity dimension factor prices for natural and aggregate factors ($WFA_{ff,a}$) and defined (XP1.1) as the product of the average price for each factor (WF_{ff}) and distribution variable for differences in average factor prices by each activity ($WFDIST_{ff,a}$); typically this distribution variable is assumed to be fixed but it can vary according to assumption about factor market behaviour.

The price definitions for and intermediate inputs that are included in the value added arm of the production nest (*fc*) are determined in the commodity markets. The equivalent average price is the price of the composite commodity (*PQD*), and since this cannot vary by activity, the distribution variable (*WFDIST*) is fixed as equal to one for all combinations. Thus, the prices paid for intermediate inputs ($WFA_{fc,a}$) are defined (XP1.2) as equal to the average commodity price, with (XP1.2c) ensuring the price is common to all activities.

Production Block Equations: Factor Prices

$$WFA_{ff,a} = WF_{ff} * WFDIST_{ff,a} \qquad \forall ff \in fcn \textbf{ AND } WFDIST0(ff,a) \qquad \text{(XP1.1)}$$

$$WFA_{ff,a} = \left(\sum_{cf \$map\_fc\_cf_{ff,cf}} PQD_{cf} \right) * WFDIST_{ff,a} \qquad \text{(XP1.2)}$$

$$\forall ff \in fc \textbf{ AND } WFDIST0(ff,a) \neq 0$$

$$WF.FX_{ff} = WF0_{ff} \qquad \not\forall ff \in fag . \qquad \text{(XP1.2b)}$$

$$WFDIST.FX_{ff,a} = WFDIST0_{ff,a} \qquad \forall ff \in fc . \qquad \text{(XP1.2c)}$$

There are two arms to the second level production nest. For aggregate value added (*QVA*) the production function is a multi-factor CES function (X9) where δ_a^{va} is the share parameter, $rhoc_a^{va}$ is the substitution parameter and AD_a^{va} is the efficiency factor. The associated first order conditions for profit maximisation (X10) determine the wage rate of factors (*WF*), where the ratio of factor payments to factor *f* from activity *a* (*WFDIST*) are included to allow for non-homogenous factors, and is derived directly from the first order condition for profit maximisation as equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity,[28] Again the

[28] The formulation in top line of (X10) implies that both the activity outputs (*QX*) and factor demands are solved simultaneously through the profit maximisation process. However, the formulation in the second line is more flexible since, *inter alia*, it allows the possibility of production rationing, i.e., if activity outputs (*QX*) were fixed, but there was still cost minimisation. Thanks are due to Sherman Robinson for the explanation as to the theoretic and practical distinction between the alternative, but mathematically equivalent, formulations.

efficiency/shift factor is defined as a variable with an adjustment mechanism (X8), where *advab* is the base values, *dabadva* is an absolute change in the base value, *ADVAADJ* is an equiproportionate (multiplicative) adjustment factor, *DADVA* is an additive adjustment factor and *adva*01 is a vector of zeros and non zeros used to scale the additive adjustment factor.

Production Block Equations: Second Level

$$ADVA_a = \left[\left(advab_a + dabadva_a\right) * ADVAADJ\right] + \left(DADVA * adva01_a\right) \qquad \text{(X8)}$$

$$QVA_a = AD_a^{va} * \left[\sum_{ff\$\left[map\_va\_f_{ff,a} f \text{ and } \delta_{ff,a}^{va}\right]} \delta_{ff,a}^{va} * ADFD_{ff,a} * FD_{ff,a}^{-\rho_a^{va}}\right]^{-1/\rho_a^{va}} \qquad \forall \rho_a^{va} \qquad \text{(X9)}$$

$$\begin{aligned} WFA_{ff,a} * \left(1 + TF_{ff,a}\right) &= PVA_a * AD_a^{va} * \left[\sum_{ff\$\delta_{ff,a}^{va}} \delta_{ff,a}^{va} * ADFD_{ff,a} * FD_{ff,a}^{-\rho_a^{va}}\right]^{-\left(\frac{1+\rho_a^{va}}{\rho_a^{va}}\right)} * \delta_{ff,a}^{va} * FD_{ff,a}^{\left(-\rho_a^{va}-1\right)} \\ &= PVA_a * QVA_a * AD_a^{va} * \left[\sum_{ff\$\delta_{ff,a}^{va}} \delta_{ff,a}^{va} * ADFD_{ff,a} * FD_{ff,a}^{-\rho_a^{va}}\right]^{-1} \\ &\quad * \delta_{ff,a}^{va} * ADFD_{ff,a}^{-\rho_a^{va}} * \delta_{ff,a}^{va} * FD_{ff,a}^{\left(-\rho_a^{va}-1\right)} \qquad \forall \delta_{f,a}^{va} \text{ and } map\_va\_ff_{ff,a} \end{aligned} \qquad \text{(X10)}$$

$$QINTD_{c,a} = ioqtdqd_{c,a} * QINT_a \qquad \forall cfan_{c,a}. \qquad \text{(X11)}$$

$$QINTD_{c,a} = \sum_{fc\$map\_cf\_fc_{c,fc}} FD_{c,a} \qquad \forall cfa_{c,a}. \qquad \text{(X12)}$$

The non-factor commodity (intermediate) demands (*QINTD*) are defined as the product of the fixed (Leontief) input coefficients of demand for commodity *cfan*, i.e,, those commodities NOT included in the factor aggregation system, by activity *a* (*ioqtdqd*), multiplied by the quantity of activity intermediate input (*QINT*) (X11). The demand for factor commodities, *cfa*, is taken directly from the 'factor demands' (FD_{cf}) determined in the factor aggregation system (X12). This ensure the demand for all commodities in production is recorded in *QINTD*.

Below the second level of the production system there can be many levels; the actual number being defined by the setup chosen by the user when configuring the mapping sets (*map_va_ff* and *map_fagg_ff*) used to control the system. All these additional levels are implemented in a single primal function and its associated first-order conditions. The primal production functions (X13) define the quantities of factors, aggregate and individual, combined to generate the aggregate factor/argument that enters into either the value added aggregate (X9) or any lower level aggregate, i.e., member of the set *fagg*. For each level there are efficiency factors ($ADFAG_{ff,a}$), factor shares ($\delta^{fd}_{ff,l,a}$) calibrated from the data and elasticities of substitution, from which the substitution parameters are derived ($\rho^{fd}_{ff,a}$), are exogenously imposed. The matching first order conditions (X14) define the wage rate for a specific factor used by a specific activity; these ratios of payments to factor *ff* from activity *a* are included to allow for non-homogenous factors where the differentiation is defined solely in terms of the activity that employs the factor. However the actual returns to a factor must be adjusted to allow for taxes on factor use ($TF_{l,a}$), although care must be taken to ensure that factor use tax rates are always zero for any aggregate factors.

The composite supplies of each commodity (*QXC*) are aggregates of the commodity outputs by each activity (*QXAC*). The default assumption is that when a commodity is produced by multiple activities it is differentiated by reference to the activity that produces the commodity; this is achieved by defining total production of a commodity as a CES aggregate of the quantities produced by each activity (X15). This provides a practical/modelling solution for two typical situations; first, where there are quality differences between two commodities that are notionally the same, e.g., modern digital vv disposable cameras, and second, where the mix of commodities within an aggregate differ between activities, e.g., a cereal grain aggregate made up of wheat and maize (corn) where different activities produce wheat and maize in different ratios. This assumption of imperfect substitution is implemented by a CES aggregator function with $adxc_c$ as the shift parameter, $\delta^{xc}_{a,c}$ as the share parameter and ρ^{xc}_c as the elasticity parameter.

The matching first order condition for the optimal combination of commodity outputs is therefore given by (X16), where *PXAC* are the prices of each commodity produced by each activity. Note how, as with the case of the value added production function two formulations are given for the first-order conditions and the second version is the default version used in the model. Further note that the efficiency/shift factor is in this case declared as a parameter; this reflects the expectation that there will be no endogenously determined changes in these shift factors.

Production Block Equations: Third and Beyond Levels

$$ADFAG_{ff,a} = \left(adfagb_{ff,a} + dabfag_{ff,a}\right) + \left(ADFAGfADJ_{ff} * ADFAGaADJ_{a}\right) \tag{X12}$$

$$FD_{ff,a} = ADFAG_{ff,a} * \left(\sum_{ffp\$\left[\delta^{fd}_{ff,ffp,a} \text{ and } map\_fagg\_f\right]f} \delta^{fd}_{ff,ffp,a} * \left(FD_{ffp,a}\right)^{\rho^{fd}_{ff,a}} \right)^{\left(-1/\rho^{fd}_{ff,a}\right)}$$
$$\forall \sum_{ffp} map\_fagg\_ff_{ff,ffp,a} \tag{X13}$$

$$\begin{aligned} FD.FX_{f,a} &= 0.0 \qquad \not\forall SAM_{f,a} \\ FD.FX_{ff,a} &= 0.0 \qquad \not\forall FD0_{ff,a} \end{aligned} \tag{X13b}$$

$$\begin{aligned} WFA_{ffp,a} &* \left(1 + TF_{ffp,a}\right) \\ &= WFA_{ff,a} * \left(1 + TF_{ff,a}\right) * FD_{ff,a} \\ &\quad * \left[\sum_{ffpp\$\left[\delta^{fd}_{ff,ffpp,a} \text{ and } map\_fagg\_ff\right]} \delta^{fd}_{ff,ffpp,a} * FD_{ffpp,a}^{-\rho^{fd}_{ff,a}} \right]^{(-1)} * \delta^{fd}_{ff,ffp,a} * FD_{ffp,a}^{\left(-\rho^{fd}_{ff,a} - 1\right)} \end{aligned}$$
$$\forall \delta^{fd}_{ff,ffpp,a}, map\_fagg\_ff_{ff,ffp,a} \tag{X14}$$

However, there are circumstances where perfect substitution may be a more appropriate assumption given the characteristics of either or both activity and commodity accounts. Thus, an alternative specification for commodity aggregation is proved where commodities produced by different activities are modeled as perfect substitutes, (X17), and the matching price condition therefore requires that *PXAC* is equal to *PXC* for relevant commodity activity combinations (X18). The choice of aggregation function is controlled by the membership of the set *cxac*, with the membership of *cxacn* being the complement of *cxac*.

Production Block Equations: Commodity Outputs

$$QXC_c = adxc_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-1/\rho^{xc}_c} \qquad \forall cx_c \text{ and } cxac_c . \qquad \text{(X15)}$$

$$PXAC_{a,c} = PXC_c * adxc_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)}$$

$$= PXC_c * QXC_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)}$$

$$\forall \delta^{xc}_{a,c} \text{ and } cxac_c \qquad \text{(X16)}$$

$$PXAC.FX_{a,c} = 0.0 \qquad \not\forall SAM_{a,c} \qquad \text{(X16b)}$$

$$QXC_c = \sum_a QXAC_{a,c} \qquad \forall cx_c \text{ and } cxacn_c . \qquad \text{(X17)}$$

$$PXAC_{a,c} = PXC_c \qquad \forall \delta^{xc}_{a,c} \text{ and } cxacn_c . \qquad \text{(X18)}$$

$$QXAC_{a,c} = IOQXACQX_{a,c} * QX_a \qquad \forall ioqxacqx_{a,c} \textbf{ AND } acetn_a . \qquad \text{(X19)}$$

$$QXAC_{a,c} = QX_a * \left(\frac{PXAC_{a,c}}{\left(PX_a * gamma^i_{a,c} * at_a^{i\,\rho^i_a} \right)} \right)^{\left(1/\left(\rho^i_a - 1\right) \right)}$$

$$\forall IOQXACQX_{a,c} \textbf{ and } acet_a \qquad \text{(X20)}$$

$$QXAC.FX_{a,c} = 0.0 \qquad \forall SAM_{a,c} \qquad \text{(X20b)}$$

Finally, it is necessary to determine the quantities of each commodity produced by each activity. There are two basic assumptions included in the model: first that secondary commodities are produced with pure by-product technologies, i.e., in a fixed ratio to the principal product, and second that activities can adjust their output mix in response to changes in the prices of the commodities they produce. The function for by-product assumption is that fixed shares of products (*IOQXACQX*) are produced by each activity according to its level of total output (*QX*); although the shares are defined as variable the user determines which rows of the matrix *IOQXACQX* are fixed when configuring the model by defining membership of the set *acet* (X19). To implement the alternative assumption, it is only necessary to specify the first order condition for a CET function; this is reported in equation (X20). However, it is also now necessary to include a market clearing condition for production; this is reported in the market clearing section below (see equation C2).

Emissions Block

The CO2 emissions associated with the use of energy commodities by each agent are recorded for activities (EM1.1) and households (EM1.2) (*CO2EMIT*). These are defined as the product of the quantity of an energy commodity consumed and the coefficients that define CO2 emissions per unit of energy commodity consumed. Note that in the model the quantities of energy commodities consumed are 'value' quantities; the coefficients are calibrated as emission per unit of 'value' quantity, which need to be noted when defining simulations.

Emissions Block Equations

$$CO2EMIT_{c,a} = QINTD_{c,a} * co2co_{c,a} \qquad \forall co2co_{c,a} \tag{EM1.1}$$

$$CO2EMIT.FX_{c,a} = 0.0 \qquad \forall co2co_{c,a} = 0.0 \tag{EM1.1b}$$

$$CO2EMIT_{c,h} = QCD_{c,h} * co2co_{c,h} \qquad \forall co2co_{c,a} \tag{EM1.2}$$

$$CO2EMIT.FX_{c,h} = 0.0 \qquad \forall co2co_{c,h} = 0.0 \tag{EM1.2b}$$

$$CO2EMIT\_TOT = \sum_{c,a\_h} CO2EMIT_{c,a\_h} \tag{EM1.3}$$

The total quantity of emissions by region ($CO2EMIT\_TOT_r$) are a summation of the CO2 emissions by each agent (EM1.3).

Factor Block

There are two sources of income for factors. First there are payments to factor accounts for services supplied to activities, i.e., domestic value added, and second there are payments to domestic factors that are used overseas, the value of these are assumed fixed in terms of the foreign currency. Factor incomes (*YF*) are therefore defined as the sum of all income to the factors across all activities (F1).

Factor Block Equations

$$YF_f = \left(\sum_a WFA_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right). \qquad (F1)$$

$$YFDISP_f = \left(YF_f * \left(1 - deprec_f \right) \right) * \left(1 - TYF_f \right). \qquad (F2)$$

$$YFINS_f = YFDISP_f. \qquad (F3)$$

$$FSISH_{insw,f} = FSI_{insw,f} \Big/ \sum_{insw} FSI_{insw,f} \qquad \forall \sum_{insw} fsia_{insw,f} \qquad (F4)$$

$$FSISH.FX_{insw,f} = 0.0 \qquad \not\forall FSISH0_{insw,f} \qquad (F4b)$$

$$INSVA_{insw,f} = FSISH_{insw,f} * YFINS_f \qquad (F5)$$

$$INSVA.FX_{insw,f} = 0.0 \qquad \not\forall INSVA0_{insw,f} \qquad (F5b)$$

Before distributing factor incomes to the institutions that supply factor services allowance is made for depreciation rates (*deprec*) and factor (income) taxes (*TYF*) so that factor income for distribution (*YFDISP*) is defined (F2).

The endogenous determination of factor incomes requires the definition of variables that control that distribution. The key assumption is that the shares of factor income (*FSISH*) distributed to institutions (*insw*) are defined by the shares of factor ownership (*FSI*), which is implemented in (F3). For coding convenience, the values of factor incomes distributed to each institution (*INSVA*) are calculated explicitly (F4); this reduces the code needed later although it increases the number of variables in the model.

<u>Household Block</u>

Household Income

Households receive income from a variety of sources (H1). Factor incomes are distributed to households in proportion to their ownership of factors ($INSVA_{h,f}$), plus inter household transfers (*HOHO*), distributed payments/dividends from incorporated enterprises (*HOENT*) and real transfers from government (*hogovconst*) that are adjustable using a scaling factor (*HGADJ*) and transfers from the rest of the world (*howor*) converted into domestic currency units.

Household Expenditure

Inter household transfers (*HOHO*) are defined (H2) as a fixed proportions of household income (*YH*) after payment of direct taxes and savings, and then household consumption expenditure (*HEXP*) is defined as household income after tax income less savings and transfers to other households (H3).

Households are then assumed to maximise utility subject to Stone-Geary (*aka* LES) utility functions. In a Stone-Geary utility function household consumption demand consists of two components; ‘subsistence’ demand (*qcdconst*) and ‘discretionary’ demand, and the equation must therefore capture both elements. Subsistence demand is defined as the volume of subsistence demand times the prices and then discretionary demand is then defined as the marginal budget shares (*beta*) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand. If the user wants to assume Cobb-Douglas utility functions, for one or more households, this can be achieved by setting the Frisch parameters equal to minus one and all the income elasticities of demand equal to one (the model code includes documentation of the calibration steps). This is typically only the case for relatively rich households where the operation of the utility function will not reduce demand below a level consistent with subsistence demand.

Household Income and Expenditure Block Equations

$$YH_h = \left(\sum_f INSVA_{h,f}\right) + \left(\sum_{hp} HOHO_{h,hp}\right) + \left(\sum_e HOENT_{h,e}\right) + \left(hogovconst_h * HGADJ * CPI\right) + \left(howor_h * ER\right). \quad \text{(H1)}$$

$$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right). \quad \text{(H2)}$$

$$HOHO_{h,hp} = 0.0 \qquad \forall SAM_{h,hp}. \quad \text{(H2b)}$$

$$HEXP_h = YH_h - HHSAV_h - \left(\sum_{hp} HOHO_{hp,h}\right). \quad \text{(H3)}$$

The second level of the utility functions is defined with CES preferences. The quantities of the aggregated commodity groups that are demanded by each household ($QCD_{cag,,h}$) are defined in the top level (LES) utility function and therefore only the first order conditions are required to determine the optimum combinations of natural commodities. This is presented as a standard FOC for a CES function which has been calibrated for shift, share and elasticity parameters based on the initial data and the, exogenous imposed, substitution elasticities that are aggregate commodity and household specific.

Household Utility Function Block Equations

$$QCD_{cc,h} = \frac{\left(\sum_h \left(\begin{array}{l} \left(PQCD_{cc} * qcdconst_{cc,h} \right) + \sum_h beta_{cc,h} \\ \quad * \left(HEXP_h - \sum_c \left(PQCD_{cag,h} * qcdconst_{cag,h} \right) \right) \end{array} \right) \right)}{PQCD_{cc}} \qquad \text{(H4)}$$

$$\forall cles_{cc} \ \textbf{AND} \ beta_{cc,h}$$

$$QCD.FX_{cc,h} = 0.0 \qquad \not\forall beta_{cc,h} \qquad \text{(H4b)}$$

$$QCD2_{cag,cc,h} = QCD_{cag,h} * \left(\frac{\left(\left(PQD_{cc} * \left(1 + TV_{cc}\right) \right) * accd_{cag,h}^{-\rho^{cd}_{cag,h}} \right)}{\left(PQCD_{cag,h} * \delta^{cd}_{cag,cc,h} \right)} \right)^{\left(-1 / \left(\rho^{cd}_{cag,h} + 1 \right) \right)} \qquad \text{(H5)}$$

$$\forall \delta^{cd}_{cag,cc,h}$$

$$QCD2.FX_{cc,ccp,h} = 0.0 \qquad \not\forall \delta^{cd}_{cc,ccp,h} \qquad \text{(H5b)}$$

$$QCD_{cc,h} = \sum_{ccp\$map\_ccag\_cc_{ccp,c}} QCD2_{ccp,c,h} \qquad \forall cces_{cc} \ \textbf{AND} \sum_{ccp\$map\_ccag\_cc_{ccp,c}} \delta^{cd}_{ccp,c,h}$$

(H6)

Enterprise Block

<u>Enterprise Income</u>

Similarly, income to enterprises (EN1) comes from the share of distributed factor incomes accruing to enterprises ($INSVA_{e,f}$) and real transfers from government (*entgovconst*) that are adjustable using a scaling factor (*EGADJ*) and the rest of the world (*entwor*) converted in the domestic currency units.

<u>Enterprise Expenditure</u>

The consumption of commodities by enterprises (*QED*) is defined (EN2) in terms of fixed volumes (*qedconst*), which can be varied via the volume adjuster (*QEDADJ*), and associated with any given volume of enterprise final demand there is a level of expenditure (*VED*); this is defined by (EN6) and creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

If *QEDADJ* is made flexible, then *qedconst* ensures that the quantities of commodities demanded are varied in fixed proportions; clearly this specification of demand is not a consequence of a defined set of behavioural relationships, as was the case for households, which reflects the difficulties inherent to defining utility functions for non-household institutions. If *VED* is fixed then the volume of consumption by enterprises (*QED*) must be allowed to vary, via the variable *QENTDADJ*.

Enterprise Block Equations

$$YE_e = \left(\sum_f INSVA_{e,f}\right) + \left(entgovconst_e * EGADJ * CPI\right) + \left(entwor_e * ER\right) \quad \text{(EN1)}$$

$$QED_{c,e} = qedconst_{c,e} * QEDADJ \quad \text{(EN2)}$$

$$HOENT_{h,e} = hoentsh_{h,e} * \left(YE_e - \left(TYE_e * YE_c\right) - ENTSAV_e - VED_e - worent_e\right) \quad \text{(EN3)}$$

$$GOVENT_e = goventsh_e * \left(\left(YE_e - \left(TYE_e * YE_c\right) - ENTSAV_e - VED_e - worent_e\right)\right) \quad \text{(EN4)}$$

$$WORENT_e = worentsh_e * \left(\left(YE_e - \left(TYE_e * YE_c\right) - ENTSAV_e - VED_c\right)\right) \quad \text{(EN5)}$$

$$VED_e = \left(\sum_c QED_{c,e} * PQD_c\right) \quad \text{(EN6)}$$

The incomes to households from enterprises, which are assumed to consist primarily of distributed profits/dividends, are defined by (EN3), where *hoentsh* are defined as fixed shares of enterprise income after payments of direct/income taxes, savings and consumption expenditure. The income to government from enterprises, which is assumed to consist primarily of distributed profits/dividends on government owned enterprises, is defined by

(EN4), where *goventsh* is defined as a fixed share of enterprise income after payments of direct/income taxes, savings and consumption expenditure. The income to the rest of the world from enterprises, based on ownership of enterprises by the rest of the world, is defined by (EN5), where *worentsh* is defined as a fixed share of enterprise income after payments of direct/income taxes, savings and consumption expenditure.

Government Block

Tax Rates

All tax rates are variables in this model. The tax rates in the base solution are defined as parameters, e.g., tmb_c are the import duties by commodity *c* in the base solution, and the equations then allow for varying the tax rates in 4 different ways. For each tax instrument there are four methods that allow adjustments to the tax rates; two of the methods use variables that can be solved for optimum values in the model according to the choice of closure rule and two methods allow for deterministic adjustments to the structure of the tax rates. The operation of this method is discussed in detail only for the equations for import duties while the other equations are simply reported.

Import duty tax rates are defined by (GT1), where tmb_c is the vector of import duties in the base solution, $dabtm_c$ is a vector of absolute changes in the vector of import duties, *TMADJ* is a variable whose initial value is ONE, *DTM* is a variable whose initial value is ZERO and $tm01_c$ is a vector of zeros and non zeros. In the base solution the values of $tm01_c$ and $dabtm_c$ are all ZERO and *TMADJ* and *DTM* are fixed as their initial values – a closure rule decision –then the applied import duties are those from the base solution. Now the different methods of adjustment can be considered in turn

1. If *TMADJ* is made a variable, which requires the fixing of another variable, and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the import duty rates necessary to satisfy model constraints, e.g., if *TMADJ* equals 1.1 then all import duties are increased by 10%.
2. If any element of *dabtm* is non zero and all the other initial conditions hold, then an absolute change in the initial import duty for the relevant commodity can be imposed using *dabtm*, e.g., if *tmb* for one element of *c* is 0.1 (a 10% import duty) and *dabtm* for that element is 0.05, then the applied import duty is 0.15 (15%).

3. If *TMADJ* is a variable, any elements of *dabtm* are non zero and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the applied import duty rates.
4. If *DTM* is made a variable, which requires the fixing of another variable, AND at least one element of *tm*01 is NOT equal to ZERO then the subset of elements of *c* identified by *tm*01 are allowed to (additively) increase by an equiproportionate amount determined by the solution value for *DTM* times the values of *tm*01. Note how in this case it is necessary to both ‘free’ a variable and give values to a parameter for a solution to emerge.

This combination of alternative adjustment methods covers a range of common tax rate adjustment used in many applied applications while being flexible and easy to use.

Export tax rates are defined by (GT2), where tme_c is the vector of export duties in the base solution, $dabte_c$ is a vector of absolute changes in the vector of export duties, *TEADJ* is a variable whose initial value is ONE, *DTE* is a variable whose initial value is ZERO and $te01_c$ is a vector of zeros and non zeros. *Ad valorem* sales tax rates are defined by (GT3) and (GT4), where tms_c/ $tmss_c$ is the vector of sales tax rates in the base solution, $dabts_c$/$dabts_c$ is a vector of absolute changes in the vector of sales taxes, *TSADJ*/*TSSADJ* is a variable whose initial value is ONE, *DTS*/*DTSS* is a variable whose initial value is ZERO and $ts01_c$/ $tss01_c$ is a vector of zeros and non zeros. Excise tax rates, levied on quantities not values, are defined by (GT5), where $texb_c$ is the vector of excise tax rates in the base solution, $dabtex_c$ is a vector of absolute changes in the vector of excise taxes, *TEXADJ* is a variable whose initial value is ONE, *DTEX* is a variable whose initial value is ZERO and $tex01_c$ is a vector of zeros and non zeros. And value added taxes (*TV*) are defined in equation GT6, where tvb_c is the vector of VAT rates in the base solution, $dabtv_c$ is a vector of absolute changes in the vector of VAT rates, *TVADJ* is a variable whose initial value is ONE, *DTV* is a variable whose initial value is ZERO and $tv01_c$ is a vector of zeros and non zeros.

Indirect tax rates on production are defined by (GT7), where txb_c is the vector of production taxes in the base solution, $dabtx_c$ is a vector of absolute changes in the vector of production taxes, *TXADJ* is a variable whose initial value is ONE, *DTX* is a variable whose initial value is ZERO and $tx01_c$ is a vector of zeros and non zeros. Taxes on factor use by each factor and activity are defined by (GT8), where $tfb_{f,a}$ is the matrix of factor use tax rates in the base solution, $dabtf_{f,a}$ is a matrix of absolute changes in the matrix of factor use taxes, *TFADJ* is a variable whose initial value is ONE, *DTFM* is a variable whose initial value is ZERO and $tf01_{f,a}$ is a matrix of zeros and non zeros.

Tax Rate Block Equations

$$TM_c = \left(\left(tmb_c + dabtm_c\right) * TMADJ\right) + \left(DTM * tm01_c\right) \quad \forall cm_c \tag{GT1}$$

$$TE_c = \left(\left(teb_c + dabte_c\right) * TEADJ\right) + \left(DTE * te01_c\right) \qquad \forall ce_c \tag{GT2}$$

$$TS_c = \left(\left(tsb_c + dabts_c\right) * TSADJ\right) + \left(DTS * ts01_c\right) \qquad \forall cd_c \textbf{ OR } cm_c \tag{GT3}$$

$$TSS_c = \left(\left(tssb_c + dabtss_c\right) * TSSADJ\right) + \left(DTSS * tss01_c\right) \qquad \forall cd_c \textbf{ OR } cm_c \tag{GT4}$$

$$TEX_c = \left(\left(texb_c + dabtex_c\right) * TEXADJ\right) + \left(DTEX * tex01_c\right) \qquad \forall cd_c \textbf{ OR } cm_c \tag{GT5}$$

$$TV_c = \left(\left(tvb_c + dabtv_c\right) * TVADJ\right) + \left(DTV * tv01_c\right) \qquad \forall cd_c \textbf{ OR } cm_c \tag{GT6}$$

$$TX_a = \left(\left(txb_a + dabtx_a\right) * TXADJ\right) + \left(DTX * tx01_a\right) \tag{GT7}$$

$$TF_{f,a} = \left(\left(tfb_{f,a} + dabtf_{f,a}\right) * TFADJ\right) + \left(DTF * tf01_{f,a}\right) \tag{GT8}$$

$$TF.FX_{ff,a} = TF0_{ff,a} \qquad \forall f_{ff} \tag{GT8b}$$

$$TYF_f = \left(\left(tyfb_f + dabtyf_f\right) * TYFADJ\right) + \left(DTYF * tyf01_f\right) \tag{GT9}$$

$$TYH_h = \left(\left(tyhb_h + dabtyh_h\right) * TYHADJ\right) + \left(DTYH * tyh01_h\right) \tag{GT10}$$

$$TYE_e = \left(\left(tyeb_e + dabtye_e\right) * TYEADJ\right) + \left(DTYE * tye01_e\right). \tag{GT11}$$

Factor income tax rates[29] are defined by (GT9), where $tyfb_f$ is the vector of factor income taxes in the base solution, $dabtyy_f$ is a vector of absolute changes in the vector of factor income taxes, *TYFADJ* is a variable whose initial value is ONE, *DTYF* is a variable whose initial value is ZERO and $tyf01_f$ is a vector of zeros and non zeros. Household income tax rates are defined by (GT10), where $tyhb_h$ is the vector of household income tax rates in the base solution, $dabtyh_h$ is a vector of absolute changes in the vector of income tax rates, *TYFADJ* is a variable whose initial value is ONE, *DTYF* is a variable whose initial value is ZERO and $tyh01_c$ is a vector of zeros and non zeros. And finally, enterprise income tax rates are defined by (GT11), where $tyeb_e$ is the vector of enterprise income tax rates in the base solution, $dabtye_e$ is a vector of absolute changes in the income tax rates, *TYEADJ* is a variable whose initial value is ONE, *DTYE* is a variable whose initial value is ZERO and $tye01_e$ is a vector of zeros and non zeros.

<u>Tax Revenues</u>

Although it is not necessary to keep the tax revenue equations separate from other equations, e.g., they can be embedded into the equation for government income (*YG*), it does aid clarity and assist with implementing fiscal policy simulations. For this model there are eight tax revenue equations. The patterns of tax rates are controlled by the tax rate variable equations. In all cases the tax rates can be negative indicating a ‘transfer’ from the government.

There are six tax instruments that are dependent upon expenditure on commodities, with each expressed as an *ad valorem* tax rate. Tariff revenue (*MTAX*) is defined (GR1) as the sum of the product of tariff rates (*TM*) and the value of expenditure on imports at world prices, the revenue from export duties (*ETAX*) is defined (GR2) as the sum of the product of export duty rates (*TE*) and the value of expenditure on exports at world prices, the two ad valorem sale tax revenues (*STAX* and *SSTAX*) are defined (GR3 and GR4) as the sum of the product of sales tax rates (*TS* and *TSS*) and the value of domestic expenditure on commodities. The excise tax revenues (*EXTAX*) are defined (GR5) as the sum of the product of excise tax rates (*TEX*) and the volume of domestic expenditure on commodities. And then there is a value added tax (GR6) where as opposed to other taxes and commodities demanded domestically the tax is only paid on final demand by households.

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29 These are defined as taxes on factor incomes that are independent of the activity that employs the factor. They could include social security type payments.

Government Tax Revenue Block Equations

$$MTAX = \sum_{c} \left(TM_c * PWM_c * ER * QM_c \right) . \qquad \text{(GR1)}$$

$$ETAX = \sum_{c} \left(TE_c * PWE_c * ER * QE_c \right) . \qquad \text{(GR2)}$$

$$STAX = \sum_{c} \left(TS_c * PQS_c * QQ_c \right) . \qquad \text{(GR3)}$$

$$SSTAX = \sum_{c} \left(TSS_c * PQS_c * QQ_c \right) . \qquad \text{(GR4)}$$

$$EXTAX = \sum_{c} \left(TEX_c * QQ_c \right) . \qquad \text{(GR5)}$$

$$VTAX = \sum_{h} \sum_{c} \left(TV_c * PQD_c * QCD_{c,h} \right) \qquad \text{(GR6)}$$

$$ITAX = \sum_{a} \left(TX_a * PX_a * QX_a \right) . \qquad \text{(GR7)}$$

$$FTAX = \sum_{f,a} \left(TF_{f,a} * WF_f * WFDIST_{f,a} * FD_{f,a} \right) . \qquad \text{(GR8)}$$

$$FYTAX = \sum_{f} \left(TYF_f * \left(YF_f * \left(1 - deprec_f \right) \right) \right) . \qquad \text{(GR9)}$$

$$DTAX = \sum_{h} \left(TYH_h * YH_h \right) + \sum_{e} \left(TYE_e * YE \right) . \qquad \text{(GR9)}$$

There is a single tax on production (*ITAX*). As with other taxes this is defined (GR7) as the sum of the product of indirect tax rates (*TX*) and the value of output by each activity evaluated in terms of the activity prices (*PX*). In addition, activities can pay taxes based on the value of employed factors – factor use taxes (FTAX) (GR8). The revenue from these taxes is defined sum of the product of factor income tax rates and the value of the factor services employed by each activity for each factor; the sum is over both activities and factors. These two taxes are the instruments most likely to yield negative revenues through the existence of production and/or factor use subsidies.

Income taxes are collected on both factors and domestic institutions. The income tax on factors (*FYTAX*) is defined (GR9) as the product of factor tax rates (*TYF*) and factor incomes for all factors, while those on institutions (DTAX) are defined (GR10) as the sum of the product of household income tax rates (*TYH*) and household incomes plus the product of the direct tax rate for enterprises (*TYE*) and enterprise income.

Government Income

The sources of income to the government account (G1) are more complex than for other institutions. Income accrues from 9 tax instruments; tariff revenues (*MTAX*), export duties (*ETAX*), value added taxes (VTAX), (general) sales taxes (*STAX* and *SSTAX*), excise taxes (*EXTAX*), production taxes (*ITAX*), factor use taxes (*FTAX*), factor income taxes (*FYTAX*) and direct income taxes (*DTAX*), which are defined in the tax equation block above. In addition, the government can receive income from its ownership of factors (*INSVA*), distributed payments/dividends from incorporated enterprises that are owned by the government (*GOVENT*) and transfers from abroad (*govwor*) converted in the domestic currency units. It would be relatively easy to subsume the tax revenue equations into the equation for government income, but they are kept separate to facilitate the implementation of fiscal policy experiments. Ultimately however the choice is a matter of personal preference.

Government Expenditure Block

The demand for commodities by the government for consumption (*QGD*) is defined (G2) in terms of fixed proportions (*qgdconst*)[30] that can be varied with a scaling adjuster (*QGDADJ*), and associated with any given volume of government final demand there is a level/value of expenditure (*VGD*) defined by (G3); this creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

Government Income and Expenditure Block Equations

[30] Alternative utility functions could be specified, e.g., Cobb-Douglas, CES, etc., but there is no substantive body of economic theory upon which such utility functions can be based. Hence the presumption of Leontief/fixed coefficient preference is a pragmatic, if simplistic, specification.

$$YG = MTAX + ETAX + STAX + SSTAX + EXTAX + VTAX + FTAX + ITAX + FYTAX + DTAX + \sum_f INSVA_{g,f} + \sum_e GOVENT_e + (govwor * ER) \quad \text{(G1)}$$

$$QGD_c = qgdconst_c * QGDADJ \quad \text{(G2)}$$

$$VGD = \left(\sum_c QGD_c * PQD_c \right) \quad \text{(G3)}$$

$$EG = \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ * CPI \right) + \left(\sum_e entgovconst_e * EGADJ * CPI \right) \quad \text{(G4)}$$

As with enterprises there are difficulties inherent to defining utility functions for a government. Changing *QGDADJ*, either exogenously or endogenously, by allowing it to be a variable in the closure conditions, provides a means of changing the behavioural assumption with respect to the 'volume' of commodity demand by the government. If the value of government final demand (*VGD*) is fixed then government expenditure is fixed and hence the volume of consumption by government (*QGD*) must be allowed to vary, via the *QGDADJ* variable. If it is deemed appropriate to modify the patterns of commodity demand by the government then the components of *qgdconst* must be changed.

Hence, total government expenditure (*EG*) can be defined (G4) as equal to the sum of expenditure by government on consumption demand at current prices, plus real transfers to households (*hogovconst*) that can be adjusted using a (multiplicative) scaling factor (*HGADJ*) and real transfers to enterprises (*entgovconst*) that can also be adjusted by a (multiplicative) scaling factor (*EGADJ*).

Savings and Investment Block

Savings

The savings rates for households (*SHH* in I1) and enterprises (*SEN* in I3) are defined as variables using the same adjustment mechanisms used for tax rates; $shhb_h$ and $senb_e$ are the savings rates in the base solution, $dabshh_h$ and $dabsen_e$ are absolute changes in the base rates, *SHADJ* and *SEADJ* are multiplicative adjustment factors, *DSHH* and *DSEN* are additive adjustment factors and $shh01_h$ and $sen01_e$ are vectors of zeros and non zeros that scale the additive adjustment factors. However, unlike the tax rate equations, each of the savings rates equations has two additional adjustment factors – *SADJ* and *DS*. These serve to allow the user to vary the savings rates for households and enterprises in tandem; this is useful when the macroeconomic closure conditions require increases in savings by domestic institutions and it is not deemed appropriate to force all the adjustment on a single institution or group of institutions.[31]

Savings by Households (*HHSAV*) and Enterprises (*ENTSAV*) are defined in (I3 and I4) respectively as savings after payment of income taxes (*TYH* and *TYE*), i.e., income taxes have first call on household and enterprise income. Total savings in the economy are defined (I5) as the sum of household and enterprise savings, plus the allowances for depreciation at fixed rates (*deprec*) out of factor income and the government budget deficit/surplus (*KAPGOV*). Finally, the current account 'deficit' (*CAPWOR*) contributes to savings in the economy. The last two terms of I5 – *KAPGOV* and *CAPWOR* – are defined below by equations in the market clearing block.

Savings Block Equations

$$SHH_h = \left(\left(shhb_h + dabshh_h\right) * SHADJ * SADJ\right) + \left(DSHH * DS * shh01_h\right) \tag{I1}$$

$$HHSAV_h = \left(YH_h * \left(1 - TYH_h\right)\right) * SHH_h . \tag{I2}$$

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[31] A similar mechanism can be easily imposed for tax rates when the user wishes to cause two or more tax instruments to move in tandem.

$$SEN_e = \left(\left(senb_e + dabsen_e \right) * SEADJ * SADJ \right) + \left(DSEN * DS * sen01_e \right) \quad \text{(I3)}$$

$$ENTSAV_e = \left(YE_e * \left(1 - TYE_e \right) \right) * SEN_e \quad \text{(I4)}$$

$$DPREC_f = \sum_f \left(YF_f * deprec_f \right) \quad \text{(I5)}$$

$$TOTSAV = \sum_f DPREC_f + \sum_h HHSAV_h + \sum_e ENTSAV_e + KAPGOV + \left(CAPWOR * ER \right) \quad \text{(I6)}$$

Investment

The presence of multiple types of capital goods implies the existence of different patterns of investment expenditures each corresponding to one type of capital good, where the set *in(i)* defines the investment patterns and each member of *i* is uniquely paired with a member of the set *k* (capital factors).[32] In essence, this implies that each capital good has a unique cost of production (a form of 'production function') defined by the quantities of each commodity required to produce a given quantity of the capital good. If the functional form for the 'production functions' are assumed to be Leontief it is possible to derive input-output coefficients that define the quantities of each input/commodity required to produce a unit of each capital good ($ioqinvd_{c,i}$).[33] Then the demand for each commodity used to produce investment goods ($QINVD_{c,i}$) as the product of the quantity/volume of each capital good ($QINV_i$) and the respective coefficients ($ioqinvd_{c,i}$); these relationships are defined in I4 and I5.

Investment Block Equations

$$QINVD_{c,i} = \left(QINV_i * ioqinvd_{c,i} \right). \quad \text{(I6)}$$

32 Note that *i* can be a single member set so the same code can be used where there is only a single investment account.

33 Alternative production functions can be easily specified, e.g., Cobb-Douglas, CES, etc., although information about the ease of substituting inputs in the production of capital goods is limited. Hence the presumption of Leontief technologies as a pragmatic, if simplistic, specification.

$$QINVD.FX_{c,i} = 0.0 \qquad \not\forall ioqinvd_{c,i} \quad . \qquad \text{(I6b)}$$
$$QINVD.FX_{c,"dstoc"} = QINVD0_{c,"dstoc"}$$

$$QINV_i = qinvb_i * IADJ \qquad \forall in_i . \qquad \text{(I7)}$$

$$INVSH\_I_i = \frac{\sum_c PQD_c * QINVD_{c,i}}{\sum_{c,i} PQD_c * QINVD_{c,i}} \qquad \forall in_i . \qquad \text{(I8)}$$

$$INVEST = \sum_{c,i} PQD_c * \left(\sum_{in} QINVD_{c,in} + dstocconst_c \right) \quad . \qquad \text{(I9)}$$

The demand for commodities for investment purposes therefore depends on the ‘technologies’ and the volumes of capital goods required. However, comparative static and recursive dynamic CGE models do not have endogenously determined investment functions that serve to define the quantities of capital goods produced. A simple, and very commonly used, dichotomy is to assume that the demand for capital goods is determined by either the amount of available investable funds – so-called savings driven assumption - or exogenously – so-called investment driven or Keynesian assumption. Assume, for purposes of exposition only, that an investment driven assumption of exogenous determination is appropriate, i.e., $QINV_i$ is fixed exogenously.

To implement the investment driven assumption, the parameters $qinvb_i$ are fixed, at the exogenously determined levels, and the scaling variable *IADJ* is fixed equal to one (I5); and hence the demand for each commodity for investment purposes are determined from I4. Given the demand for each commodity and the prices for each commodity (PQD_c) the value of investment expenditures to produce each capital good *i* ($INVEST*INVSH\_I_i$) is the product of prices and quantities (I6). The total value of all investments (*INVEST*) is then defined as the summation of the expenditures on each capital good (I7), and market clearing for investable funds is ensure by the equality of total savings (*TOTSAV*) and investment (see C20 below).

If a savings driven assumption is adopted, then the value of *INVEST* is determined by total savings (*TOTSAV*) and the parameters $qinvb_i$ determine the ratio of capital goods produced with the scaling variable *IADJ* providing (multiplicative) equiproportionate changes in the volumes of each capital good. The scaling variable *IADJ* adjusts the volumes of capital goods produced so that the expenditures on each capital good ($INVEST*INVSH\_I_i$) exhaust the available investable funds (*INVEST*); thus in such a setting I7 operates as a market clearing equation.

The members of the set *i* include the agent that gathers together investable funds from savings by domestic and foreign agents (‘*i_s*’) and distributes those funds across different investment activities. One such investment activity is stock changes (‘*dstoc*’); thus stock changes can be included within this formulation.

In a comparative static context, the specification of different patterns of investment expenditures is relevant *if and only if* the analyst has information that indicates that the average pattern of investment expenditures will change due to the simulation. If the relative volumes of investment in capital goods is invariant, then the system operates as if there is a single investment account, i.e., the system *de facto* collapses back to the ‘standard’ approach in the STAGE family of models.

Foreign Institutions Block

The economy also employs foreign owned factors whose services must be recompensed. It is assumed that these services receive proportions of the factor incomes available for distribution, (W1).

Foreign Institutions Block Equations

$$YFWOR_f = \sum_w INSVA_{w,f} \quad . \qquad \text{(W1)}$$

Market Clearing Block

The market clearing equations ensure the simultaneous clearing of all markets. In this model there are six relevant markets: factor and commodity markets and enterprise, government, capital and rest of world accounts. Market clearing with respect to activities has effectively been achieved by (X16), wherein the supply and demand for domestically produced commodities was enforced, while the demand system and the specification of expenditure relationships ensures that the household markets are cleared.

The description immediately below refers to A default set of closure rules/market clearing conditions for this model; this choice of default should NOT be interpreted as defining THE appropriate settings when using the model. The model user needs to take responsibility for the clearing conditions: a subsequent section explores alternative closure rules//market clearing configurations available with this model.

Factor Market Clearing

Adopting an initial assumption of full employment, which the model closure rules will demonstrate can be relaxed, amounts to requiring that the factor market is cleared by equating factor demands (*FD*) and factor supplies by institutions (*FSI*) for all factors (C1)

Factor Market Clearing Block Equations

$$\sum_{insw} FSI_{insw,f} = \sum_{a} FD_{f,a} \qquad \forall \sum_{a} FD0_{f,a} . \tag{C1}$$

There is however no reason to suppose that the proportionate changes in the amount of labour time devoted to leisure and non-leisure activities will be identical across households. Even if the elasticities controlling the operation of the RHGs utility functions are the same there are differences in the levels of household incomes and preferences, i.e., there will be differences in the shift and share parameters of the utility functions. Thus, the presence of a labour-leisure trade-off means that the labour/factor supplies by institution (*FSI*) will be endogenously determined variables and hence the functional distribution of income can change.

Importantly, the factors (labour) used by institutions to produce leisure can only be supplied by the specific institution. Thus, the factor quantities supplied by each institution to produce leisure (*FSIL*) must be defined so as to be activity, and its paired RHG, and factor specific This is defined in equation C2 where the mapping (*map_hh_alei*) pairs leisure activities (*alei*) with RHGs (*hh*). Then the market clearing condition for the factor supplies by institution (*FSI*) and the demand for factors by non-leisure activities (*FD*) and leisure activities (*FSIL*) are determined as residuals.

Factor Mobility Equations

$$WFMOB_{f,fp} = \frac{WF_f}{WF_{fp}} \quad \forall fmob_{f,fp} \text{ and } \sum_a FD0_{f,a} \,. \tag{C2}$$

$$FSIM\_F_{f,fp,insa} = \left[fsi\_upd_{insa,fp} * \left(\frac{WFMOB_{f,fp}}{WFMOB\_upd_{f,fp}} \right)^{etaff_{f,fp,insa}} \right] - fsi\_upd_{insa,fp} \quad \forall map\_fmob\_fmob_{f,fp,insa} \text{ and } FSI_{insa,f} \text{ and } FSI_{insa,fp} \tag{C3}$$

$$FSIOUT_{insa,f} = \sum_{fp\$ fmobin_f} FSIM\_F_{fp,f,insa} \quad \forall fmobin_f \text{ and } \sum_{fp} map\_fmob\_fmob_{fp,f,insa} \tag{C4}$$

$$FSIIN_{insa,f} = \sum_{fp\$ fmobin_f} FSIM\_F_{f,fp,insa} \quad \forall fmobin_f \text{ and } \sum_{f} map\_fmob\_fmob_{fp,f,insa} \tag{C5}$$

$$FSI_{insa,f} = fsi\_upd_{insa,f} - FSIOUT_{insa,f} + FSIIN_{insa,f} \quad \forall \sum_{fp,fpp} map\_fmob\_fmob_{fp,fpp,insa} \tag{C6}$$

The factor mobility options provide further possibilities for changes in the factor supplies by institution. These options allow the model user to relax the restrictive assumption that labour types are rigidly segmented such that some labour owned by an institution can transition into an alternative labour category in response to changes in relative wage rates. This system ensures that the quantities of labour in natural units, e.g., person hours, owned by each institution are constant, *ceteris paribus*,[34] but the mix can adjust.

[34] In model variants that allow the size of the 'workforce', i.e., those employed and unemployed by activities within the SNA production boundary, to vary, e.g., where there is a labour-leisure trade-off or

The incentives for institutions to transition some labour from one category to another are assumed to be changes in the relative prices of different categories of labour (C2). The pair-wise relative prices of labour types are expressed as the ratios of the average wage rate in the destination (*f*) categories and the average wage rate in the source (*fp*) categories with the responsiveness of labour to transition being (exogenously) defined by the elasticity $etaff_{f,fp,insa}$. If the elasticity is zero then labour cannot transition and the non-zero elements in the matrix of elasticities define membership of the set $map\_fmob\_fmob_{f,fp,insa}$, which defines the destination (*f*) and source (*fp*) categories by institution (*insa*). The quantities of labour that transition by institutions (FSIM_F) is defined as the quantity of workers that transition relative to the number of workers in the source (*fp*) category ($fsi\_upd_{insa,f}$) (C3). In comparative static applications $fsi\_upd_{insa,f}$ is equal to numbers owned by each institution in the base period. In recursive dynamic applications the numbers owned by each institution are updated (*upd*) after each solution phase. This is implemented for each transition path that is identified as open, by $map\_fmob\_fmob_{f,fp,insa}$, and for each case where ownership of both source and destination is non-zero.

The quantities of labour exiting a source ($FSIOUT_{ins,f}$) and the quantities entering a destination ($FSIIN_{ins,f}$) are recorded (C4 and C5) and the factor supplies by institution ($FSI_{insa,f}$) are accordingly updated (C6).

The model is coded so that all factors are potentially pair-wise mobile, whereas in practice it would be expected that this code is applied for labour categories. Care needs to be taken in assigning the pathways that are open, e.g., a transition from unskilled agricultural worker to senior managers outside of agriculture may not be deemed a reasonable short-term assumption. Mobility of factor between broad groups (labour, capital and land) are NOT sensible so care should be taken to exclude such pathways.

Commodity Market Clearing

Market clearing for the composite commodity markets requires that the supplies of the composite commodity (*QQ*) are equal to total of domestic demands for composite commodities, which consists of intermediate demand (*QINTD*), household (*QCD* and *QCD*2), enterprise (*QED*) and government (*QGD*) and investment (*QINVD*) final demands (C13).

'surplus' labour, the quantities of labour by institution can vary. These variants do not conflict with these options.

Note how the market clearing condition with respect to final demand by households has to be formulated so as to avoid double counting by ensuring that no aggregate commodities enter into the definition (RHS) of domestic demand. Since the markets for domestically produced commodities are also cleared (X16) this ensures a full clearing of all commodity markets. Similarly, it is necessary to ensure clearing of the production of differentiated commodities by activities when activities can adjust their output mixes in response to changes in relative commodity prices; this is done in equation (C12).

Commodity Market Clearing Block Equations

$$QXAC_{a,c} = IOQXACQXV_{a,c} + QX_a \tag{C12}$$

$$QQ_c = QTTD_c + \sum_a QINTD_{c,a} + \sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + \sum_{in} QINVD_{in,c} + dstocconst_c \tag{C13}$$

Macroeconomic Closure Block

Making savings a residual for each account clears the two institutional accounts that are not cleared elsewhere – government and rest of the world. Thus the government account clears (C14) by defining government savings (*KAPGOV*) as the difference between government income and other expenditures, i.e., a residual. The rest of world account clears (C15) by defining the balance on the capital account (*CAPWOR*) as the difference between expenditure on imports, of commodities and factor services, and total income from the rest of the world, which includes export revenues and payments for factor services, transfers from the rest of the world to the household, enterprise and government accounts, i.e., it is a residual.

Macroeconomic Closure Block Equations

$$KAPGOV = YG - EG. \tag{C14}$$

$$CAPWOR = \left[\begin{array}{l} \left(\sum_{cm} PWM_{cm} * QM_{cm} \right) + \left(\sum_{f} \frac{YFWOR_f}{ER} \right) \\ + \left(\sum_{e} \frac{WORENT_e}{ER} \right) \end{array} \right] - \left[\begin{array}{l} \left(\sum_{ce} PWE_{ce} * QE_{ce} \right) + \left(\sum_{f} factwor_f \right) \\ + \left(\sum_{h} howor_h \right) + \sum_{e} entwor_e + govwor \end{array} \right]. \quad \text{(C15)}$$

Absorption Closure

The total value of domestic final demand (*VFDOMD*) is defined (C16) as the sum of the expenditures on final demands by households and other domestic institutions (enterprises, government and investment). Note again that the value of final demand must exclude the demand for aggregate commodities to avoid double counting.

It is also useful to express the values of final demand by each non-household domestic institution as a proportion of the total value of domestic final demand; this allows the implementation of what has been called a 'balanced macroeconomic closure'.[35] Hence the share of the value of final demand by enterprises (C17) can be defined as a proportion of total final domestic demand, and similarly for government's value share of final demand (C18) and for investment's value share of final demand (C19).

If the share variables (*VEDSH*, *VGDSH* and *INVESTSH*) are fixed then the quantity adjustment variables on the associated volumes of final demand by domestic non-household institutions (*QEDADJ*, *QGDADJ* and *IADJ* or *S*ADJ*) must be free to vary. On the other hand if the volume adjusters are fixed the associated share variables must be free so as to allow the value of final demand by 'each' institution to vary.

Absorption Closure Block Equations

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[35] The adoption of such a closure rule for this class of model has been advocated by Sherman Robinson and is a feature, albeit implemented slightly differently, of the IFPRI standard model.

$$VFDOMD = \left(\sum_{c,h} PQCD_c * QCD_{c,h} \right) + \left(\sum_{e,c} PQD_c * QED_{e,c} \right) + \left(\sum_{c} PQD_c * QGD_c \right) + \left(\sum_{in,c} PQD_c * QINVD_{in,c} \right) + \left(\sum_{c} PQD_c * dstoccont_c \right) \tag{C16}$$

$$VEDSH_e = VED_e / VFDOMD \tag{C17}$$

$$VGDSH = VGD / VFDOMD \tag{C18}$$

$$INVESTSH = INVEST / VFDOMD \tag{C19}$$

Slack

The final account to be cleared is the capital account. Total savings (*TOTSAV*), see I3 above, is defined within the model and hence there has been an implicit presumption in the description that the total value of investment (*INVEST*) is driven by the volume of savings. This is the market clearing condition imposed by (C20). But this market clearing condition includes another term, *WALRAS*, which is a slack variable that returns a zero value when the model is fully closed and all markets are cleared, and hence its inclusion provides a quick check on model specification.

SLACK Block Equations

$$TOTSAV = INVEST + WALRAS \tag{C20}$$

GDP

It is not necessary to include a variable in the model for GDP, since GDP is a simple summary 'variable' that can be calculated from the simulation results. However, it is convenient in some circumstances, e.g., while benchmarking a recursive dynamic model, to include GDP as a variable. In this model GDP is included as a variable that is calculated from the expenditure side, i.e., domestic absorption (valued a purchaser prices)[36] plus exports (valued at basic prices) less imports (valued at basic prices), (C21). For the same reasons, domestic absorption (*ABSORP*) is also included as a variable (C22).

GDP Block Equations

$$\begin{aligned} GDP = & \left(\sum_{c,h} PQCD_c * QCD_{c,h}\right) + \left(\sum_{e,c} PQD_c * QED_{e,c}\right) \\ & + \left(\sum_{c} PQD_c * QGD_c\right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c}\right) \\ & + \left(\sum_{c} PQD_c * dstocconst_c\right) \\ & + \left(\sum_{c} PWE_c * QE_c * ER\right) - \left(\sum_{c} PWM_c * QM_c * ER\right) \end{aligned} \quad \text{(C21)}$$

$$\begin{aligned} ABSORP = & \left(\sum_{c,h} PQCD_c * QCD_{c,h}\right) + \left(\sum_{e,c} PQD_c * QED_{e,c}\right) \\ & + \left(\sum_{c} PQD_c * QGD_c\right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c}\right) \\ & + \left(\sum_{c} PQD_c * dstocconst_c\right) \end{aligned} \quad \text{(C22)}$$

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36 Again, note the need to avoid double counting that would occur if aggregate commodities were included.

4. Model Closure Conditions or Rules

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of the main options available with this formulation of the model by reference to the accounts to which the rules refer.

Foreign Exchange Account Closure

The closure of the rest of the world account can be achieved by fixing either the exchange rate variable (AC1a) or the balance on the current account (AC1b). Fixing the exchange rate is appropriate for countries with a fixed exchange rate regime whilst fixing the current account balance is appropriate for countries that face restrictions on the value of the current account balance, e.g., countries following structural adjustment programmes. It is a common practice to fix a variable at its initial level by using the associated parameter, i.e., ***0, but it is possible to fix the variable to any appropriate value.

The model is formulated with the world prices for traded commodities declared as variables, i.e., PWM_c and PWE_c. If a strong small country assumption is adopted, i.e., the country is assumed to be a price taker on all world commodity markets, and then all world prices will be fixed. When calibrating the model the world prices will be fixed at their initial levels, (AC1c), but this does not mean they cannot be changed as parts of experiments.

However, the model allows a relaxation of the strong small country assumption, such that the country may face a downward sloping demand curve for one or more of its export commodities. Hence the world prices of some commodities are determined by the interaction of demand and supply on the world market, i.e., they are variables. This is achieved by limiting the range of world export prices that are fixed to those for which there are no export demand function, (AC1d), by selecting membership of the set *cedn*.

Foreign Exchange Market Closure Equations

$$ER = \overline{ER} \qquad \text{(AC1a)}$$

$$CAPWOR = \overline{CAPWOR} \qquad \text{(AC1b)}$$

$$PWE_c = \overline{PWE_c}$$
$$PWM_c = \overline{PWM_c} \qquad \text{(AC1c)}$$

$$PWE_{cedn} = \overline{PWE_{cedn}} \qquad \text{(AC1d)}$$

<u>Capital Account Closure</u>

To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. There are multiple ways of achieving this result. For instance this can be achieved by fixing either the saving rates for households or the volumes of commodity investment. This involves fixing either the savings rates adjusters (AC2a) or the investment volume adjuster (AC2c). Note that fixing the investment volume adjuster (AC2b) means that the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*). Note also that only one of the savings rate adjusters should be fixed; if *SADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of households and enterprises, if *SHADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of households, and if *SEADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of enterprises. Alternatively savings rates can be adjusted through the additive adjustment factors (DS, DSHH, DSEN) with the same relationships between the savings rates of different classes of institutions (AC2b). Note that there are other sources of savings. The magnitudes of these other savings sources can also be changed through the closure rules (see below).

Capital Account Closure Equations

$$SADJ = \overline{SADJ}$$
$$SHADJ = \overline{SHADJ}$$
$$SEADJ = \overline{SEADJ} \qquad \text{(AC2a)}$$

$$DS = \overline{DS}$$
$$DSHH = \overline{DSHH}$$
$$DSEN = \overline{DSEN} \qquad \text{(AC2b)}$$

$$IADJ = \overline{IADJ} \qquad \text{(AC2c)}$$

$$INVEST = \overline{INVEST} \qquad \text{(AC2d)}$$

$$INVESTSH = \overline{INVESTSH} \qquad \text{(AC2e)}$$

Fixing savings, and thus deeming the economy to be savings-driven, could be considered a Neo-Classical approach. Closing the economy by fixing investment could be construed as making the model reflect the Keynesian investment-driven assumption for the operation of an economy.

The model includes a variable for the value of investment (*INVEST*), which can also be used to close the capital account (AC2d). If *INVEST* is fixed in an investment driven closure, then the model will need to adjust the savings rates to maintain equilibrium between the value of savings (*TOTSAV*) and the fixed value of investment. This can only be achieved by changes in the volumes of commodities demanded for investment (*QINVD*) or their prices (*PQD*). But the prices (*PQD*) depend on much more than investment, hence the main adjustment must take place through the volumes of commodities demanded, i.e., *QINVD*, and therefore the volume adjuster (*IADJ*) must be variable, as must the savings rate adjuster (*SADJ*).

Alternatively the share of investment expenditure in the total value of domestic final demand can be fixed, (AC2e), which means that the total value of investment is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *INVEST* equal to some level.

<u>Enterprise Account Closure</u>

Fixing the volumes of commodities demand by enterprises, (AC3a), closes the enterprise account. Note that this rule allows the value of commodity expenditures by the enterprise account to vary, which *ceteris paribus* means that the value of savings by enterprises (*CAPENT*) and thus total savings (*TOTSAV*) vary. If the value of this adjuster is changed, but left fixed, this imposes equiproportionate changes on the volumes of commodities demanded.

Enterprise Account Closure Equations

$$QEDADJ = \overline{QEDADJ} \qquad \text{(AC3a)}$$

$$VED = \overline{VED} \, . \qquad \text{(AC3b)}$$

$$VEDSH = \overline{VEDSH} \qquad \text{(AC3c)}$$

$$HEADJ = \overline{HEADJ} \qquad \text{(AC3d)}$$

If *QEDADJ* is allowed to vary then another variable must be fixed; the most likely alternative is the value of consumption expenditures by enterprises (*VED*) (AC3b). This would impose adjustments through equiproportionate changes in the volumes of commodities demanded, and would feed through so that enterprise savings (*CAPENT*) reflecting directly the changes in the income of enterprises (*YE*). Alternatively the share of enterprise expenditure in the total value of domestic final demand can be fixed, (AC3c), which means that the total value of enterprise consumption expenditure (*VED*) is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *VED* equal to some level.

Finally, the scaling factor for enterprise transfers to households (*HEADJ*) needs fixing (AC3d).

<u>Government Account Closure</u>

The closure rules for the government account are slightly more tricky because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income and expenditure are 'fixed', government savings must be free to adjust.

Thus in the base specification all the tax rates (variables) are fixed by declaring the base tax rates as parameters and then fixing all the multiplicative and additive tax rate scaling factors (AC4a – AC4r).

Tax Rate Adjustment Closure Equations

$$TMADJ = \overline{TMADJ} \quad \text{(AC4a)}$$

$$TEADJ = \overline{TEADJ} \quad \text{(AC4b)}$$

$$TSADJ = \overline{TSADJ} \quad \text{(AC4c)}$$

$$TQSADJ = \overline{TQSADJ} \quad \text{(AC4d)}$$

$$TVADJ = \overline{TVADJ} \quad \text{(AC4e)}$$

$$TEXADJ = \overline{TEXADJ} \quad \text{(AC4f)}$$

$$TXADJ = \overline{TXADJ} \quad \text{(AC4g)}$$

$$TFADJ = \overline{TFADJ} \quad \text{(AC4h)}$$

$$TYADJ = \overline{TYADJ} \quad \text{(AC4i)}$$

$$TYEADJ = \overline{TYEADJ} \quad \text{(AC4j)}$$

$$TYHADJ = \overline{TYHADJ} \quad \text{(AC4k)}$$

$$DTM = \overline{DTM} \quad \text{(AC4l)}$$

$$DTE = \overline{DTE} \quad \text{(AC4m)}$$

$$DTS = \overline{DTS} \quad \text{(AC4n)}$$

$$DTQS = \overline{DTQS} \quad \text{(AC4o)}$$

$$DTV = \overline{DTV} \quad \text{(AC4p)}$$

$$DTEX = \overline{DTEX} \quad \text{(AC4q)}$$

$$DTX = \overline{DTX} \quad \text{(AC4r)}$$

$$DTF = \overline{DTF} \quad \text{(AC4s)}$$

$$DTYF = \overline{DTYF} \quad \text{(AC4t)}$$

$$DTYH = \overline{DTYH} \quad \text{(AC4u)}$$

$$DTYE = \overline{DTYE} . \quad \text{(AC4v)}$$

$$MTAX = \overline{MTAX} . \quad \text{(AC4v)}$$

$$ETAX = \overline{ETAX} . \quad \text{(AC4v)}$$

$$STAX = \overline{STAX} . \quad \text{(AC4v)}$$

$$QSTAX = \overline{QSTAX} . \quad \text{(AC4v)}$$

$$VTAX = \overline{VTAX} \quad \text{.} \qquad \text{(AC4v)}$$

$$EXTAX = \overline{EXTAX} \quad \text{.} \qquad \text{(AC4v)}$$

$$FTAX = \overline{FTAX} \quad \text{.} \qquad \text{(AC4v)}$$

$$ITAX = \overline{ITAX} \quad \text{.} \qquad \text{(AC4v)}$$

$$FYTAX = \overline{FYTAX} \quad \text{.} \qquad \text{(AC4v)}$$

$$DTAX = \overline{DTAX} \quad \text{.} \qquad \text{(AC4v)}$$

Consequently changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (GR1 to GR8). The two other sources of income to the government are controlled by parameters, *govvash* and *govwor*, and therefore are not a source of concern for model closure.[37]

Also note that because there are equations for the revenues by each tax instrument (GR1 to GR8) it is straightforward to adjust the tax rates to achieve a given volume of revenue from each tax instrument; this type of arrangement is potentially useful in circumstances where is is argued/believed that binding constraints upon the revenue possibilities from specific tax instruments.

In the base specification government expenditure is controlled by fixing the volumes of commodity demand (*QGD*) through the government demand adjuster (*QGDADJ*) in (AC4s). Alternatively either the value of government consumption expenditure (*VGD*) can be fixed, (AC4t), or the share of government expenditure in the total value of domestic final demand (*VGDSH*) can be fixed, (AC4u). The scaling factor on the values of transfers to households and enterprises through the household (*HGADJ*) and enterprise (*EGADJ*) adjusters, (AC4v and AC4w) also need to be fixed.

Government Expenditure Closure Equations

37 The values of income from non-tax sources can of course vary because each component involves a variable.

$$QGDADJ = \overline{QGDADJ} \qquad \text{(AC4s)}$$

$$VGD = \overline{VGD} \qquad \text{(AC4t)}$$

$$VGDSH = \overline{VGDSH}\,. \qquad \text{(AC4u)}$$

$$HGADJ = \overline{HGADJ} \qquad \text{(AC4v)}$$

$$EGADJ = \overline{EGADJ} \qquad \text{(AC4w)}$$

$$CAPGOV = \overline{CAPGOV} \qquad \text{(AC4x)}$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not directly control. Hence the equilibrating condition is that government savings, the internal balance, is not fixed.

If however the model requires government savings to be fixed (AC4x), then either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

In its simplest form this can be achieved by allowing one of the previously fixed adjusters (AC4a to AC4w) to vary. Thus if the sales tax adjuster (*TSADJ*) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables *TMADJ* and *KAPGOV* would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the sales tax mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates it is necessary to change the relevant tax parameters. Typically such changes would be implemented in policy experiment files rather than within the closure section of the model.

Numéraire

The model specification allows for a choice of two price normalisation equations (AC5a and AC5b), the consumer price index (CPI) and a producer price index (PPI). A *numéraire* is needed to serve as a base since the model is homogenous of degree zero in prices and hence only defines relative prices.

Numéraire Closure Equations

$$CPI = \overline{CPI} \qquad \text{(AC5a)}$$

$$PPI = \overline{PPI}\,. \qquad \text{(AC5b)}$$

Factor Market Closure

The factor market closure rules are more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply (*FSI*) of and total demand for factors (*FD*) equate (AC6a). The total supplies of each factor are determined exogenously and hence defines the first set of factor market closure conditions. The demands for factor *f* by activity *a* and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages, by allowing for the possibility that the payments to notionally identical factors might vary across activities through the variable that captures the 'sectoral proportions for factor prices'. These proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, and are therefore assumed fixed, (AC6b). Finally while it may seem that factor prices must be limited to positive values the actual bounds placed upon the average factor prices, (AC6c) are plus or minus infinity. This is a consequence of the use of the PATH solver.

Basic Factor Market Closure Equations

$$\sum_{insw} FSI_{insw,f} = \overline{FS}_f \tag{AC6a}$$

$$WFDIST_{f,a} = \overline{WFDIST_{f,a}} \, . \tag{AC6b}$$

$$\begin{aligned} \text{Min } WF_f &= -\text{infinity} \\ \text{Max } WF_f &= +\text{infinity} \end{aligned} \tag{AC6c}$$

Factor Immobility and/or Unemployment Closures

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as factors. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations (AC6d) where *fact* indicates the specific factor and *activ* a specific activity. This block of equations includes all the variables that were declared for the model with reference to factors plus an extra equation for *WFDIST*, i.e., $WFDIST_{fact,activ} = \overline{WFDIST}_{fact,activ}$, whose role will be defined below. The choice of which equations are binding and which are not imposed will determine the factor market closure conditions.

Factor Block Equations

$$
\begin{aligned}
FS_{fact} &= \overline{FS_{fact}} \\
WFDIST_{fact,a} &= \overline{WFDIST_{fact,a}} \\
\text{Min } WF_{fact} &= -\text{infinity} \\
\text{Max } WF_{fact} &= +\text{infinity} \\
FD_{fact,a} &= \overline{FD_{fact,a}} \\
WF_{fact} &= \overline{WF_{fact}} \\
WFDIST_{fact,activ} &= \overline{WFDIST_{fact,activ}} \\
\text{Min } FS_{fact} &= -\text{infinity} \\
\text{Max } FS_{fact} &= +\text{infinity}
\end{aligned}
\qquad \text{(AC6d)}
$$

As can be seen the first four equations in the block (AC6d) are the same as those in the 'Full Factor Mobility and Employment Closure'; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the 'Full Factor Mobility and Employment Closure'. Assume that this set of conditions represents a starting point, i.e., the first four equations are binding and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, i.e., the condition ($FD_{fact,a} = \overline{FD_{fact,a}}$) must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., the condition (AC6b) must now be relaxed.

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition (AC6a) is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e., (AC6b), which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.[38]

Factor Market Closure Equations

$$FD_{fact,a} = \overline{FD_{fact,a}} \quad \text{(AC6e)}$$

$$WFDIST_{fact,a} = \overline{WFDIST_{fact,a}} \quad \text{(AC6f)}$$

$$FS_{fact} = \overline{FS_{fact}} \quad \text{(AC6g)}$$

$$WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}} \quad \text{(AC6h)}$$

$$WF_{fact} = \overline{WF_{fact}} \quad \text{(AC6i)}$$

$$FS_{fact} = \overline{FS_{fact}} \quad \text{(AC6j)}$$

$$\text{Min } FS_{fact} = 0$$
$$\text{Max } FS_{fact} = +\text{infinity} \quad \text{(AC6k)}$$

Start again from the closure conditions for full factor mobility and employments and then assume that there is unemployment of one or more factors in the economy; typically this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing the condition that factor prices are fixed (AC6i) and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing (AC6a). It

38 It can be important to ensure a sensible choice of reference activity. In particular this is important if a factor is not used, or little used, by the chosen activity.

is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions (AC6k) can be imposed.[39]

Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

Factor Market Clearing Equations

$$FD_{f,activ} = \overline{FD_{f,activ}} \tag{AC6l}$$

$$WFDIST_{f,activ} = \overline{WFDIST_{f,activ}} \tag{AC6m}$$

If all factors used by an activity are fixed, this requires imposing the conditions that factor demands are fixed, (AC6l), where *activ* refers to the activity of concern. But the returns to these factors in this activities must now be allowed to vary, i.e., the conditions (AC6m) must now be relaxed. In this case the condition fixing the total supply of the factor is not redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to manage the model by gathering together factor market conditions that are inspired by activity characteristics after

39 If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supersedes previous conditions.

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Appendix 1: STAGE Model Genealogy

The STAGE model started life in the mid-1990s. After initial (futile) struggles with the Cameroon CGE model then in the GAMS library Sherman Robinson gave Scott a copy of the single country CGE model developed for the US Department of Agriculture's (USDA) Economic Research Service (ERS) under Sherman's leadership (Robinson *et al*., 1990; Kilkenny, 1991). The USDA ERS model was based on an input-output representation of the inter industry transactions that limited the applicability of the model for the analyses of the decisions made by multi-product activities. This concern was raised with Sherman and Hans Lofgren in the late 1990s[40]; this problem was addressed by Hans and Sherman and a copy of the solution was shared with Scott. The developments by Hans and Sherman at IFPRI ultimately resulted in the production of the IFPRI standard model in 2001 (Lofgren *et al*., 2001). Consequently the IFPRI standard and STAGE models share a common heritage and a number of features although there also differences.

The USDA ERS model provided the basis for the PROVIDE project model (McDonald, 2003). This model also included the treatment of multi-product activities developed for the IFPRI Standard model but included different treatments of margins and differences that reflected issues relevant to South Africa at that time. Valuable contributions to the PROVIDE model were made by Cecilia Punt, Melt van Schoor, Lindsay Chant and Kalie Pauw. Melt van Schoor also made major contributions through the development of the SAMGator and SeeResults interfaces. An energy version of the PROVIDE model was developed with Jonah Tlhalefang and was used in Jonah's PhD thesis at the University of Sheffield. The 'final'/most developed version of the PROVIDE model appeared in Cecilia Punt's PhD thesis from the University of Stellenbosch, which among other things, included explicit modelling of changes in the composition of outputs by activities.

The PROVIDE project model developed into the STAGE model as part of the process of developing the GLOBE model from 2002 with Karen Thierfelder. Karen had also started her modelling career using the USDA ERS model and the NAFTA model in the 1990s.

The GLOBE model used a simplified variant of the STAGE model as the basis for the development of the within country/region behavioural equations. This process generated some

40 The issue had become relevant when estimating the implications of BSE (McDonald and Roberts, 1998).

changes in behavioural relationships, code structure, methods for analyzing results and notation. Consequently in 2005 the STAGE 1 model was consolidated from previous models and made open source with some revisions from 2009. The STAGE 2 model was a developed as a series of variants on a core model between 2009 and 2019. It included contributions made with Karen Thierfelder, Cecilia Punt, Emanuele Ferrari, Dorothee Flaig, Emerta Aragie, Jonas Luckmann and Arndt Feuerbacher.

The STAGE 3 model synthesizes many of the ideas included in variants of STAGE 2. The introduction of a system of generalized production and consumption systems means that many of the variants of STAGE 2 are redundant. The model also includes the scope to include, *inter alia*, labour-leisure trade-offs, activities outside of the conventional SNA production boundary, environmental concerns, demographics, and an enhanced modelling of investment decisions. Many of these developments enhance the use of the model in recursive dynamic mode.

STAGE is part of a suite of models that include two global models (ANARRES and R23 models) and a range of teaching models – the SMOD suite. All these model use a (overwhelmingly) common set of notation and formats.

Appendix 2: Parameter, Variable and Equation Lists STAGE

The parameter and variable listings are in alphabetic order, and are included for reference purposes. The parameters listed below are those used in the behavioural specifications/equations of the model, in addition to these parameters there are two further sets of parameters. The first extra set of parameters is used in model calibrated and for deriving results; there is one such parameter for each variable and they are identified by appending a ‘0’ (zero) to the respective variable name. The second extra set are parameters used as intermediate parameters used as part of the calibration process or parameters used to check the calibrations.

Parameter List

Parameter Name	Parameter Description
ac(c)	Shift parameter for Armington CES function
adfag(ff,a)	Shift parameter for factor and activity specific efficiency
adva01(a)	0-1 par for flexing of shift parameter on functions for QVA
advab(a)	Shift parameter for CES production functions for QVA
adx01(a)	0-1 par for flexing of shift parameter on functions for QX
adxb(a)	Shift parameter for CES production functions for QX
adxc(c)	Shift parameter for commodity output CES aggregation
alphah(c,h)	Expenditure share by commodity c for household h
at(c)	Shift parameter for Armington CET function
ati(a)	Shift parameter for Armington CET function
beta(c,h)	Marginal budget shares
co2co(c,sac)	CO2 emission coeff tonnes CO2 per tonne oil equivalent
comactco(c,a)	use matrix coefficients
comhoav(c,h)	Household consumption shares
comtotsh(c)	Share of commodity c in total commodity demand
dabadva(a)	Change in base shift parameter on functions for QVA
dabadx(a)	Change in base shift parameter on functions for QX
dabsen(e)	Change in base Enterprise saving rates
dabshh(h)	Change in base Household saving rates
dabte(c)	Change in base export taxes on comm'y imported from region w
dabtex(c)	Change in base excise tax rate
dabtf(ff,a)	Change in base factor us tax rate on activities
dabtm(c)	Change in base tariff rates on comm'y imported from region w
dabts(c)	Change in base sales tax rate
dabtss(c)	Change in base sales tax 2 rate
dabtv(c)	Change in base value added tax rate
dabtx(a)	Change in base indirect tax rate
dabtye(e)	Change in base direct tax rate on enterprises
dabtyf(f)	Change in base direct tax rate on factors
dabtyh(h)	Change in base direct tax rate on households
delta(c)	Share parameter for Armington CES function
deltafd(ff,ff,a)	CES Share parameters for Aggregated FD fag using ff by a
deltava(ff,a)	Share parameters for CES production functions for QVA
deltax(a)	Share parameter for CES production functions for QX
deltaxc(a,c)	Share parameters for commodity output CES aggregation
deprec(f)	accounting depreciation rate by factor f
dirpay(h)	Agricultural support by direct payment to Agric households
dstocconst(c)	Stock change demand volume

econ(c)	constant for export demand equations
entgovconst(e)	Government transfers to enterprise e
entvash(e,f)	Share of income from factor f to enterprise e
entvashchk(e,f)	Share of income from factor f to enterprise e
entwor(e)	Transfers to enterprise e from world (constant in foreign currency)
eta(c)	export demand elasticity
factwor(f)	Factor payments from RoW (constant in foreign currency)
frisch(h)	Elasticity of the marginal utility of income
gamma(c)	Share parameter for Armington CET function
gammai(a,c)	Share parameter for Armington CET function per activity
goventsh(e)	Share of entp' income after tax save and consump to govt
govvash(f)	Share of income from factor f to government
govvashchk(f)	Share of income from factor f to government
govwor	Transfers to government from world (constant in foreign currency)
hexp_sub0(h)	Subsistence consumption expenditure in base
hexps(h)	Subsistence consumption expenditure
hoentconst(h,e)	transfers to hhold h from enterprise e (nominal)
hoentsh(h,e)	Share of entp' income after tax save and consump to h'hold
hogovconst(h)	Transfers to hhold h from government (nominal but scalable)
hohoconst(h,hp)	interhousehold transfers
hohosh(h,hp)	Share of h'hold h after tax and saving income transferred to hp
hovash(h,f)	Share of income from factor f to household h
hovashchk(h,f)	Share of income from factor f to household h
howor(h)	Transfers to household from world (constant in foreign currency)
invconst(c,i)	Investment demand volume for investment of type i
ioqintqx(a)	Agg intermed quantity per unit QX for Level 1 Leontief agg
ioqinvd(c,i)	Input of commodity c per unit of investment of type i
ioqinvdCHK(i)	Check on Input of commodity c per unit of investment of type i
ioqtdqd(c,a)	intermediate input output coefficients
ioqtdqtt(c,m)	quantity of commodity c used to produce a unit of margin m
ioqttqe(m,c)	quantity of margin m used per unit of export demand QE
ioqttqq(m,c)	quantity of margin m used per unit of domestic demand QQ
ioqvaqx(a)	Agg value added quant per unit QX for Level 1 Leontief agg
ioqxacqx(a,c)	Share of commodity c in output by activity a
nest_fd(ff,ff,a)	All nests below VA
nest_va(ff,a)	Value added nest
predeltax(a)	dummy used to estimated deltax
pwse(c)	world price of export substitutes
qcdconst(c,h)	Volume of subsistence consumption
qcdconst_neg(c,h)	Negative subsistence consumption indicates a calibration error
qedconst(c,e)	Enterprise demand volume
qgdconst(c)	Government demand volume
qinvb(i)	Investment volume by investment type i in base
rhoc(c)	Elasticity parameter for Armington CES function
rhocva(a)	Elasticity parameter for CES production function for QVA
rhocx(a)	Elasticity parameter for CES production function for QX
rhocxc(c)	Elasticity parameter for commodity output CES aggregation
rhofd(ff,a)	Elasticity parameter for CES prodn fns for Aggregated FD
rhot(c)	Elasticity parameter for Output Armington CET function
rhoti(a)	Elasticity parameter for Output Armington CET function
sen0(e)	Initial Enterprise saving rates
sen01(e)	0-1 par for potential flexing of Enterprise saving rates
senb(e)	Base Enterprise saving rates
shh0(h)	Initial Household saving rates
shh01(h)	0-1 par for potential flexing of Household saving rates
shhb(h)	Base Household saving rates
sumelast(h)	Weighted sum of income elasticities
te01(c)	0-1 par for potential flexing of export taxes on comm'ies
teb(c)	Export subsidy rate
tex01(c)	0-1 par for potential flexing of excise tax rates

texb(c)	Excise tax rates
tf01(ff,a)	0-1 par for potential flexing of factor use tax rates
tfb(ff,a)	Factor use tax rate
tm01(c)	0-1 par for potential flexing of Tariff rates on comm'ies
tmb(c)	Tariff rate on commodity c
totdemd(c)	total commodity demand at purchaser prices
totsupp(c)	total commodity supply at basic prices
ts01(c)	0-1 par for potential flexing of sales tax rates
tsb(c)	Sales tax rates
tss01(c)	0-1 par for potential flexing of sales tax 2 rates
tssb(c)	Sales tax 2 rates
tv01(c)	0-1 par for potential flexing of value added tax rates
tvb(c)	Value added tax rates
tx01(a)	0-1 par for potential flexing of indirect tax rates
txb(a)	Indirect tax rate on activity a
tye01(e)	0-1 par for potential flexing of direct tax rates on e'rises
tyeb(e)	Direct tax rate on enterprises
tyf01(f)	0-1 par for potential flexing of direct tax rates on factors
tyfb(f)	Factor Income tax rate
tyh01(h)	0-1 par for potential flexing of direct tax rates on h'holds
tyhb(h)	Direct tax rate on household h
use(c,a)	use matrix transactions
vddtotsh(c)	Share of value of domestic output for the domestic market
worvash(f)	Share of income from factor f to RoW
worvashchk(f)	Share of income from factor f to RoW
yhelast(c,h)	Normalised) household income elasticities

Variable List

Variable Name	Variable Description
ABSORP	Absorption
ADFD(ff,a)	Shift parameter for factor and activity specific efficiency
ADVA(a)	Shift parameter for CES production functions for QVA
ADVAADJ	Scaling Factor for Shift parameter on CES functions for QVA
ADX(a)	Shift parameter for CES production functions for QX
ADXADJ	Scaling Factor for Shift parameter on CES functions for QX
CAPWOR	Current account balance
CO2EMIT(c,sac)	CO2 emissions by energy commodity by user
CO2EMIT_TOT	Total CO2 emissions
CPI	Consumer price index
DADVA	Partial scaling factor for Shift parameter on CES functions for QVA
DADX	Partial scaling factor for Shift parameter on CES functions for QX
DS	Partial household and enterprise savings rate scaling factor
DSEN	Partial enterprise savings rate scaling factor
DSHH	Partial household savings rate scaling factor
DTAX	Direct Income tax revenue
DTE	Partial Export tax rate scaling factor
DTEX	Partial Excise tax rate scaling factor
DTF	Uniform adjustment to factor use tax by activity
DTM	Partial Tariff rate scaling factor
DTS	Partial Sales tax rate scaling factor
DTSS	Partial Sales tax 2 rate scaling factor
DTV	Partial value added tax rate scaling factor
DTX	Partial Indirect tax rate scaling factor
DTY	Partial direct tax on hold and enterprise rate scaling factor
DTYE	Partial direct tax on enterprise rate scaling factor
DTYF	Partial direct tax on factor rate scaling factor
DTYH	Partial direct tax on household rate scaling factor
EG	Expenditure by government
EGADJ	Transfers to enterprises by government Scaling Factor
ENTSAV(e)	Enterprise savings
ER	Exchange rate (domestic per world unit)
ETAX	Export tax revenue
EXTAX	Excise tax revenue
FD(ff,a)	Demand for factor f by activity a
FS(ff)	Supply of factor f
FSI(insa,f)	Factor supplies from institution ins by factor f
FSISH(insa,f)	Shares of factor f supplied by institution ins
FTAX	Factor use tax revenue
FYTAX	Factor Income tax revenue
GDP	GDP from Expenditure
GOVENT(e)	Government income from enterprise e
HEADJ	Scaling factor for enterprise transfers to households
HEXP(h)	Household consumption expenditure
HGADJ	Scaling factor for government transfers to households
HHSAV(h)	Household savings
HOENT(h,e)	Household Income from enterprise e
HOHO(h,hp)	Inter household transfer
IADJ	Investment scaling factor
INSVA(insa,f)	Factor income after deprecn distribution to institn ins
INVEST	Total investment expenditure
INVESTSH	Value share of investment in total final domestic demand
INVSH_I(i)	Shares of savings to investment of type i
IOQXACQXV(a,c)	Share of commodity c in output by activity a
ITAX	Indirect tax revenue
KAPGOV	Government Savings
MTAX	Tariff revenue

PD(c)	Consumer price for domestic supply of commodity c
PE(c)	Domestic price of exports by activity a
PINT(a)	Price of aggregate intermediate input
PM(c)	Domestic price of competitive imports of commodity c
PPI	Producer (domestic) price index
PQCD(c)	Purchaser price of composite commodity cc private final demand
PQD(c)	Purchaser price of composite commodity c
PQS(c)	Supply price of composite commodity c
PTT(m)	Price of trade and transport margin m
PVA(a)	Value added price for activity a
PWE(c)	World price of exports in dollars
PWM(c)	World price of imports in dollars
PX(a)	Composite price of output by activity a
PXAC(a,c)	Activity commodity prices
PXC(c)	Producer price of composite domestic output
QCD(c,h)	Household consumption by commodity c
QD(c)	Domestic demand for commodity c
QE(c)	Domestic output exported by commodity c
QED(c,e)	Enterprise consumption by commodity c
QEDADJ	Enterprise demand volume Scaling Factor
QGD(c)	Government consumption demand by commodity c
QGDADJ	Government consumption demand scaling factor
QINT(a)	Aggregate quantity of intermediates used by activity a
QINTD(c,a)	Demand for intermediate inputs by commodity & activity
QINV(i)	Investment volume by investment type i
QINVD(c,i)	Investment demand by commodity c for investment type i
QM(c)	Imports of commodity c
QQ(c)	Supply of composite commodity c
QTT(m)	Quantity of trade and transport margin m
QTTD(c)	Intermediate input use for trade and transport margin m
QVA(a)	Quantity of aggregate value added for level 1 production
QX(a)	Domestic production by activity a
QXAC(a,c)	Domestic commodity output by each activity
QXC(c)	Domestic production by commodity c
SADJ	Savings rate scaling factor for BOTH households and enterprises
SEADJ	Savings rate scaling factor for enterprises
SEN(e)	Enterprise savings rates
SHADJ	Savings rate scaling factor for households
SHH(h)	Household savings rates
SSTAX	Sales tax 2 revenue
STAX	Sales tax revenue
TE(c)	Export taxes on exported comm'y c ad valorem
TEADJ	Export subsidy Scaling Factor
TEX(c)	Excise tax rate per unit quantity
TEXADJ	Excise tax rate scaling factor
TF(ff,a)	Tax rate on factor use
TFADJ	Factor Use Tax Scaling Factor
TM(c)	Tariff rates ad valorem
TMADJ	Tarrif rate Scaling Factor
TOTSAV	Total savings
TS(c)	Sales tax rate ad valorem
TSADJ	Sales tax rate scaling factor
TSS(c)	Sales tax 2 rate ad valorem
TSSADJ	Sales tax 2 rate scaling factor
TV(c)	Value added tax rate
TVADJ	Value added tax rate scaling factor
TX(a)	Indirect tax rate
TXADJ	Indirect Tax Scaling Factor
TYADJ	Household and Enterprise Income Tax Scaling Factor
TYE(e)	Direct tax rate on enterprises

TYEADJ	Enterprise income tax Scaling Factor
TYF(f)	Direct tax rate on factor income
TYFADJ	Factor Tax Scaling Factor
TYH(h)	Direct tax rate on households
TYHADJ	Household Income Tax Scaling Factor
UNEMP(f)	Unemployed factors
VED(e)	Value of enterprise e consumption expenditure
VEDSH(e)	Value share of Ent consumption in total final domestic demand
VFDOMD	Value of final domestic demand
VGD	Value of Government consumption expenditure
VGDSH	Value share of Govt consumption in total final domestic demand
VTAX	Value added tax revenue
WALRAS	Slack variable for Walras's Law
WF(ff)	Price of factor f
WFA(ff,a)	Activity specific factor prices
WFDIST(ff,a)	Sectoral proportion for factor prices
YE(e)	Enterprise incomes
YF(f)	Income to factor f
YFDISP(f)	Factor income for distribution after depreciation
YFINS(f)	Factor income for distribution to domestic non govt institutions
YFWOR(f)	Foreign factor income
YG	Government income
YH(h)	Income to household h