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STAGE

STAGE_t: Production System

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Outline

- CES or Leontief functions at top level
  - Allows option
  - Allows for activities with v small cost share from intermediates
- Multi product activities
  - Input Output Table: each activity produces one commodity and each commodity is produced by one activity
  - Supply and Use Table: each activity can produce many commodities and each commodity can be produced by many activities
  - Commodities produced by activities
    - Simple by-product assumption (fixed shares of output)
    - Commodities differentiated by source activity (CET)
      - Commodities differentiated by source activity (CES)

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Why use Supply and Use Tables?

- IOT are derived from SUT using simple linear transformations (SNA method)
- In STAGE multi-product relationships are explicitly modelled not subsumed in the data generation process
- Supplementing SUT data is relatively straightforward
  - SUT data structures used by national accounts agencies
    - Hence SUT data are more frequently available
    - Activity and commodity definitions are clear
  - Augmenting SUT data
    - Factor use data
    - Emission data

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RSA 'Domestic Output' Table

	cagric	cmins	cfood	ctext	cpetchem	cmprod	cmach	cveh	comanu	cutil	ccns	ctrad	cserv
aagric	63,248	0	0	0	0	0	0	0	0	0	717	0	0
amins	0	111,574	0	0	0	203	0	0	15	0	3,608	0	339
afood	133	0	95,838	0	636	20	0	0	51	0	0	2,465	5,588
atext	0	0	1	32,321	580	0	0	8	33	0	0	272	1,128
apetchem	0	0	225	634	137,533	1,136	87	510	1,361	0	0	3,408	7,282
amprod	0	0	0	6	692	113,454	1,963	1,385	478	0	0	411	4,988
amach	0	0	0	0	29	778	26,420	556	397	0	0	596	1,127
aveh	0	0	0	325	73	813	929	70,726	668	0	0	2,415	2,201
aomanu	0	0	0	317	1,918	138	713	187	91,964	0	0	719	4,080
autil	0	0	0	0	0	0	0	0	0	44,862	1,050	0	39
acns	0	0	0	0	0	0	0	0	0	0	87,551	101	843
atrad	0	0	0	0	0	0	0	0	0	0	0	355,855	3,906
aserv	0	0	0	0	0	0	0	0	0	0	0	485	581,697

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Top Level of Production Nest

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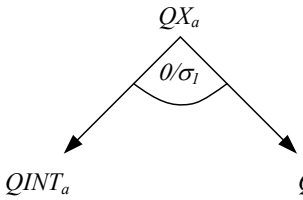
Top Level Production Nest: CES



- When σ_1 is >0
- Standard 2 argument CES cw FOC

$$QX_a = AD_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + \left(1-\delta_a^x\right) QINT_a^{-rhoc_a^x} \right)^{-\frac{1}{rhoc_a^x}} \quad \forall aqx_a$$

```
QX(a)  =E= ADX(a)*{deltax(a)*QVA(a)**(-rhocx(a))
          + (1-deltax(a))*QINT(a)**[-rhocx(a)]}
                **(-1/rhocx(a)) ;
```

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{\left(1-\delta_a^x\right)} \right]^{\frac{1}{\left(1+rhoc_a^x\right)}} \quad \forall aqx_a$$

Set of activities with CES

```
QVA(a)  =E= QINT(a)*{[PINT(a)/PVA(a))*(deltax(a)/
                (1-deltax(a))]}**[1/(1+rhocx(a))] ;
```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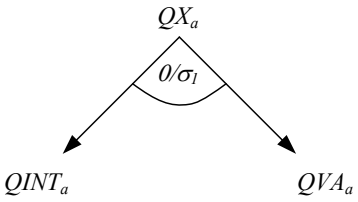
Top Level Production Nest: Leontief



- When σ_1 is =0
- Aggregate Value Added (*QVA*) and Intermediates (*QINT*) are fixed per unit of output (*io*****)

$$QVA_a = ioqvaqx_a * QX_a \qquad \forall aqxn_a$$

```
QVA(a) =E= ioqvaqx(a) * QX(a) ;
```

$$QINT_a = ioqintqx_a * QX_a \qquad \forall aqxn_a$$

Set of activities with LEONTIEF

```
QINT(a) =E= ioqintqx(a) * QX(a) ;
```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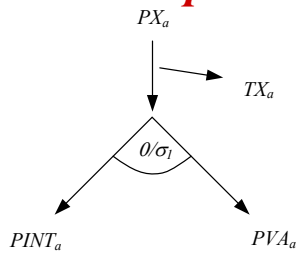
Top Level Production Nest: Prices



- When σ_1 is >0 AND = 0
- Exploits Eulers theorem applied to linear homogenous functions
- NB: price received by the activity for payment of inputs excludes (*ad valorem*) production taxes (*TX*)

Set of ALL activities

$$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a) \qquad \forall a$$

```
PX(a)*(1 - TX(a))*QX(a)
    =E= [(PVA(a)*QVA(a)) + (PINT(a)*QINT(a))] ;
```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Second Level of Production Nest

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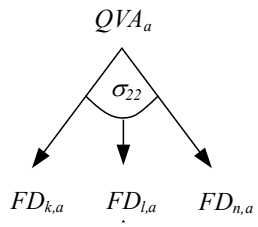
Second Level Nest: Value Added 1



- General form (*n*-argument) CES function
- Elasticity of substitution between pairs of factors identical
- *ADFD* are stock-flow relationships
- Mix of 'natural' and 'aggregate' factors

$$QVA_a = AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}}$$

ONLY implemented for map_va_ff AND *deltava* > 0

```
QVA(a) =E= ADVA(a)
          *(SUM(ff$[map_va_ff(ff,a) AND deltava(ff,a)],
                    deltava(ff,a)
          *(ADFD(ff,a)*FD(ff,a))**(-rhocva(a))))
                **(-1/rhocva(a)) ;
```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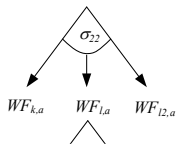
Second Level Nest: Value Added 2



- First order condition
- Different factoring to yield *QVA*

$$WFA_{f,a} * \left(1 + TF_{f,a}\right)$$

NB

$$= PVA_a * AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}} \right]^{-\left(\frac{1+\rho_a^{va}}{\rho_a^{va}}\right)} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)}$$

NB

$$= PVA_a * QVA_a * AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1}$$

$$* \delta_{f,a}^{va} * ADFD_{f,a}^{-\rho_a^{va}} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)}$$

ONLY implemented for *deltava* > 0

```
WFA(ff,a)*(1 + TF(ff,a))
    =E= PVA(a)*QVA(a)
        *{SUM[ffp$deltava(ffp,a),deltava(ffp,a)
        *[ADFD(ffp,a)*FD(ffp,a)]**(-rhocva(a))]}**(-1)
        *deltava(ff,a)*ADFD(ff,a)**(-rhocva(a))
         *FD(ff,a)**(-rhocva(a)-1) ;
```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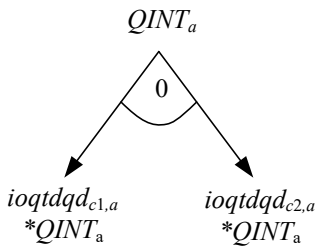
Second Level Nest: Intermediate Inputs



- When σ_1 is = 0.0
- Intermediate inputs are fixed per unit (*io******) of Aggregate Intermediates (*QINT*)
- Total intermediate demand by commodity (*QINTD*) is defined
- Intermediate commodity demand by activity (*c,a*) is not a variable

For *aqxn* from top level Leontief

```
QINTD(c) =E= SUM(a,ioqtdqd(c,a)*QINT(a)) ;
```

For *aqx* from top level FOC

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Another Level of Production Nest

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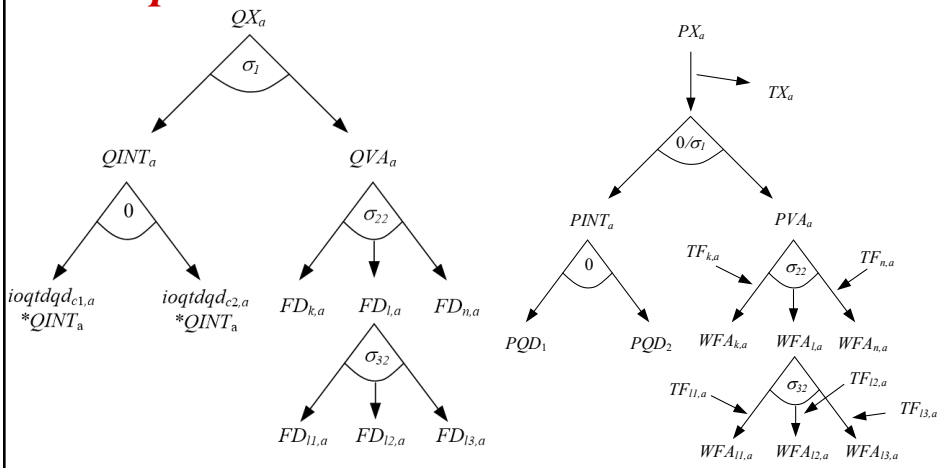
Simple Three Level Production Function



NB: in this case $WFA_{ff,a}$ defined as price paid by activity *a* to factor *f*

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Value Added Nest

```
QVAPRODFN(a)$rhocva(a)..
QVA(a) =E= ADVA(a)
           *(SUM(ff$[map_va_ff(ff,a) AND deltava(ff,a)],
                 deltava(ff,a)
           *(ADFD(ff,a)*FD(ff,a))**(-rhocva(a))))
                  **(-1/rhocva(a)) ;

QVAFOC(ff,a)$[map_va_ff(ff,a) AND deltava(ff,a)]..
 WFA(ff,a)*(1 + TF(ff,a))
        =E= PVA(a)*QVA(a)
            *{SUM[ffp$deltava(ffp,a),deltava(ffp,a)
            *[ADFD(ffp,a)*FD(ffp,a)]**(-rhocva(a))]}**(-1)
            *deltava(ff,a)*ADFD(ff,a)**(-rhocva(a))
                  *FD(ff,a)**(-rhocva(a)-1) ;
```

ff(sac)	factors and aggregates
f(ff)	natural factor accounts
fag(ff)	aggregate factors
map_va_ff(ff,a)	factor inputs to QVA

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Level 3 Production Nest

```
FDPRODFN(ff,a)$SUM[ffp,map_fagg_ff(ff,ffp,a)]..
FD(ff,a) =E= ADFAG(ff,a)
             *{SUM[ffp${map_fagg_ff(ff,ffp,a) AND deltafd(ff,ffp,a)},
               deltafd(ff,ffp,a) *FD(ffp,a)**(-rhofd(ff,a))]}
                  **[-1/rhofd(ff,a)] ;

FDFOC(ff,ffp,a)$[map_fagg_ff(ff,ffp,a) AND deltafd(ff,ffp,a)]..
WFA(ffp,a)*[1 + TF(ffp,a)]
       =E= {WFA(ff,a) *[1 + TF(ff,a)] * FD(ff,a)}
           *{SUM[ffpp${map_fagg_ff(ff,ffpp,a) AND deltafd(ff,ffpp,a)},
             deltafd(ff,ffpp,a)*FD(ffpp,a)**(-rhofd(ff,a))]}
             **(-1)*deltafd(ff,ffp,a)*FD(ffp,a)
                **[-rhofd(ff,a)-1] ;
```

ff(sac)	factors and aggregates
f(ff)	natural factor accounts
fag(ff)	aggregate factors
map_fagg_ff(ff,ff,a)	factor inputs to FD

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Commodity Outputs

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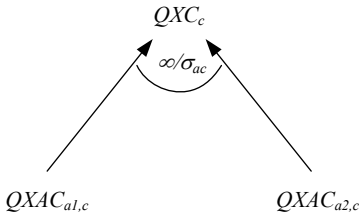
Supply of Commodities by Activities: CES 1



- When $\sigma_{ac} > 0$ but $< \infty$
- *n*-argument case
- Determined by need to satisfy *QXC*

$$QXC_c = adxc_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-1/\rho_c^{xc}}$$

commodities produced domestically

$\forall cx_c$ and $cxac_c$

commodities differentiated by activity

```
QXC(c) =E= adxc(c)*{SUM[a$deltaxc(a,c),deltaxc(a,c)
          *QXAC(a,c)**(-rhocxc(c))]}**(-1/rhocxc(c))
```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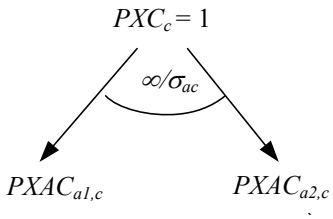
Supply of Commodities by Activities: CES 2



- When $\sigma_{ac} > 0$ but $< \infty$
- Standard *n*-argument FOC with *QXC* factored

$$PXAC_{a,c} = PXC_c * QXC_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)}$$

$$* \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)} \quad \forall cxac_c$$

commodities differentiated by activity

```
PXAC(a,c) =E= PXC(c)*QXC(c)
                *(SUM(ap$deltaxc(ap,c),deltaxc(ap,c)
                *QXAC(ap,c)**(-rhocxc(c))))**(-1)
                *deltaxc(a,c)*QXAC(a,c)**(-rhocxc(c)-1) ;
```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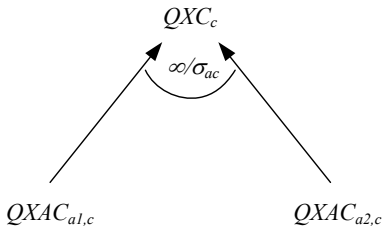
Supply of Commodities by Activities: Homo



- When $\sigma_{ac} = \infty$
- 2-argument illustration
- Determined by need to satisfy *QXC*

commodities produced domestically

$$QXC_c = \sum_a QXAC_{a,c} \qquad \forall cx_c \text{ and } cxacn_c$$

commodities **NOT** differentiated by activity

```
QXC(c) =E= SUM(a,QXAC(a,c)) ;
```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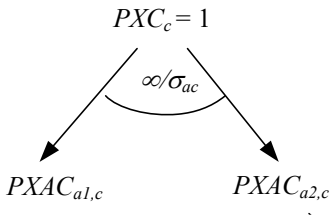
Supply of Commodities by Activities: Homog



- When $\sigma_{ac} = \infty$
- By definition of homogenous commodities price received by each source ($PXAC$) is identical to that of the composite (PXC)

$$PXAC_{a,c} = PXC_c \qquad \forall cxacn_c$$

commodities **NOT** differentiated by activity

```
PXAC(a,c) =E= PXC(c) ;
```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Outputs of Activities

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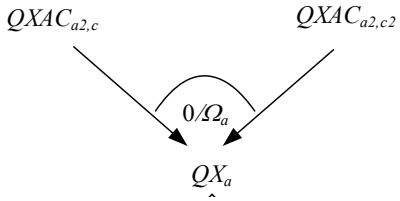
Outputs by Activities 1i



- Commodities produced by activities in **fixed shares** (*ioqxacqx*)
- A by-product assumption

$$QXAC_{a,c} = IOQXACQX_{a,c} * QX_a$$

$$\forall ioqxacqx_{a,c} \textbf{ and } acetn_a$$

```
ACTOUT(a,c)$(ioqxacqx(a,c) AND acetn(a))..
   QXAC(a,c) =E= IOQXACQX(a,c) * QX(a) ;
```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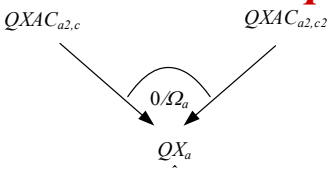
Outputs by Activities 1ii



- A by-product assumption

If QX_a increases/decreases then all *c* produced by *a* ($QXAC_{c,a}$) increase/decrease in proportion

Irrespective of price changes for individual commodities

Hence, in simulations activity *a* can 'choose' to increase the amount of commodity *c* it produces EVEN if the price it receives for *c* (*PXAC*) decreases

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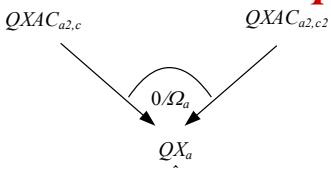
Outputs by Activities 2i



- Only need the first order condition to determine $QXAC_{a,c}$ since other variables determined elsewhere

$$QXAC_{a,c} = QX_a * \left(\frac{PXAC_{a,c}}{\left(PX_a * gamma_{a,c}^{i} * at_a^{i\,\rho_a^i} \right)} \right)^{\left(1/\left(\rho_a^i - 1 \right) \right)}$$

$$\forall ioqxacqx_{a,c} \textbf{ and } acet_a$$

```
ACTOUTFOC(a,c)$(ioqxacqx(a,c) AND acet(a))..
QXAC(a,c) =E= QX(a)*{PXAC(a,c)/[PX(a)*gammai(a,c)*ati(a)
                **(rhoti(a))]}
                      **(1/(rhoti(a)-1)) ;
```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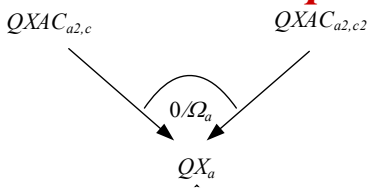
Outputs by Activities 2ii



- Commodities produced by activities in **variable shares** (*IOQXACQXV*)
- Assumption – activities are price responsive wrt commodity output

$$PX_a = \sum_c IOQXACQXV_{a,c} * PXC_c$$

```
PX(a) =E= SUM(c,IOQXACQXV(a,c)*PXAC(a,c)) ;
```

Production equilibrium

$$QXAC_{a,c} = IOQXACQXV_{a,c} * QX_a$$

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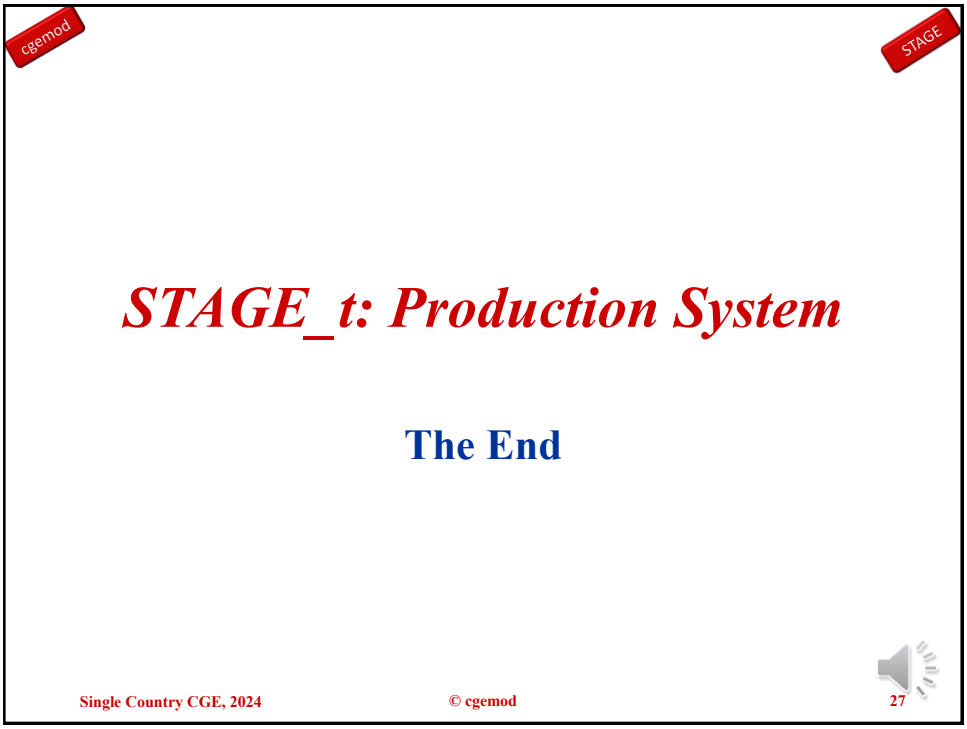
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