# *A Two Sector Closed Economy CGE Model: Part 2*



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## *Outline*

- *Introduction*
- *Economic Data & Behaviour*
- The Model in Algebra and GAMS
  - Prices
  - Production
  - Factors
  - *Households*
  - *Government*
  - *Saving-Investment*
  - *Other Equations*
- *Model Closure & Market Clearing*



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# *The Model in GAMS*



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## *Formal/Algebraic Statement*

- VARIABLES are in upper case;
- Prefixes: *P* – prices; *Q* – quantities; *W* - factor prices; *F* - factor quantities;
- parameters are in lower case, except those used to intialise variables;
- elasticities and associated parameters are given Greek names;
- parameter names have a two-character suffix/prefix which distinguishes their definition, e.g., ***sh*, *io*** and ***av*

$c = \{\text{primary, secondary}\}$

$a = \{\text{agriculture, industry}\}$

$f = \{\text{labour, capital}\}$

$h = \{\text{urban, rural}\}$

$g = \{govt\}$

$sac = \{c, a, f, h, g, i\_s, total\}$



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## *Key Price Equations*

**Acc$^g$ Identity**

$$PQD_c * \left( QINTD_c + \sum_h QCD_{c,h} + QINVD_c + QGD_c \right) = PQS_c * QQ_c + SAM_{g,c}$$

$$PQD_c * QQ_c = PQS_c * QQ_c + SAM_{g,c}$$

$PQS_c * QQ_c = PX_a * QX_a$ $\quad \forall c = a$ **Acc$^g$ Identity**

$QQ_c = QX_a$ $\quad \forall c = a$ **Supply = Demand**

$PQS_c = PX_a = 1$ $\quad \forall c = a$ **Basic Prices**

**In GAMS** ➡

$$PX_a = \sum_c ioqqqx_{a,c} * PQS_c$$

PX(a) =E= SUM(c,ioqqqx(a,c)*PQS(c));



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## *Price Normalisation Rule*

$PQS_c = PX_a = 1$ $\quad \forall c = a$ **Basic Prices**

# ALL prices are derived relative to BASIC PRICES

**If price data exist they would be used to determine the basic prices of the system**



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## *Purchaser Prices*

$$PQD_c * QQ_c = PQS_c * QX_a + SAM_{g,c} \qquad \forall c = a$$
$$QQ_c = QX_a \qquad \forall c = a$$

$$PQD_c * QQ_c = PQS_c * QQ_c + SAM_{g,c}$$ **Acc$^g$ Identity**

$$SAM_{g,c} = \left(ts_c * PQS_c * QQ_c\right)$$ **Behavioural assumption**

$$PQD_c * QQ_c = PQS_c * QQ_c + \left(ts_c * PQS_c * QQ_c\right)$$
$$PQD_c = PQS_c + \left(ts_c * PQS_c\right) = PQS_c * \left(1 + ts_c\right)$$ **Acc$^g$ Identity**

**In GAMS** ➡

$$PQD_c = PQS_c * \left(1 + ts_c\right)$$
PQD(c) =E= PQS(c)*(1+ts(c)) ;



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## *Activity Prices – NO Intermediate Inputs*

$$PX_a * QX_a = \sum_f SAM_{f,a} + SAM_{g,a} \qquad \forall a$$
$$= \sum_f WF_f * FD_{f,a} + SAM_{g,a} \qquad \forall a$$ **Acc$^g$ Identity**

$$SAM_{g,a} = \left(tx_a * PX_a * QX_a\right)$$ **Behavioural assumption**

$$PX_a * QX_a = \sum_f WF_f * FD_{f,a} + \left(tx_a * PX_a * QX_a\right)$$
$$= PVA_a * QX_a + \left(tx_a * PX_a * QX_a\right)$$
$$PX_a = PVA_a + \left(tx_a * PX_a\right)$$
$$PVA_a = PX_a * \left(1 - tx_a\right)$$ **Acc$^g$ Identity**

**Implies** ➡

$$PVA_a = PX_a * \left(1 - tx_a\right)$$
PVA(a)=E=PX(a)*(1-tx(a))

**Amount per unit of output available to pay factors**



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## Activity Prices – with Intermediate Inputs

$$PX_a * QX_a = SAM_{c,a} + \sum_f WF_f * FD_{f,a} + SAM_{g,a} \qquad \forall a$$

**Acc$^g$ Identity**

$$SAM_{c,a} = \sum_c \left( a^x_{c,a} * QX_a * PQD_c \right)$$

$$SAM_{g,a} = \left( tx_a * PX_a * QX_a \right)$$

**Behavioural assumptions**

$$PX_a * QX_a = \sum_c \left( a^x_{c,a} * QX_a * PQD_c \right) + \left( PVA_a * QX_a \right) + \left( tx_a * PX_a * QX_a \right)$$

$$PX_a = \sum_c \left( a^x_{c,a} * PQD_c \right) + PVA_a + \left( tx_a * PX_a \right)$$

$$PVA_a = PX_a * \left( 1 - tx_a \right) - \sum_c \left( a^x_{c,a} * PQD_c \right)$$

**Acc$^g$ Identity**



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## Activity Prices – with Intermediate Inputs

$$PVA_a = PX_a * \left( 1 - tx_a \right) - \sum_c \left( a^x_{c,a} * PQD_c \right)$$

**Cost of production taxes per unit of output**

**In GAMS** ➡

```
PVA(a) =E= PX(a)*(1 - tx(a))
- SUM(c,PQD(c) * ioqintdqx(c,a)) ;
```

**Amount per unit of output available to pay factors**

**Cost of intermediate inputs per unit of output**



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## *Production Block Equations*

$$QX_a = ad_a \prod_f \left(FD_{f,a}\right)^{\alpha_{f,a}}$$ **Production function**

**In GAMS** ➡

$$QX_a = ad_a \prod_f \left(FD_{f,a}\right)^{\alpha_{f,a}}$$

```
QX(a)  =E= ad(a)*PROD(f,FD(f,a)**alpha(f,a)) ;
```

**Given the specification of PVA the production function is unchanged**

**BUT the factor demand equations are CHANGED**



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## *Factor Demand Equations*

**Profit function**

$$\Pi_a = \left(PX_a * QX_a\right) - \sum_c \left(a^x_{c,a} * PQD_c * QX_a\right) - \left(tx_a * PX_a * QX_a\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right]$$

**Price definition**

$$PVA_a * QX_a = \left\{\left(PX_a * QX_a\right) - \sum_c \left(a^x_{c,a} * PQD_c * QX_a\right) - \left(tx_a * PX_a * QX_a\right)\right\}$$

**Profit function**

$$\Pi_a = \left(PVA_a * QX_a\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right]$$

$$= \left(PVA_a * \left(\alpha^x_a . FD^{\beta_{l,a}}_{l,a} . FD^{\beta_{k,a}}_{k,a}\right)\right) - \left[\left(WF_l * FD_{l,a}\right) + \left(WF_k * FD_{k,a}\right)\right]$$



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# *Factor Demand Equations*

**Partially differentiate**
**Set equal to zero**
**Solve for factor price**

**1st Order Condition**

$$WF_l = \left( PVA_a * \left( \alpha_a^x . \beta_{l,a} . FD_{l,a}^{(\beta_{l,a}-1)} . FD_{k,a}^{\beta_{k,a}} \right) \right) = PVA_a * \beta_{l,a} * \frac{QX_a}{FD_{l,a}}$$

$$\Rightarrow WF_l * FD_{l,a} = \beta_{l,a} . \left( PVA_a * QX_a \right)$$

**In GAMS** ➡

$$FD_{f,a} = \frac{QX_a * PVA_a * \alpha_{f,a}}{WF_f}$$

```
WF(f)*WFDIST(f,a)*FD(f,a) =E= QX(a)*PVA(a)*alpha(f,a) ;
```



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# *Intermediate Input Demand*

$$QINTD_c = \sum_a ioqintdqx_{c,a} * QX_a$$

where $ioqintdqx_{c,a}$ is the quantity of input $c$ used per unit output of $a$

**In GAMS** ➡

```
QINTD(c) =E= SUM(a,ioqintdqx(c,a)*QX(a)) ;
```

The purchaser prices of intermediate demand (*PQD*) are known

**Therefore values of intermediate inputs are known if *QX* is known**



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## *Factor Incomes*

$$YF_f = \sum_a WF_f * FD_{f,a}$$

**Acc$^g$ Identity**

**In GAMS ➡**

$$YF_f = \sum_a WF_f * FD_{f,a}$$

```
YF(f) =E= SUM(a,WF(f)*WFDIST(f,a)*FD(f,a)) ;
```



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## *The End*

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