



ANARRES v1: A SAM Based Global CGE Model

Scott McDonald¹

Address for correspondence:

Scott McDonald,
cgemod,
14 Moncrieffe Road
Sheffield,
S7 1HR, UK.
Email: scott@cgemod.org.uk
Tel: +44 789 979 3671

Abstract

This paper provides a technical description of a global computable general equilibrium (CGE) model that is calibrated from a Social Accounting Matrix (SAM) representation of the Global Trade Analysis Project (GTAP) database. An important feature of the model is the treatment of nominal and real exchange rates and hence the specification of multiple numéraire. Another distinctive feature of the model is the use of a ‘dummy’ region, known as ANARRES, that allows for the recording of inter-regional transactions where either the source or destination are not identified.

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¹ Scott McDonald is retired former Professor of Economics.

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1. Introduction

This paper provides a technical description of a variant of a Social Accounting Matrix (SAM) based Global Computable General Equilibrium (CGE) model that has been calibrated using data derived from the Global Trade Analysis Project's (GTAP) database. The model is a member of a family of CGE models that model trade relationships using principles described in the 1-2-3 model (de Melo and Robinson, 1989; Devarajan, *et al.*, 1990). More specifically this model is a direct descendant of an early US Department of Agriculture model (see Robinson *et al.*, 1990) and NAFTA (see Robinson *et al.*, 1993). However numerous features of this model stem from other developments in CGE modelling over the last 15 years; some of these sources of inspiration are direct and easily identified, e.g., analysis used in the World Development Report, 1995 (Lewis, Robinson, and Wang), the IFPRI standard model (Lofgren *et al.*, 2002) and the PROVIDE Project model (McDonald, 2003), others are indirect and easily identified, e.g., the GTAP model (Hertel, 1997), while others are both direct and indirect but less easily identified; a substantial debt is owed to the community of CGE modellers. In addition the model owes a lot to the development of the SAM approach to national accounting, e.g., Stone (1962a and b) and Pyatt (1991), and the SAM approach to modelling, e.g., Pyatt (1987), Drud *et al.*, (1986).

The underlying approach to multi-region modelling for this CGE model is the construction of a series of single country CGE models that are linked through their trading relationships. As is common with all known CGE models the price systems in the model are linear homogenous and hence the focus is upon movements in relative, rather than absolute, prices. Consequently each region in the model has its own numéraire price, typically the consumer price index (CPI), and a nominal exchange rate, while the model as a whole requires a numéraire, which is an exchange rate index for a number of reference regions. As such this model contains a fundamentally different philosophical approach to global modelling to that found in the GTAP model.² Behind this difference lies a deep theoretical debate about how comparative static and finite horizon dynamic CGE models should value transfers associated with the capital account of the balance of payments (see Robinson, 2004).

² The GTAP model does not contain nominal exchange rates and has a single global numéraire.

A distinctive feature of the model is the use of a ‘dummy’ region, known as GILD, that allows for the recording of inter-regional transactions where either the source or destination are not identified. Examples of such transactions include trade and transportation margins and data on remittances. The GILD construct provides a general method for dealing with any transactions data where full bilateral information is missing.

The rest of this paper is organised as follows. Section 2 reviews the data used in the model; this section also provides a brief description of how the data were transformed from the GTAP database into a SAM. This is followed in section 3 by a descriptive overview of the model and then, in section 4, by a formal description of the model’s equations. The description in section 4 is based upon a default setting for the model closure rules; one of the model’s key features is the flexibility of the closure rules and consequently section 5 considers the alternatives built into the model’s basic structure. All global CGE models are large and therefore present a series of potential implementation problems; section 6 briefly reviews some of the programmes that have been developed to support the basic model and provides some guidelines for use of this class of model. This is followed by some concluding comments that primarily focus upon planned model developments. Finally there are a series of technical appendices relating to aspects of model formulation, calibration of use – details about the content of these appendices is provided at the start of the appendices.

2. Model Data

The data used in the model were derived from the GTAP database (see Hertel, 1997) using a three dimensional Social Accounting Matrix (SAM) method for organising the data. Details of the method used to generate a SAM representation are reported in McDonald and Thierfelder (2004a) while a variety of reduced form representations of the SAM and methods for augmenting the GTAP database are reported in McDonald and Thierfelder (2004b) and McDonald and Sonmez (2004) respectively. Detailed descriptions of the data are provided elsewhere so the discussion here is limited to the general principles.

2.1 Global Social Accounting Matrix

The Global SAM can be conceived of as a series of single region SAMs that are linked through the trade accounts; it is particularly valid in the context of the GTAP database to note that the ONLY way in which the regions are linked directly in the database is through

commodity trade transactions although there are some indirect links through the demand and supply of trade and transport services. Specifically the value of exports, valued free on board (*fob*) from source x to destination y must be exactly equal to the value of imports valued fob to destination y from source x , and since this holds for all commodity trade transactions the sum of the differences in the values of imports and exports by each region must equal zero. However, the resultant trade balances do not fully accord with national accounting conventions because other inter regional transactions are not recorded in the database (see McDonald and Sonmez, 2004). A description of the transactions recorded in a representative SAM for a typical region in the database is provided in Table 1.

A SAM is a transactions matrix; hence each cell in a SAM simply records the values of the transactions between the two agents identified by the row and column accounts. The selling agents are identified by the rows, i.e., the row entries record the incomes received by the identified agent, while the purchasing agents are identified by the columns, i.e., the column entries record the expenditures made by agents. As such a SAM is a relatively compact form of double entry bookkeeping that is complete and consistent and can be used to present the National Accounts of a country in a single two-dimensional matrix (see UN, 1993, for a detailed explanation of the relationship between conventional and SAM presentations of National Accounts). A SAM is complete in the sense that the SAM should record ALL the transactions within the production boundary of the National Accounts, and consistent in the sense that income transactions by each and every agent are exactly matched by expenditure transactions of other agents. A fundamental consequence of these conditions is that the row and column totals of the SAM for each region must be identical, and hence the SAM provides a complete characterisation of current account transactions of an economy as a circular (flow) system. In the context of a global SAM the complete and consistent conditions need extending to encompass transactions between regions; this simply requires that each and every import transaction by a region must have an identical counterpart export transaction by another region. This is enough to ensure that the resultant global SAM provides a characterisation of current account transactions of the global economy as a circular (flow) system.

Table 1 Social Accounting Matrix for a Region in the Global Social Accounting Matrix

	Commodities	Activities	Factors	Households	Government	Capital	Margins	Rest of World	Totals
Commodities	0	Combined Intermediate Use Matrix	0	Private Consumption	Government Consumption	Investment Consumption	Exports of Margins (<i>fob</i>)	Exports of Commodities (<i>fob</i>)	Total Demand for Commodities
Activities	Domestic Supply Matrix	0	0	0	0	0	0	0	Total Domestic Supply by Activity
Factors	0	Expenditure on Primary Inputs	0	0	0	0	0	0	Total Factor Income
Households	0	0	Distribution of Factor Incomes	0	0	0	0	0	Total Household Income
Government	Taxes on Commodities	Taxes on Production Taxes on Factor Use	Direct/Income Taxes	Direct/Income Taxes	0	0	0	0	Total Government Income
Capital	0	0	Depreciation Allowances	Household Savings	Government Savings	0	Balance on Margins Trade	Foreign Savings	Total Savings
Margins	Imports of Trade and Transport Margins	0	0	0	0	0	0	0	Total Income from Margin Imports
Rest of World	Imports of Commodities (<i>fob</i>)	0	0	0	0	0	0	0	Total Income from Imports
Totals	Total Supply of Commodities	Total Expenditure on Inputs by Activities	Total Factor Expenditure	Total Household Expenditure	Total Government Expenditure	Total Investment	Total Expenditure on Margin Exports	Total Expenditure on Exports	

Given these definitions of a SAM the transactions recorded in a SAM are easily interpreted. In Table 1 the row entries for the commodity accounts are the values of commodity sales to the agents identified in the columns, i.e., intermediate inputs are purchased by activities (industries etc.), final consumption is provided by households, the government and investment demand and export demand is provided by the all the other regions in the global SAM and the export of margin services. The commodity column entries deal with the supply side, i.e., they identify the accounts from which commodities are purchased so to satisfy demand. Specifically commodities can be purchased from either domestic activities – the domestic supply matrix valued inclusive of domestic trade and transport margins – or they can be imported – valued exclusive of international trade and transport margins. In addition to payments to the producing agents – domestic or foreign – the commodity accounts need to make expenditures with respect to the trade and transport services needed to import the commodities and any commodity specific taxes.

The GTAP database provides complete coverage of bi lateral transactions in commodities – these are valued free on board (*fob*) - but only provides partial coverage of transactions in trade and transport margins. Specifically the imports of trade and transport margins by each region are directly associated with the imports of specific commodities, hence for each commodity import valued *fob* the source and destination regions are identified and the value of each trade and transport margin service used is identified. The sum of the values of trade and transport services and the *fob* value of the commodity imports represent the carriage insurance and freight (*cif*) paid value of each imported commodity. But the source regions of the trade and transport services are NOT identified, and similarly the values of exports of trade and transport services by a region do NOT identify the destination regions. To overcome this lack of information an artificial region called GILD is included in the database. This region collects together all the exports of trade and transport services by other regions as its imports and then exports these to other regions to satisfy their demand for the use of trade and transport services associated with commodity imports. By construction, the value of imports by GILD for every trade and transport margin service must exactly equal the value of exports for the corresponding trade and transport service. However, this does not mean that the trade balance between GILD and each and every region must exactly balance, rather it requires that the sum of GILD trade balances with other regions is exactly equal to zero.

An important feature of the construction of a SAM can be deduced from the nature of the entries in the commodity account columns. By definition the column and row totals must equate and these transaction totals can be expressed as an implicit price times a quantity, and the quantity of a commodity supplied must be identical to the quantity of a commodity demanded. The column entries represent the expenditures incurred in order to supply a commodity to the economy and hence the implicit price must be exactly equal to the average cost incurred to supply a commodity. Moreover since the row and column totals equate and the quantity represented by each corresponding entry must be same for the row and column total the implicit price for the row total must be identical to average cost incurred to supply the commodity. Hence the column entries identify the components that enter into the formation of the implicit prices in the rows, and therefore identify the price formation process for each price in the system. Typically a SAM is defined such that the commodities in the rows are homogenous and that all agents purchase a commodity at the same price.

Total income to the activity accounts is identified by the row entries. In the simple representation of production in the GTAP database each activity makes a single commodity and each commodity is made by a single activity, which means that the domestic supply matrix is a diagonal (square) matrix. The expenditures on inputs used in production are recorded in the activity columns. Activities use intermediate inputs, which in this version of the database are record as composites of domestically produced and imported commodities, primary inputs and pay taxes on production and factor use. For each region the sum of the payments to primary inputs and on production and factor use taxes by activity is equal to the activity's contribution to the value added definition of GDP while the sum over activities equals the region's value added measure of GDP.

The remaining accounts relate to the institutions in the SAM. All factor incomes are distributed to the single private household after making allowance for depreciation of physical capital and the payment of direct (income) taxes on factor incomes. Incomes from factor sales are also the sole source of income to the household account. Three categories of expenditures by the household account are recorded; direct (income) taxes, savings and consumption. The government receives incomes from commodity taxes, production taxes and direct taxes on factor and household incomes, and uses that income to pay for consumption and for savings. In the basic form of the database government savings are set to zero for all regions; this stems from the reduced form representation of intra institutional transactions provided by the GTAP

database (see McDonald and Thierfelder, 2004b).³ There are therefore five sources of savings in each region: depreciation, household/private savings, government savings, balances on trade in margin services and balances on trade in commodities, but only a single expenditure activity – investment (commodity) demand.

As should be apparent from the description of the SAM for a representative region the database is strong on inter regional transactions but relatively parsimonious on intra-regional transactions.

2.2 Other GTAP Data

In addition to the transactions data the GTAP database contains other data that can be used with this model, and/or variants of the model. The most obviously useful data are the import and primary factor elasticity data used in the GTAP model; the programme used to derive an aggregation of the SAM also contains a routine for aggregating these elasticities for use in this model. However, the GTAP elasticities are only a subset of the elasticities used in this model and it is therefore necessary to provide other elasticities even when using the GTAP elasticity data.

Other data of interest to modellers include estimates of energy usage and emissions and land use (carbon sinks). None of these data are used in this variant of the model.

2.3 Database Dimensions

The dimensions of the SAM are determined by the numbers of accounts within each aggregate group identified in Table 1, while the actual numbers of accounts in each group of accounts are defined for version 5.4 and 6.0 of the GTAP database in Table 2. Given the large number of accounts in the SAMs for each region and the relatively large number of regions the total number of cells in the global SAM is very large, although only slightly over 10 percent of the cells actually contain non zero entries; nevertheless this still means that the GTAP database contains some 4 million transaction values, which implies that there are some 8 million possible prices and quantities that can be deduced from the database. Even allowing for the implications of adopting the law of one price for transactions in the rows of each region's SAM and for other ways of reducing the numbers of independent prices and

³ McDonald and Sonmez (2004) demonstrate that it is straightforward to overcome this limitation of the database. The model described in this paper operates whether the government savings are zero or non-zero.

quantities that need to be estimated in a modelling environment, it is clear that the use of the GTAP database without aggregation is likely to generate extremely large models (in terms of the number of equations/variables). Consequently, except in exceptional circumstances all CGE models that use the GTAP data operate with aggregations of the database.

Table 2 Dimensions of the Global Social Accounting Matrix

Account Groups	Sets	Total Number of Accounts		
		GTAP 5.4	GTAP 6.0	GTAP 7.1
Commodities	C	57	57	57
Activities	A	57	57	57
Factors	F	5	5	5
Taxes	$(2*r)+(1*f)+3$	164	182	232
Other Domestic Institutions	3	3	3	3
Margins	$3*r$	234	261	336
Trade	R	78	87	112
Total		598	652	802
Total Number of Cells in the Global SAM		27,893,112	36,984,048	72,038,848

3 Overview of the Model

3.1 Behavioural Relationships

The within regional behavioural relationships are fairly standard in this variant of the model; it is easy to make them more elaborate but the focus in this variant of the model is upon international trade relationships. The activities are assumed to maximise profits using technology characterised by Constant Elasticity of Substitution (CES) and/or Leontief production functions between aggregate primary inputs and aggregate intermediate inputs, with CES production functions over primary inputs and Leontief technology across intermediate inputs. The household maximises utility subject to preferences represented by a Stone-Geary utility function, i.e., a linear expenditure system, having first paid income taxes and having saved a fixed proportion of after tax income.⁴

The Armington assumption is used for trade. Domestic output is distributed between the domestic market and exports according to a two-stage Constant Elasticity of Transformation

⁴ With appropriate parameter specification the LES collapses to a Cobb-Douglas specification.

(CET) function. In the first stage a domestic producer allocates output to the domestic or export market according to the relative prices for the commodity on the domestic market and the composite export commodity, where the composite export commodity is a CET aggregate of the exports to different regions – the distribution of the exports between regions being determined by the relative export prices to those regions. Consequently domestic producers are responsive to prices in the different markets – the domestic market and all other regions in the model – and adjust their volumes of sales according relative prices. The elasticities of transformation are commodity and region specific. The CET functions across exports can be switched off so that export supplies are determined by import demands.⁵

Domestic demand is satisfied by composite commodities that are formed from domestic production sold domestically and composite imports. This process is modelled by a three-stage CES function. At the bottom stage one composite import commodity is a CES aggregate of imports from different regions with the quantities imported from different regions being responsive to relative prices and another composite import commodity is a Leontief aggregate of imports from different regions with the quantities being fixed proportions of the volume of import demand. This second composite commodity is introduced for the treatment of imports whose volumes are small and can, as a consequence have large terms of trade effects. The second level is a Leontief aggregate of the two composite commodities formed at the bottom level, which defines the volumes of composite imports. The top stage defines a composite consumption commodity as a CES aggregate of a domestic commodity and a composite import commodity with the mix being determined by the relative prices. The elasticities of substitution are commodity and region specific.⁶ Hence the optimal ratios of imports to domestic commodities and exports to domestic commodities are determined by first order conditions based on relative prices. The price and quantity systems are described in greater detail below

All commodity and activity taxes are expressed as *ad valorem* tax rates, while income taxes are defined as fixed proportions of household incomes. Import duties and export taxes apply to imports and exports, while sales taxes are applied to all domestic absorption, i.e., imports are subject to sequential import duties and sales taxes. Production taxes are levied on the value of output by each activity, while activities also pay taxes on the use of specific factors. Factor income taxes are charged on factor incomes after allowance for depreciation

⁵ Switching off the CET function allows the model to function in a similar manner to the GTAP model.

⁶ This is different to the GTAP model where the elasticities are only commodity specific.

after which the residual income is distributed to households. Income taxes are taken out of household income and then the households are assumed to save a proportion of disposable income. This proportion is either fixed or variable according to the closure rule chosen for the capital account.

Government expenditure consists of commodity (final) demand, which is assumed to be in fixed proportions in real/volume terms. Hence government saving, or the internal balance, is defined as a residual. However, the closure rules for the government account allow for various permutations. In the base case it is assumed that the tax rates and volume of government demand are fixed and government savings are calculated as a residual. However, the tax rates can all be adjusted using various forms of scaling factors; hence for instance the value of government savings can be fixed and one of the tax scalars can be made variable thereby producing an estimate of the constrained optimal tax rate. If the analyst wishes to change the relative tax rates across commodities (for import duties, export taxes and sales taxes) or across activities (for production taxes) then the respective tax rate parameters can be altered via a second adjuster. Equally the volume of government consumption can be changed by adjusting the closure rule with respect the scaling adjuster attached to the volumes of government consumption. The patterns of government expenditure are altered by changing the parameters that controls the pattern of government expenditure (*qgdconst*).

Table 3 Behavioural Relationships for a Global CGE Model

	Commodities	Activities	Factors	Households	Government	Capital	Margins	Rest of World	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Stone-Geary Utility Functions	Fixed Exogenously	Fixed Shares of Savings	Three-Stage CET Functions	Three-Stage CET Functions	Consumer Commodity Price
Activities	Total Supply from Domestic Production	0	0	0	0	0	0	0	Activity Prices
Factors	0	Two-stage CES Production Functions	0	0	0	0	0	0	Factor Prices
Households	0	0	Fixed Shares of Factor Income	0	0	0	0	0	
Government	Ad valorem tax rates	Ad valorem tax rates on Output and Factor Use	Average tax rates	Average tax rates	0	0	0	0	
Capital	0	0	Shares of Factor Incomes	Shares of household income	Government Savings (Residual)	0	Current Account 'Deficit' on Margins Trade	Current Account 'Deficit'	
Margins	Fixed Technical Coefficients	0	0	0	0	0	0	0	
Rest of World	Three-Stage CES Functions	0	0	0	0	0	0	0	
Prices	Producer Prices Domestic and World Prices for Imports	Value Added Prices							

Table 4 Transactions Relationships for a for a Global CGE Model

	Commodities	Activities	Factors	Households
Commodities	0	$(PQD_c * QINTD_c)$	0	$(PQD_c * QCD_c)$
Activities	$(PDS_c * QDS_c)$	0	0	0
Factors	0	$(WF_f * FD_{f,a})$	0	0
Households	0	0	$\sum_f \left(hvas h_{h,f} * YF_f \right)$	0
Government	$\left(TM_{w,c} * PWM_{w,c} * QMR_{w,c} * ER \right)$ $\left(TE_{w,c} * PWE_{w,c} * QER_{w,c} * ER \right)$ $(TS_c * PQS_c * QQ_c)$	$(TX_a * PX_a * QX_a)$ $\left(TF_{f,a,r} * WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r} \right)$	$\left(TYF_{f,r} * \left(YF_{f,r} - \left(deprec_{f,r} * YF_{f,r} \right) \right) \right)$	$(TYH_h * YH_h)$
Capital	0	0	$(deprec_{f,r} * YF_{f,r})$	$\left(YH_{h,r} * (1 - TYH_{h,r}) \right) * (SHH_{h,r})$
Margins	$(PT_c * QT_{w,c})$	0	0	0
Rest of World	$\left(PWMFOB_{w,c} * QMR_{w,c} * ER \right)$	0	0	0
Total	$(PQD_c * QQ_c)$	$(PX_a * QX_a)$	YF_f	YH

Table 4 (cont) Transactions Relationships for a for a Global CGE Model

	Government	Capital	Margins	RoW
Commodities	$(PQD_c * QGD_c)$	$(PQD_c * QINVD_c)$	$\begin{pmatrix} PWE_{c,w} * QER_{c,w} \\ *ER \end{pmatrix}$	$\begin{pmatrix} PWE_{c,w} * QER_{c,w} \\ *ER \end{pmatrix}$
Activities	0	0	0	0
Factors	0	0	0	0
Households	0	0	0	0
Government	0	0	0	0
Capital	$(YG - EG)$	0	$(KAPREG * ER)$	$(KAPREG * ER)$
Margins	0	0	0	0
Rest of World	0	0	0	0
Total	YG	$INVEST$	0	0

Total savings come from the households, the internal balance on the government account and the external balance on the trade account. The external balance is defined as the difference between the value of total exports and total imports, converted into domestic currency units using the exchange rate. In the base model it is assumed that the exchange rates are flexible and hence that the external balances are fixed. Alternatively the exchange rates can be fixed and the external balances can be allowed to vary. Expenditures by the capital account consist solely of commodity demand for investment. In the base solution it is assumed that the shares of investment in total domestic final demand are fixed and that household savings rates adjust so that total expenditures on investment are equal to total savings, i.e., the closure rule presumes that savings are determined by the level of investment expenditures. The patterns of investment volume are fixed, and hence the volume of each commodity changes equiproportionately according to the total values of domestic final demand. It is possible to fix the volumes of real investment and then allow the savings rates, by households, to vary to maintain balances in the capital account, and it is possible to change the patterns of investment by changing the investment parameters (*qinvdconst*).

3.2 Price and Quantity Systems for a Representative Region

3.2.1 Price System

The price system is built up using the principle that the components of the ‘price definitions’ for each region are the entries in the columns of the SAM. Hence there are a series of explicit accounting identities that define the relationships between the prices and thereby determine the processes used to calibrate the tax rates for the base solution. However, the model is set up using a series of linear homogeneous relationships and hence is only defined in terms of relative prices. Consequently as part of the calibration process it is necessary set some of the prices equal to one (or any other number that suits the modeller) – this model adopts the convention that prices are normalised at the level of the CES and CET aggregator functions *PQS*, the supply price of the domestic composite consumption commodity and *PXC*, the producer price of the composite domestic output. The price system for a typical region in a 4-region global model is illustrated by Figure 1 – note that this representation abstracts from the ANARRES region.

The user can adjust the definition of a ‘small’ source region when configuring the model; the definition of a ‘large’ source region is then defined as the complement. The region specific import prices are expressed in terms of the domestic currency units after paying for trade and transport services and any import duties. Thus a destination region is assumed to purchase a commodity in a source economy where the price is defined in “world dollars” at the basket exchange rate and is valued free on board (*fob*), i.e., *PWMFOB*. The carriage insurance and freight (*cif*) price (*PWM*) is then defined as the *fob* price plus trade and transport margin services (*margcor*) times the unit price of margin services (*PT*). The *cif* prices are related to the domestic price of imports by the addition of any import duties (*TM*) and then converted into domestic currency units using the nominal exchange rate (*ER*).

The prices for commodities by activities (*PXC*) are determined by the domestic prices (*PD*) and the composite export prices (*PE*). The composite export prices are a CET aggregates of the export prices received by the source economy for exports to specific destinations (*PER*). The prices of the composite exports are determined as aggregates of the domestic prices paid for exports by all those regions that demand exports from this economy under the maintained assumption that exports are differentiated both by their destination region and the ‘regional’ group that the destination region is part of; hence there is a two-stage sub aggregation process whereby exports to like groups so regions are aggregated to form intermediate aggregates of exported commodities, which are then aggregated to form the composite export commodities. This allows for a degree of differentiation by both destination and commodity. The prices paid by the destination regions (*PWE*) are net of export taxes (*TE*) and are expressed in the currency units of the model’s reference region by use of the nominal exchange. Notice how the export prices by region of destination (*PER*), and the intermediate aggregates, are all normalised on 1, but the seeming counterpart of normalising import prices by source region (*PMR*) are not normalised on 1. The link between the regions is therefore embedded in the identification of the quantities exchanged rather than the normalised prices and is a natural consequence of the normalisation process. The CET function can be switched off so that the domestic and export commodities are assumed to be perfect substitutes; this is the assumption in the GTAP model and is an option in this model.

The price system also contains a series of equilibrium identities. Namely the *fob* export price (*PWE*) for region *x* on its exports to region *y* must be identical to the *fob* import price (*PWMFOB*) paid by region *y* on its imports from region *x*. These equilibrium identities are indicated by double headed arrows.

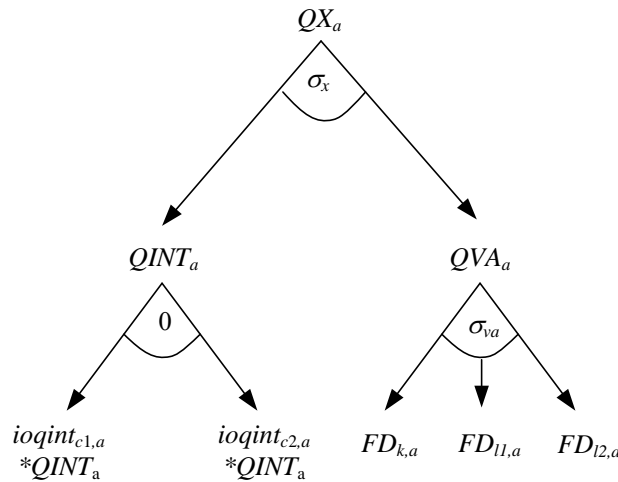
The composite consumption commodity is then allocated between domestic intermediate demands ($QINTD$), private consumption demand (QCD), government demand (QGD) and investment demand ($QINVD$).

On the output side, domestic output by activity (QX) is identical to domestic commodity output (QXC). Domestically produced commodities are then allocated between the domestic market (QD) and composite export commodities (QE) under the maintained assumption of imperfect transformation. Exports are allocated between the different destination regions (QER) under the maintain assumption of imperfect transformation.

3.2.3 Production System

The production system is set up as a two-stage nest of CES production functions. At the top level aggregate intermediate inputs ($QINT$) are combined with aggregate primary inputs (QVA) to produce the output of an activity (QX). This top level production function can take either CES or Leontief form, with CES being the default and the elasticities being activity **and** region specific.⁸ Aggregate intermediate inputs are a Leontief aggregation of the individual intermediate inputs where the input-output coefficients ($ioqint$) are defined in terms of input quantities relative to the aggregate intermediate input. The value added production function is a standard CES function over all primary inputs, with the elasticities being activity **and** region specific. The operation of this aggregator function can, of course, be influenced by choices over the closure rules for the factor accounts.

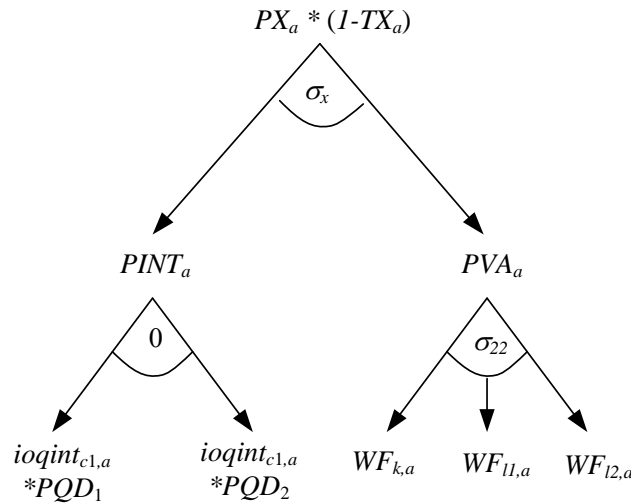
Figure 4 **Production Quantity System for a Typical Region**



⁸ The model allows the user to specify the share of intermediate input cost in total cost below which the Leontief alternative is automatically selected.

In the price system for production the value added prices (PVA) are determined by the activity prices (PX), the production tax rates (TX), the input-output coefficients ($ioqint$) and the commodity prices (PQD). The activity prices are a one to one mapping of the commodity prices received by activities (PXC); this is a consequence of the supply matrix being a square diagonal matrix.

Figure 5 **Production Price System for a Typical Region**

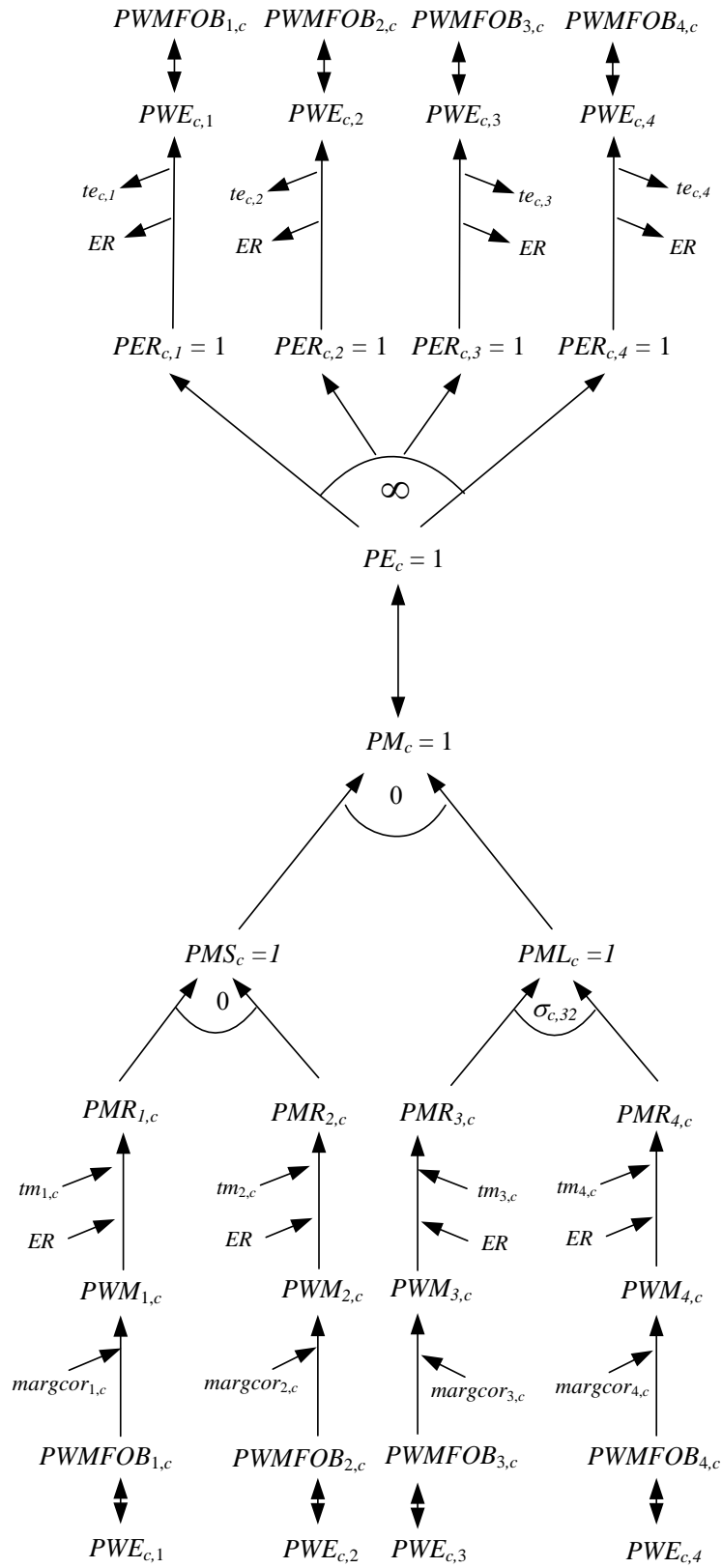


3.3 The ANARRES Region

An important feature of the model is the use of the concept of a region known as ANARRES. While the GTAP database contains complete bilateral information relating to the trade in commodities, i.e., in all cases transactions are identified according to their region of origin and their region of destination, this is not the case for trade in margins services associated with the transportation of commodities. Rather the GTAP database identifies the demand, in value terms, for margin services associated with imports by all regions from all other regions but does not identify the region that supplies the margin services associated with any specific transaction. Consequently the data for the demand side for margin services is relatively detailed but the supply side is not. Indeed the only supply side information is the total value of exports of margin services by each region. The ANARRES construct allows the model to get around this shortage of information, while simultaneously providing a general method for dealing with any other transactions data where full bilateral information is missing.

Figure 6

Price System for the GILD Region



The price system for the ANARRES region is illustrated in Figure 6. On the import side ANARRES operates like all other regions. The commodities used in trade and transport

services are assumed to be differentiated by source region and the proportion of imports accounted for by the source region. Thus a two-level Leontief and CES aggregation nest is used. It is assumed that imports of trade and transport services can *potentially* incur trade and transport margins (*margcor*) and face tariffs (*TM*); in fact the database does not include any transport margins or tariff data for margin services in relation to the destination region, although they can, and do, incur export taxes levied by the exporting region.

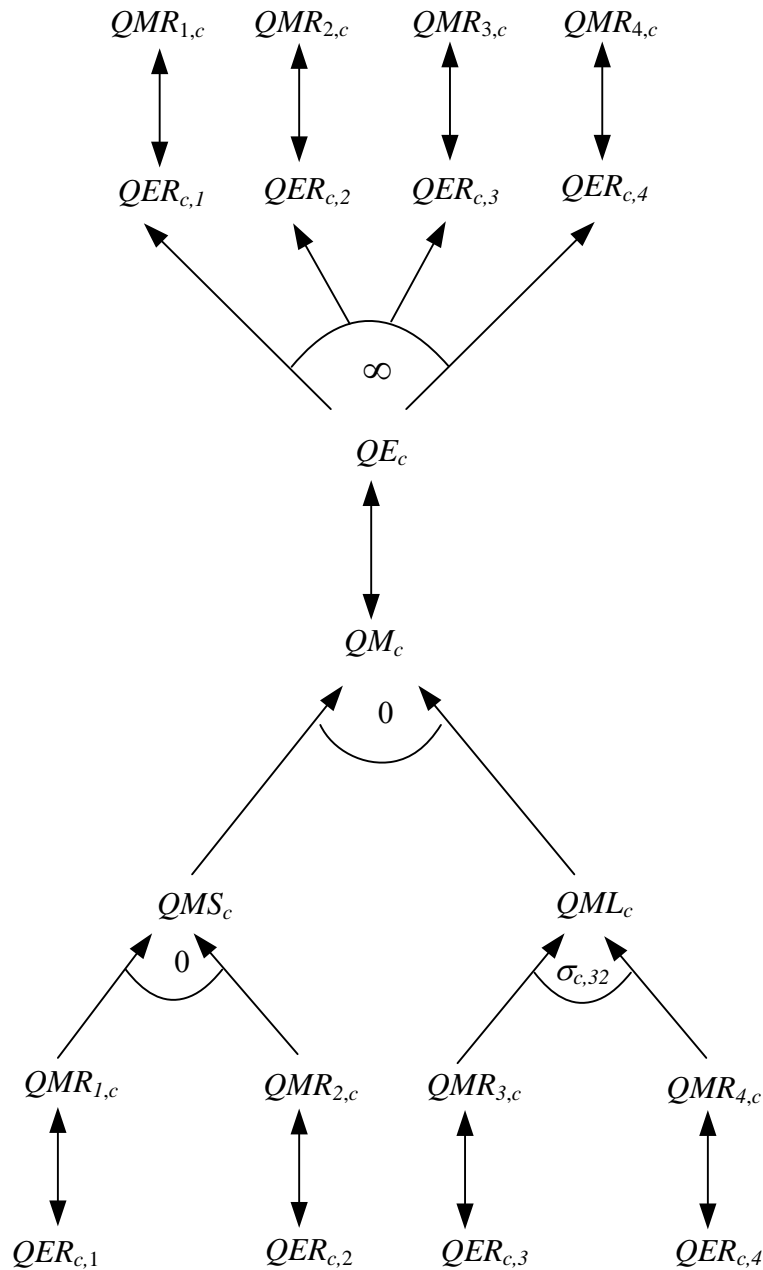
The export side is slightly different. In effect the GILD region is operating as a method for pooling differentiated commodities used in trade and transport services and the only differences in the use of trade and transport services associated with any specific import are the quantities of each type of trade service used and the mix of types of trade services. Underlying this is the implicit assumption that each type of trade service used is homogenous, and should be sold therefore at the same price. Hence the export price system for GILD needs to be arranged so that GILD exports at a single price, i.e., there should be an infinite elasticity of substitution between each type of trade service exported irrespective of its destination region. Therefore the average export price (*PE*) should equal the price paid by each destination region (*PER*), which should equal the export price in world currency units (*PWE*) and will be common across all destinations (*PT*).

The linked quantity system contains the same asymmetry in the treatment of imports and exports by ANARRES (see Figure 7). The imports of trade and transport commodities are assumed to be differentiated by region and the proportion of imports accounted for by the source region, hence the elasticity of substitution is greater than or equal to zero but less than infinity, while the exports of trade and transport commodities are assumed to be homogenous and hence the elasticities of transformation are infinite.

One consequence of using a GILD region for trade and transport services is that ANARRES runs trade balances with all other regions. These trade balances relate to the differences in the values of trade and transport commodities imported from GILD and the value of trade and transport commodities exported to GILD; however, the sum of GILD 'trade balances with other regions must be zero since GILD is an artificial construct rather than a real region. But the demand for trade and transport services by any region is determined by technology, i.e., the coefficients *margcor*, and the volume of imports demanded by the destination region. This means that the prices of trade and transport commodities only have an indirect effect upon their demand – the only place these prices

enter into the import decision as a variable is as a partial determinant of the difference between the *FOB* and *CIF* valuations of other imported commodities. Consequently, the primary market clearing mechanism for the GILD region comes through the quantity of trade and transport commodities it chooses to import.

Figure 7 **Quantity System for the GILD Region**



The GILD concept has other potential uses in the model. All transactions between regions for which there is an absence of full bilateral information can be routed through the GILD region. While this is not a ‘first best’ solution, it does provide a ‘second best’ method by which augmented versions of the GTAP database can be used to enrich the analyses of

international trade in a global model prior to availability of full bilateral transactions data (see McDonald and Sonmez (2006) for an application).

4. Formal Description of the Model

This formal description of the model proceeds in five stages with three of them in this section and the fourth, relating to model closure rules, being detailed in the next section. The fifth stage is in Appendix A1 where alternative specifications for the Armington and CET functions are detailed. For this section, the description begins with identification of the sets used in the model, this is followed by details of each equation (block) in the model and ends with a table that summarises all the equations and identifies the associated variables, the counts for equations and variables and identifies whether the equation is implemented or not for the ANARRES region.

4.1 Model Sets

Rather than writing out each and every equation in detail it is useful to start by defining a series of sets; thereafter if a behavioural relationship applies to all members of a set an equation only needs to be specified once. The natural choice for this model is a set for all the transactions by each region (*sac*) plus a series of sets that group commodities, activities, factors, import duties, export taxes, trade margins, trade and finally some individual accounts relating to domestic institutions. The outer set for any region is defined as

$$sac = \{c, a, f, h, tmr, ter, tff, g, i, owatpmarg, ww, total\}$$

and the following are the basic sets for each region in this model

$$\begin{aligned}
c(sac) &= \{\text{commodities}\} \\
a(sac) &= \{\text{activities}\} \\
f(sac) &= \{\text{factors}\} \\
h(sac) &= \{\text{households}\} \\
tmr(sac) &= \{\text{import duties}\} \\
ter(sac) &= \{\text{export taxes}\} \\
tff(sac) &= \{\text{factort taxes}\} \\
g(sac) &= \{\text{saltax, prodtax, factax, dirtax, Govt}\} \\
i(sac) &= \{\text{kap}\} \\
owatpmarg(sac) &= \{\text{trade and transport margins}\} \\
ww(sac) &= \{\text{rest of the world - trade partners and aggregates}\} \\
wm(ww) &= \{\text{rest of the world - aggregates}\} \\
w(ww) &= \{\text{rest of the world - trade partners}\}
\end{aligned}$$

Various subsets of a and c are declared and then assigned on the basis of certain characteristics of the data set used to calibrate the specific implementation of the model, so-called dynamic sets. The subsets of a used in the model are

$$\begin{aligned}
acx(a, r) &= \{\text{activities purchased domestically}\} \\
acxn(a, r) &= \{\text{activities NOT purchased domestically}\} \\
aqx(a, r) &= \{\text{Activities with CES function at Level 1 of nest}\} \\
aqxn(a, r) &= \{\text{Activities with Leontief function at Level 1 of nest}\} \\
aleon(a) &= \{\text{activities with Leontief top level prodn function}\}
\end{aligned}$$

while the subsets of c used in this model are

$$\begin{aligned}
ct(c, r) &= \{\text{trade margin commodities}\} \\
ctn(c, r) &= \{\text{non-trade margin commodities}\} \\
ct2(c, r) &= \{\text{trade margin commodities used for Globe}\} \\
ctn2(c, r) &= \{\text{non-trade margin commodities used for Globe}\} \\
ce(c, r) &= \{\text{export commodities}\} \\
cen(c, r) &= \{\text{non-export commodities}\} \\
cer(c, r, w) &= \{\text{export commodities by region}\} \\
cern(c, r, w) &= \{\text{non-export commodities by region}\} \\
cm(c, r) &= \{\text{imported commodities}\} \\
cmn(c, r) &= \{\text{non-imported commodities}\} \\
cmr(w, c, r) &= \{\text{imported commodities by region}\} \\
cmrn(w, c, r) &= \{\text{non-imported commodities by region}\} \\
cmrs(w, c, r) &= \{\text{small shares imported commodities by aggregate region}\} \\
cmrsn(w, c, r) &= \{\text{non-small shares imported commodities by aggregate region}\} \\
cms(c, r) &= \{\text{commodities small shares}\} \\
cmrl(w, c, r) &= \{\text{large shares imported commodities by aggregate region}\} \\
cmrln(w, c, r) &= \{\text{non-large shares imported commodities by aggregate region}\} \\
cml(c, r) &= \{\text{commodities large shares}\} \\
cmrn2(c, r, w) &= \{\text{non-imported commodities by aggregate region}\} \\
cx(c, r) &= \{\text{commodities produced domestically}\} \\
cxn(c, r) &= \{\text{commodities NOT produced domestically AND imported}\} \\
cd(c, r) &= \{\text{commodities produced AND demanded domestically}\} \\
cdn(c, r) &= \{\text{commodities NOT produced AND demanded domestically}\} \\
cintd(c, r) &= \{\text{commodities WITH intermediate demand by region}\} \\
cintdn(c, r) &= \{\text{commodities WITHOUT intermediate demand by region}\}
\end{aligned}$$

and a subset of w is needed to allow for GILD

$$wgn(w) = \{\text{Rest of world without Globe}\}.$$

It is also necessary to define a set of regions, r , for which there are two subsets

$$\begin{aligned}
rgn(r) &= \{\text{all regions excluding Globe}\} \\
ref(r) &= \{\text{reference regions for global numeraire}\} \\
rleon(r) &= \{\text{regions with Leontief top level prodn function}\}
\end{aligned}$$

A macro SAM that facilitates checking various aspects of model calibration and operation is used in the model and this needs another set, ss ,

$$ss = \left\{ commdty, activity, valuad, hholds, \right. \\ \left. tmtax, tetax, govt, kapital, margs, world, totals \right\}.$$

The model also makes use of a series of mapping files that are used to link sets. These are

$$\begin{aligned}
map_w_tmr(w, tmr) &= \{\text{Tariff mapping}\} \\
map_tmr_w(tmr, w) &= \{\text{Tariff mapping reverse}\} \\
map_w_ter(w, ter) &= \{\text{Export tax mapping}\} \\
map_ter_w(ter, w) &= \{\text{Export tax mapping reverse}\} \\
map_c_w_marg(c, w, owatpmarg) &= \{\text{Trade margin mapping of owatpmarg to ct2 and w}\} \\
map_marg_w(owatpmarg, w) &= \{\text{Trade margin mapping of w to owatpmarg}\} \\
mapr_w(r, w) &= \{\text{Region to trade partner mapping}\} \\
mapw_r(w, r) &= \{\text{Region to trade partner mapping}\} \\
map_f_tff(f, tff) &= \{\text{Factor taxes to factors}\} \\
map_tff_f(tff, ff) &= \{\text{Factor taxes to factors reverse}\} \\
map_aagg_a(aagg, a) &= \{\text{Mapping from activities to aggregate activities}\} \\
map_cagg_c(cagg, c) &= \{\text{Mapping from commodities to aggregate commodities}\} \\
map_wagg_w(wagg, w) &= \{\text{Mapping from regions to aggregate regions}\} \\
map_ragg_r(ragg, r) &= \{\text{Mapping from regions to aggregate regions}\}
\end{aligned}$$

Finally various other sets are declared to facilitate model operation. These are

$$\begin{aligned}
sacn(sac) &= \{\text{SAM accounts without totals}\} \\
ssn(ss) &= \{\text{Macro SAM accounts without totals}\} \\
cons &= \{\text{set for programme control parameters}\}
\end{aligned}$$

4.1.1 Reserved Names

The model uses a number of names that are reserved; these are

DIRTAX	Direct Taxes
SALTAX	Sales Taxes
PRODTAX	Production Taxes
FACTAX	Factor Taxes

4.1.2 Conventions

The equations for the model are set out in 9 ‘blocks’ each of which can contain a number of sub blocks. The equations are grouped under the following headings:

1. TRADE BLOCK
 - a. Exports Block
 - b. Imports Block
2. COMMODITY PRICE BLOCK
3. NUMERAIRE PRICE BLOCK
4. PRODUCTION BLOCK
 - a. Production
 - b. Intermediate Input Demand
 - c. Commodity Output
 - d. Activity Output
5. FACTOR BLOCK
6. HOUSEHOLD BLOCK
 - a. Household Income
 - b. Household Expenditure
7. GOVERNMENT BLOCK
 - a. Government Tax Rates
 - b. Government Tax Revenues
 - c. Government Income
 - d. Government Expenditure Block
8. KAPITAL BLOCK
 - a. Savings Block
 - b. Investment Block
9. MARKET CLEARING BLOCK
 - a. Factor Accounts
 - b. Commodity Accounts
 - c. Investment and Savings Accounts

- d. Commodity Trade Accounts
- e. Margin Trade Accounts
- f. Absorption Closure
- g. Slack

This grouping is carried throughout the model code, i.e., it is followed for the parameter declaration and calibration, variable declaration and variable initialization sections. This modularization of the code is adopted for ease of reading and altering the model rather than being a requirement of the model.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modeling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- The standard prefixes for variable names are: P for price variables, Q for quantity variables, W for factor prices, F for factor quantities, E for expenditure variables, Y for income variables, and V for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.
- A series of variables are declared that allow for the equiproportionate multiplicative adjustment of groups of variables. These variables are named using the convention $**ADJ$, where $**$ is the variable series they adjust.
- A series of variables are declared that allow for the additive adjustment of groups of variables. These variables are named using the convention $D**$, where $**$ is the variable series they adjust.
- All parameters are in lower case, except those paired to variables that are used to initialise variables.
- Parameter names have a two or five character suffix which distinguishes their definition, e.g., $**sh$ is a share parameter, $**av$ is an average and $**const$ is a constant parameter.
- For the Armington (CES) functions all the share parameters are declared with the form $delta**$, all the shift/efficiency parameters are declared with the form $ac**$, and all the elasticity parameters are declared with the form $rho**$, where $**$ identifies the function in which the parameter operates.

- For the CET functions all the share parameters are declared with the form γ^{**} , all the shift/efficiency parameters are declared with the form α^{**} , and all the elasticity parameters are declared with the form ρ^{**} , where $**$ identifies the function in which the parameter operates.
- All coefficients in the model are declared with the form α^{****} , where $****$ consists of two parts that identify the two variables related by the coefficient.
- The index ordering follows the specification in the SAM: row, column, and then r to indicate the region. For example, exports from region r to region w would be $QER_{c,w,r}$ because region r 's export data in its SAM is found in the commodity row (c) and the trade partner column (w). Likewise, imports in region r from region w are designated, $QMR_{w,c,r}$ because region r 's import data in its SAM is found in the trade partner row (w) and the commodity column (c).
- All sets have another name, or alias, given by the set name followed by "p". For example, the set of commodities may be called c or cp .

4.2 Equations for the Model

The model equations are reported and described by blocks/groups below and then they are summarised in table A4 in the appendix.

4.2.1 Exports Block Equations

The treatment of exports is complicated by the incorporation of the facility to treat export commodities as imperfect and by the need to accommodate the special case of exports (of trade and transport services) that are homogenous from GILD. The presumption of imperfect substitution is the default presumption in this model; reasons for this decision being its symmetry with the Armington assumption on the imports side, the amelioration of the terms of trade effects associated with the Armington assumption and a belief that in general there is differentiation between commodities supplied to domestic and export markets. However there are proponents of the arguments for treating exports as perfect substitutes and there are clearly cases where such an assumption may be appropriate, e.g., supplies of unprocessed mineral products.⁹ A formulation of the model allows the CET functions to be switched off at either or both levels of the export supply nest for specific commodities and/or for specific regions.

⁹ The GTAP model assumes perfect substitution and historically it has been argued that perfect substitution is appropriate for Australia.

The domestic prices of commodity exports, c , by destination, w , and source, r , region (PER) are defined as the product of world prices of exports (PWE) – also defined by commodity and destination and source region, the source region's exchange rate (ER) and one minus the export tax rate (TE) (X1). The possibility of non-traded commodities means that the equations for the domestic prices of exports (and imports) are only implemented for those commodities that are traded; this requires the use of a dynamic set, cer , which is defined by those commodities that are exported in the base data. Also notice that the world prices of exports (PWE) are defined as variables; in a global model the small country trade assumption is not valid since, by definition, world prices are endogenous and therefore ALL regions are treated as 'large' producers of a commodity.

Export Block Equations 1

$$PER_{c,r,w} = PWE_{c,r,w} * ER_r * (1 - TE_{c,r,w}) \quad \forall c \in cer \quad (X1)$$

$$PE_{c,r} * QE_{c,r} = \sum_w PER_{c,r,w} * QER_{c,r,w} \quad \forall c \in ce, r \in rgn \quad (X2)$$

$$PER_{c,r,w} = PE_{c,r} \quad \forall c \in ct2, r \notin rgn, w \in wgn \quad (X3)$$

$$QXC_{c,r} = at_{c,r} \cdot \left(\gamma_{c,r} * QE_{c,r}^{\rho_{c,r}^t} + (1 - \gamma_{c,r}) * QD_{c,r}^{\rho_{c,r}^t} \right)^{1/\rho_{c,r}^t} \quad (X4)$$

$$\forall c \in (cd \cap ce), r \in rgn$$

$$QE_{c,r} = QD_{c,r} \left[\frac{PE_{c,r}}{PD_{c,r}} * \frac{(1 - \gamma_{c,r})}{\gamma_{c,r}} \right]^{\frac{1}{(\rho_{c,r}^t - 1)}} \quad \forall c \in (cd \cap ce), r \in rgn \quad (X5)$$

$$QXC_{c,r} = QD_{c,r} + QE_{c,r} \quad \forall (c \in (cd \cap cen), r \in rgn)$$

$$\text{OR } \forall (c \in (cdn \cap ce), r \in rgn) \quad (X6)$$

The prices of the composite export commodities can then be expressed as simple volume weighted averages of the of the export prices by region, where $PE_{c,r}$ and $QE_{c,r}$ the price and quantity of the composite export commodity c from region r , and the weights are the volume shares of exports and are variable. This comes from the fact that a CET function is liner homogenous and hence Eulers theorem can be applied. Notice however that (X2) is only implemented of the set rgn , i.e., the region GILD is excluded. Rather the composite export price for trade margin commodities from GILD is defined in X3, which indicates that it is

assumed that the trade margin commodities exported by ANARRES are perfect substitutes for each other, i.e., the same price is paid for each trade margin commodity by ALL purchasing regions.

Domestic commodity outputs (QXC) are either exported (QE) as composite commodities or supplied to the domestic market (QD). The allocation of output between the domestic and export markets is determined by the output transformation functions, Constant Elasticity of Transformation (CET) functions, (X4) with the optimum combinations of QE and QD determined by first-order conditions (X5). In this version of the model primal forms of the CET is used with associated first-order conditions. However, some commodities are non-traded and therefore X4 and X5 are implemented if and only if the commodity is traded. This means that domestic commodity outputs are undefined for non-traded commodities, but by definition the quantity supplied to the domestic market is the amount produced, and it is necessary to cover the possibility that a commodity may be produced domestically and exported but not consumed domestically. These two sets of possibilities are covered by X6.

Export Block Equations 2

$$QER_{c,w,r} = QE_{c,r} * \left(\frac{PER_{c,w,r}}{\left((PE_{c,r} * \gamma_{c,w,r}^r * at_{c,r}^r)^{\rho_{c,r}^e} \right)} \right)^{\left(\frac{1}{\rho_{c,r}^e - 1} \right)} \quad \forall c \in cer2, r \in rgn \quad (X7)$$

$$QE_{c,r} = \sum_w QER_{c,r,w} \quad \forall c \in ct, r \notin rgn. \quad (X8)$$

$$QE_{c,r} = QM_{c,r} \quad \forall c \in ct2, r \notin rgn. \quad (X9)$$

These quantity equations deal however only with the composite export commodities, i.e., hypothetical commodities whose roles in the model are to act as neutral intermediaries that enter into the first-order conditions that determine the optimal mix between domestic use and exports of domestic commodity production (X5). In the model the composite export commodities are themselves CET aggregates of commodity exports to different regions (QER), and the appropriate first order condition is given by (X7). Initially the formulation of X7 is not intuitive but, as demonstrated in Appendix A1, the formulation is a straightforward manipulation of a more conventional representation; this form is used because it improves

model performance. Note however that (X7) does not define the exports of trade margin commodities BY ANARRES; this is because these commodities are assumed to be perfect substitutes and therefore simple addition is adequate, this is done by X8.

Finally, there is a need for an equilibrium conditions for trade by ANARRES. Since ANARRES is an artificial construct whose sole role in the model is to gather exports, whose destinations are unknown and supply imports whose sources are unknown, and vice versa, it must always balance its trade within each period. Thus, the volume of exports of trade margin commodities by GILD must be exactly equal to the volume imports of trade margin commodities, see X9.

4.2.2 Imports Block Equations

The prices of imported commodities are made up of several components. The export price in foreign currency units – valued free on board (fob) ($PWMFOB$) – plus the cost of trade and transport services, which gives the import price carriage insurance and freight (*cif*) paid (PWM), plus any import duties; all of which are then converted into domestic currency units (PMR). Clearly the import price value *fob* ($PWMFOB$) is identical to the export price valued *fob* (PWE) – this condition is imposed in the market clearing block (see below) – and hence the *cif* price is defined in M3, where *margcor* is the quantity of trade and transport services required to import a unit of the imported commodity and PT is the price of trade and transport services. Embedded in the definition of the coefficient *margcor* is the explicit assumption that transporting a commodity from a specific source to a specific destination requires the use of a specific quantity of services – the actual cost of these services can vary according to changes in the prices of the trade and transport services or the quantity of services required to transport a particular commodity.

The domestic prices of imports from a region (PMR) are then defined as the product of world prices of imports (PWM) – after payment for carriage, insurance and freight (*cif*) - the exchange rate (ER) and one plus the import tariff rate (TM). The possibility of non-traded commodities means that the equations for the domestic prices of imports are only implemented for those commodities that are traded; this requires the use of a dynamic set, *cmr*, which is defined by those commodities that are imported by a region from another region in the base data.

Import Block Equations 1

$$PML_{c,r} = \frac{\sum_{w \in cmr_{w,c,r}} (PMR_{w,c,r} * QMR_{w,c,r})}{QML_{c,r}} \quad \forall (c,r) \in cml \quad (M1)$$

$$PMS_{c,r} = \sum_w ioqmrqms_{w,c,r} * PMR_{w,c,r} \quad \forall (c,r) \in cms \quad (M2)$$

$$PWM_{w,c,r} = PWMFOB_{w,c,r} + \left(\sum_{cp \in ct_{cp,r}} margcor_{w,cp,c,r} * PT_{cp,r} \right) \quad (M3)$$

$$\forall c \in cmr$$

$$PMR_{w,c,r} = PWM_{w,c,r} * ER_r * (1 + TM_{w,c,r}) \quad (M4)$$

$$\forall c \in cmr$$

$$PM_{c,r} * QM_{c,r} = (PML_{c,r} * QML_{c,r}) + (PMS_{c,r} * QMS_{c,r}) \quad (M5)$$

$$\forall (c,r) \in cm_{c,r}$$

The model treats imports that account for ‘small’ shares of imports of a commodity by a region differently from those that account for ‘large’ shares of imports of a commodity by a region.¹⁰ This is because the operations of CES functions depend not only on the elasticity of substitution but also on the shares of arguments: commodities with ‘small’ trade shares can have a disproportionately large impact on the terms of trade in such aggregation functions. For commodities with ‘small’ trade shares it is assumed that they account for fixed proportions ($ioqmrqms_{w,c,r}$) of the total volume of imports of a commodity: the justification for this assumption rest upon a (vaguely defined) specific factor specification. The composite price of ‘small’ share imports (PMS) is therefore a quantity share weighted aggregate of the landed prices (M2). The composite price of ‘large’ share imports (PML) is a simple volume weighted averages of the of the import prices by region: this derives from the fact that a CES function is liner homogenous and hence Eulers theorem can be applied (M1).

The prices of the composite import commodities can also be expressed as a simple volume weighted averages of the of the import prices by region, (M5) where $PM_{c,r}$ and $QM_{c,r}$ are the price and quantity of the composite import commodity c by region r , and the weights

¹⁰ The definition of ‘small’, and hence by complement ‘large’, is selected by the user when calibrating the model.

are the volume shares of imports and are variable. Notice however that (M5) is only controlled by the set cm , in contrast to (X2) – the composite export price – which was also controlled by the set rgn , i.e., the region ANARRES was excluded. This reflects the fact that the region ANARRES does import commodities using the same trading assumption as other regions but only exports homogenous trade and transport services, which explains the need for the equation (X3).

Import Block Equations 2

$$QQ_{c,r} = ac_{c,r} \left(\delta_{c,r} * QM_{c,r}^{-\rho_{c,r}^e} + (1 - \delta_{c,r}) * QD_{c,r}^{-\rho_{c,r}^e} \right)^{\left(\frac{1}{\rho_{c,r}^e} \right)} \quad \forall c \in (cx \cap cm), r \in rgn \quad (M6)$$

$$QM_{c,r} = QD_{c,r} * \left(\left(\frac{PD_{c,r}}{PM_{c,r}} \right) * \left(\frac{\delta_{c,r}}{(1 - \delta_{c,r})} \right) \right)^{\frac{1}{(1 + \rho_{c,r}^e)}} \quad \forall c \in (cx \cap cm), r \in rgn \quad (M7)$$

$$QQ_{c,r} = QD_{c,r} + QM_{c,r} \quad \forall c \in (cx \cap cmn), r \in rgn \quad (M8)$$

OR $\forall c \in (cxn \cap cm), r \in rgn$

$$QMS_{c,r} = ioqmsqm_{c,r} + QM_{c,r} \quad \forall c \in cms \quad (M9)$$

$$QML_{c,r} = ioqmlqm_{c,r} + QM_{c,r} \quad \forall c \in cml \quad (M10)$$

$$QMR_{w,c,r} = ioqmrqms_{w,c,r} + QMS_{c,r} \quad \forall c \in cmrs \quad (M11)$$

$$QMR_{w,c,r} = QML_{c,r} * \left(\frac{PMR_{w,c,r} * acr_{c,r}^{\rho_{c,r}^m}}{PML_{c,r} * \delta_{w,c,r}^r} \right)^{\left(\frac{-1}{(\rho_{c,r}^m + 1)} \right)} \quad \forall c \in cmrl \quad (M12)$$

$$QT_{w,c,r} = \sum_{cp} (QMR_{w,cp,r} * margcor_{w,cp,w,r}) \quad \forall c \in ct2, r \in rgn \quad (M13)$$

The composite supply of the imported commodity (QM) is a Leontief aggregate of imports with ‘small’ (QMS) and ‘large’ (QML) shares using appropriately defined input-output coefficients (M9 and M10). Similarly the quantities imported of the ‘small’ share commodities by source region are defined by fixed (input-output) coefficients (M11). The use of a two stage Leontief nest in this case ensures no substitution possibilities between the

aggregate imports QMS and QML ; if some substitution possibilities are required the nesting system used for production can be adapted.

The composite imports of commodities with ‘large’ shares are defined as CES aggregates of the imports from different regions (QMR) (M12). The first order conditions come from the price definition terms for composite imports, PML (M1) and are only implemented for those cases where there were import transactions in the base period – this is controlled by the set cmr . Initially the formulation of M12 is not intuitive but, as demonstrated in Appendix A1, the formulation is a straightforward manipulation of a more conventional representation; this form, which is effectively a set of first-order conditions, is used because it reduces the number of equations in the model.

The composite (consumption) commodities are then a mixture of composite imports (QM) and domestic demand from domestic production (QD). The mixtures between the domestic and import supplies are determined by the substitution functions, Constant Elasticity of Substitution (CES) functions, (M6) with the optimal combinations of QM and QD being determined by first-order conditions, (M7). But, some commodities are non-traded and therefore M6 and M7 are implemented only if the commodity is traded, which leaves QQ undefined for non-traded commodities. By definition if there are no imports the quantity demanded by the domestic market is the amount produced, and if there is no domestic production the amount demanded is the amount imported. These two sets of possibilities are both covered by M8.

Also associated with any imported commodity is a specific quantity of trade and transport services. These services are assumed to be required in fixed quantities per unit of import by a specific region from another specific region, (M13) where the $margcor$ are the trade and transport coefficients associated with a unit (quantity) import by region r from region w . This is only implemented for trade and transport commodities ($ct2$) and for regions that ‘actually’ import goods (rgn).

4.2.3 Commodity Price Block Equations

The composite price equations (CP1, CP2 and CP3) are derived from the first order conditions for tangencies to consumption and production possibility frontiers. By exploiting Euler’s theorem for linearly homogeneous functions the composite prices can be expressed as expenditure identities rather than dual price equations for export transformation and import

aggregation, such that $PQS_{c,r}$ is the weighted average of the producer price of a commodity, when $PD_{c,r}$ is the producer price of domestically produced commodities and $PM_{c,r}$ the domestic price of the composite imported commodity, (CP1) where $QD_{c,r}$ the quantity of the domestic commodity demanded by domestic consumers, $QM_{c,r}$ the quantity of composite imports and $QQ_{c,r}$ the quantity of the composite commodity. Notice how the commodity quantities are the weights. This composite commodity price (CP1) does not include sales taxes, which create price wedges between the purchaser price of a commodity ($PQD_{c,r}$) and the producer prices ($PQS_{c,r}$). Hence the purchaser price is defined as the producer price plus the sales taxes (CP2).

This formulation means that the sales taxes are levied on all sales on the domestic market, irrespective of the origin of the commodity concerned.

Commodity Price Block Equations

$$PQS_{c,r} = \frac{(PD_{c,r} * QD_{c,r}) + (PM_{c,r} * QM_{c,r})}{QQ_{c,r}} \quad \forall c \in (cd \cup cm), r \in rgn \quad (CP1)$$

$$PQD_{c,r} = PQS_{c,r} * (1 + TS_{c,r}) \quad \forall c \in (cd \cup cm), r \in rgn \quad (CP2)$$

$$PXC_{c,r} = \frac{(PD_{c,r} * QD_{c,r}) + (PE_{c \in ce, r} * QE_{c \in ce, r})}{QXC_{c,r}} \quad \forall c \in cx, r \in rgn \quad (CP3)$$

The composite output price for a commodity, $PXC_{c,r}$, is also derived by exploiting Euler's theorem for linearly homogeneous functions, and is given by (CP3) where $PD_{c,r}$ is the domestic producer price for the output of commodities supplied to the domestic market, $QD_{c,r}$ is the supply of output to the domestic market, $QE_{c,r}$ is the quantity exported by activities, and $QXC_{c,r}$ is the quantity of domestic production by commodity.

4.2.4 Numéraire Price Block

It is also necessary to define a price numéraire for each region; for this model two alternative numéraire are defined so as to allow the modeler some discretion as to the choice of

numéraire.¹¹ The consumer price indices (CPI_r) are defined as base weighted sum of the commodity prices, where the weights are the value shares of each commodity in final demand ($comtotsh_{c,r}$), (N1).

While the domestic producer price indices (PPI_r) are defined as the weighted sums of the commodity prices received by producers on the domestic market, where the weights are the value shares of each commodity supplied by domestic producers to the domestic market ($vddtotsh_{c,r}$), (N2). This provides a convenient alternative price normalisation term; if the exchange rate is also fixed it serves to fix the real exchange rate.

Numéraire Price Block Equations

$$CPI_r = \sum_c comtotsh_{c,r} * PQD_{c,r} \quad \forall r \in rgn \quad (N1)$$

$$PPI_r = \sum_c vddtotsh_{c,r} * PD_{c,r} \quad \forall r \in rgn \quad (N2)$$

$$ERPI = \sum_{ref} tradtotsh_{ref} * ER_{ref} \quad (N3)$$

Notice how both price indices are controlled to be implemented only for those regions that have consumption and production activities. Hence the ANARRES does not have its own price indices; rather the price indices for ANARRES are those of the reference region(s) in the model.

The exchange rate numéraire (N3) is defined as an exchange rate index for the set ref , which allows the user to define the reference region/regions for the exchange rate. In an earlier version of the model the exchange rate of a single region – usually the USA or the aggregate including the USA – was fixed and served as the numéraire. This alternative is still available if the membership of ref is limited to a single region but experience suggests that model performance is improved by including several regions in ref ; typically this is done with a basket of regions containing the OECD members but other alternatives are possible.

¹¹ The price index not used as the numéraire provides useful additional information when interpreting the results.

Fixed country trade balances must be seen as specified in “real” terms defined by the global numéraire. So, if the US exchange rate is fixed to one, the global numéraire is defined as US dollars, and all trade balances can be seen as “real” variables defined in terms of the value of US exports. If the weighted exchange rate for a group of regions is chosen as global numéraire, trade balances can be seen as a “claim” against a weighted average of exports by the group of regions.

4.2.5 Production Block Equations

The output price by activity ($PX_{a,r}$) is defined by the shares of commodity outputs produced by each activity, (P1.1) where, for this case, the weights ($ioqxcqx_{a,c,r}$) are equal to one where the commodities and activities match and zero otherwise, i.e., there is a one to one mapping between the commodity and activity accounts. The weights are derived from the information in the supply or make matrix.¹²

The value of output by activity is defined as the activity price ($PX_{a,r}$) less production taxes ($TX_{a,r}$) times the volume of output ($QX_{a,r}$). This revenue must be divided between payments to primary inputs – the price of value added ($PVA_{a,r}$) times the quantity of value added ($QVA_{a,r}$) – and intermediate inputs – the price of aggregate intermediate inputs ($PINT_{a,r}$) times the volume of aggregate intermediate inputs ($QINT_{a,r}$) (P1.2). Given the assumption that intermediate inputs are used in fixed (volume) proportions, the price of aggregate intermediate inputs ($PINT_{a,r}$) is defined as the weighted average price of the intermediate inputs where the weights are the (normalised) input-output coefficients (P1.3).

The default top level production function (P1.5), is a CES aggregation of aggregate primary and intermediate inputs, where the first order conditions for profit maximization (P1.6) determine the optimal ratio of the inputs. The efficiency factor ($ADX_{a,r}$) and the factor shares parameters ($\delta_{a,r}^x$) are calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ($\rho_{a,r}^x$), are exogenously imposed. Note in this case the efficiency factor is declared as variable and is determined by (P1.4), where $adx0_{a,r}$ is the vector of efficiency factors in the base solution, $dabadx_{a,r}$ is a vector of absolute changes in the vector of efficiency factors, $ADXADJ_r$ is a variable whose initial value is ONE, $DADX_r$

¹² When using GTAP data, $ioqxcqx_{a,c,r}$ is always a diagonal matrix.

is a variable whose initial value is ZERO and $adx01_c$ is a vector of zeros and non zeros.¹³ In the base solution the values of $adx0_{a,r}$ and $dabadx_{a,r}$ are all ZERO and $ADXADJ_r$ and $DADX_r$ are fixed as their initial values – a closure rule decision – then the applied efficiency factors are those from the base solution. This formulation allows flexibility in the formulation of the efficiency parameter that is especially useful in the context of a dynamic model – the structure of the equation is identical to that used for the tax rate equations and a description of its operation is provided when describing the tax rate equations.

¹³ Typically the values are either one or zero, i.e., the adjustment factor is switched on or off. Non zero values other than one switch on the adjustment factor and allow a more complex set of adjustments although it is important to be careful about the rationale for such a set of adjustments.

Production Block Equations – Top Level

$$PX_{a,r} = \sum_c ioqxcqx_{a,c,r} * PXC_{c,r} \quad \forall r \in rgn \quad (P1.1)$$

$$PX_{a,r} * (1 - TX_{a,r}) * QX_{a,r} = (PVA_{a,r} * QVA_{a,r}) + (PINT_{a,r} * QINT_{a,r}) \quad \forall r \in rgn \quad (P1.2)$$

$$PINT_{a,r} = \sum_c ioqint_{c,a,r} * PQD_{c,r} \quad \forall r \in rgn \quad (P1.3)$$

$$ADX_{a,r} = ((adx0_{a,r} + dabadx_{a,r}) * ADXADJ_r) + (DADX_r * adx01_{a,r}) \quad (P1.4)$$

$$QX_{a,r} = ADX_{a,r} * \left[\delta_{a,r}^x * (QVA_{a,r})^{-\rho_{a,r}^x} + (1 - \delta_{a,r}^x) * (QINT_{a,r})^{-\rho_{a,r}^x} \right]^{-1/\rho_{a,r}^x} \quad \forall r \in rgn, a \in aqx \quad (P1.5)$$

$$QVA_{a,r} = QINT_{a,r} * \left(\left(\frac{PINT_{a,r}}{PVA_{a,r}} \right) * \left(\frac{\delta_{a,r}^x}{(1 - \delta_{a,r}^x)} \right) \right)^{\left(\frac{1}{(1 + \rho_{a,r}^x)} \right)} \quad \forall r \in rgn, a \in aqx \quad (P1.6)$$

$$QINT_{a,r} = \sum_a ioqintqx_{a,r} * QX_{a,r} \quad \forall r \in rgn, a \in aqxn \quad (P1.7)$$

$$QVA_{a,r} = \sum_a ioqvaqx_{a,r} * QX_{a,r} \quad \forall r \in rgn, a \in aqxn \quad (P1.8)$$

$$QXC_{c,r} = \sum_a ioqxcqx_{a,c,r} * QX_{a,r} \quad \forall r \in rgn \quad (P1.9)$$

The production function (P1.5) is only implemented for members of the set *aqx*; for its complement, *aqxn*, the CES function is replaced by Leontief functions. These require that aggregate intermediate inputs (P1.7) and aggregate values added (P1.8) are fixed proportions of the volumes of output. If there are no intermediate inputs used by an activity the top level functions is automatically Leontief, and the user is able to determine the minimum costs share of intermediate inputs below which the Leontief assumption is imposed automatically after the user defines the share – the default is 10 percent.

Finally it is necessary to define the relationship between activity and commodity outputs, which is the counterpart to the price equation linking commodity and activity prices (P1.1). This is defined as a simple linear relationship whereby the commodity output is defined as the sum of the quantities of each commodity produced by each activity, (P1.9). But

given the underlying data, whereby each activity only produces a single commodity and $ioqxqxc$ is an identity matrix, (P1.9) simply maps activities to commodities and *visa versa*.

Production Block Equations – Second Level

$$ADVA_{a,r} = \left((adva0_{a,r} + dabadv_{a,r}) * ADVAADJ_r \right) + \left(DADVA_r * adva01_{a,r} \right) \quad (P2.1)$$

$$QVA_{a,r} = ADVA_{a,r} * \left(\sum_f \delta_{f,a,r}^{va} * (ADFD_{f,a,r} * FD_{f,a,r})^{-\rho_{a,r}^{va}} \right)^{\left(\frac{-1}{\rho_{a,r}^{va}} \right)} \quad (P2.2)$$

$\forall r \in rgn$

$$\begin{aligned} & WF_{f,r} * WFDIST_{f,a,r} * (1 + TF_{f,a,r}) \\ &= PVA_{a,r} * QVA_{a,r} * \left[\sum_f \delta_{f,a,r}^{va} * (ADFD_{f,a,r} * FD_{f,a,r})^{-\rho_{a,r}^{va}} \right]^{(-1)} \\ & \quad * \delta_{f,a,r}^{va} * (ADFD_{f,a,r})^{-\rho_{a,r}^{va}} * (FD_{f,a,r})^{(-\rho_{a,r}^{va}-1)} \end{aligned} \quad (P2.3)$$

$\forall r \in rgn, \delta_{f,a,r}^{va}$

$$QINTD_{c,r} = \sum_a ioqint_{c,a,r} * QINT_{a,r} \quad \forall r \in rgn, cintd_{c,r} \quad (P2.4)$$

The second level production functions are CES aggregation function over the all factors that are demanded by each activity ($FD_{f,a,r}$), with efficiency factors ($ADVA_{a,r}$) and the factor shares ($\delta_{f,a,r}^{va}$) calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ($\rho_{a,r}^{va}$), are exogenously imposed (P2.2). The associated first-order conditions for optimal factor combinations are derived from equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity, (P2.3). The actual wage rate for a specific factor used by a specific activity is defined as the average wage rate for that factor ($WF_{f,r}$) times a factor and activity specific factor ‘efficiency’ parameter ($WFDIST_{f,a,r}$); these ratios of payments to factor f from activity a are included to allow for non-homogenous factors where the differentiation is defined solely in terms of the activity that employs the factor. However the actual returns to a factor must be adjusted to allow for taxes on factor use ($TF_{f,r}$).

The efficiency factors are declared as variables (P2.1), where $adva0_{a,r}$ is the vector of efficiency factors in the base solution, $dabadva_{a,r}$ is a vector of absolute changes in the vector of efficiency factors, $ADVAADJ_r$ is a variable whose initial value is ONE, $DADVA_r$ is a variable whose initial value is ZERO and $adva01_{a,r}$ is a vector of zeros and non-zeros.¹⁴ In the base solution the values of $adva0_{a,r}$ and $dabadva_{a,r}$ are all ZERO and $ADVAADJ_r$ and $DADVA_r$ are fixed as their initial values – a closure rule decision – then the applied efficiency factors are those from the base solution.

Since production uses intermediate inputs, it is also necessary to specify the demand for intermediate inputs ($QINTD_{c,r}$). This is done from the perspective of commodity demands, i.e., it is summed over activities to produce the demand for intermediate inputs by commodity rather than by activity (P2.4).

4.2.6 Factor Block Equations

The total income received by each factor account (YF_f) is defined as the summation of the earnings of that factor across all activities (F1). However, only a proportion of total factor income is available for distribution to the domestic institutional accounts ($YFDIST_{f,r}$). First allowance must be made for depreciation, which it is assumed takes place at fixed rates ($deprec_{f,r}$) relative to factor incomes and the payment of factor income taxes ($TYF_{f,r}$) (F2).

Factor Block Equations

$$YF_{f,r} = \sum_a WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r} \quad \forall r \in rgn \quad (F1)$$

$$YFDIST_{f,r} = \left(YF_{f,r} - (deprec_{f,r} * YF_{f,r}) \right) * (1 - TYF_{f,r}) \quad \forall r \in rgn \quad (F2)$$

4.2.7 Household Block Equations

Households acquire income from only one source in this model; the sale of factor services. Therefore household income ($YH_{h,r}$) is defined simply as the sum of factor incomes available

¹⁴ Typically the values are either one or zero, i.e., the adjustment factor is switched on or off. Non zero values other than one switch on the adjustment factor and allow a more complex set of adjustments although it is important to be careful about the rationale for such a set of adjustments.

for distribution. In this variant allowance is made for the possibility of multiple households by including a set of distribution parameters ($hvas_{h,f,r}$) that are defined as the shares of each factor demanded in the economy that is supplied by each household. In the case of one household all the shares equal one.

Household Block Equations

$$YH_{h,r} = \sum_f hvas_{h,f,r} * YFDIST_{f,r} \quad \forall r \in rgn \quad (H1)$$

$$HEXP_{h,r} = (YH_{h,r} * (1 - TYH_{h,r})) * (1 - SHH_{h,r}) \quad \forall r \in rgn \quad (H2)$$

$$QCD_{c,h,r} * PQD_{c,r} = (PQD_{c,r} * qcdconst_{c,h,r}) + \beta_{c,h,r} * \left(HEXP_{h,r} - \sum_c PQD_{c,r} * qcdconst_{c,h,r} \right) \quad \forall r \in rgn \quad (H3)$$

Household consumption demand is derived in two stages. In the first stage (H2) household consumption expenditures ($HEXP_{h,r}$) are defined as household incomes after the payment of direct taxes and savings and inter household transfers. Note how the saving rates are defined as proportions of after tax incomes that are saved; this is important for the calibration of the income tax and savings parameters.

The household utility functions are assumed to be Stone-Geary, i.e., a linear expenditure system, which means that household consumption demand consists of two components, ‘subsistence’ demand ($qcdconst_{c,h,r}$) and ‘discretionary’ demand, and the equation must therefore capture both elements. Discretionary demand is defined as the marginal budget shares (β) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand. The quantities of each commodity demanded by the household are then defined by the shares of household consumption expenditure. The Stone-Geary function collapses to a Cobb-Douglas utility function if all the income elasticities of demand are set equal to one and the Frisch parameter, the elasticity of the marginal utility of income, is also set to one. One advantage of the Cobb-Douglas specification is that it results in the changes in the values for household consumption

expenditures ($HEXP_{h,r}$) being equal to the changes in an equivalent variation measure of household welfare.

4.2.8 Government Tax Block Equations

There are seven tax instruments. Each is defined as a simple *ad valorem* rate dependent upon the values of imports, exports, sales, production and factor use by activities and the levels of factor and household and income. The ‘tax’ rates are all declared as variables.

All tax rates are variables in this model and for each tax instrument a series of factors are declared to facilitate policy experiments. The tax rates in the base solution are defined as parameters, e.g., $tm0_{w,c,r}$ are the import tariff rates by commodity c imported from region w in region r in the base solution, and the tax rate equations then allow for varying the tax rates in four different ways. Two of the methods use variables that can be solved for optimum values in the model according to the choice of closure rule and two methods allow for deterministic adjustments to the structure of the tax rates. The operations of these methods are discussed in detail only for the equations for import duties while the other equations are simply reported.

Government Tax Rates Equations

$$TM_{w,c,r} = \left((tm0_{w,c,r} + dabtm_{w,c,r}) * TMADJ_r \right) + (DTM_r * tm01_{w,c,r}) \quad \forall c \in cmr \quad (T1.1)$$

$$TE_{c,w,r} = \left((te0_{c,w,r} + dabte_{c,w,r}) * TEADJ_r \right) + (DTE_r * te01_{c,w,r}) \quad \forall c \in cer \quad (T1.2)$$

$$TS_{c,r} = \left((ts0_{c,r} + dabts_{c,r}) * TSADJ_r \right) + (DTS_r * ts01_{c,r}) \quad (T1.3)$$

$$\forall r \in rgn, c \in (cd \cup cm)$$

$$TX_{a,r} = \left((tx0_{a,r} + dabtx_{a,r}) * TXADJ_r \right) + (DTX_r * tx01_{a,r}) \quad \forall r \in rgn \quad (T1.4)$$

$$TYF_{f,r} = \left((tyf0_{f,r} + dabtyf_{f,r}) * TYFADJ_r \right) + (DTYF_r * tyf01_{f,r}) \quad \forall r \in rgn \quad (T1.5)$$

$$TYH_{h,r} = \left((tyh0_{h,r} + dabtyh_{h,r}) * TYHADJ_r \right) + (DTYH_r * tyh01_{h,r}) \quad \forall r \in rgn \quad (T1.6)$$

$$TF_{f,a,r} = \left((tf0_{f,a,r} + dabtf_{f,a,r}) * TFADJ_r \right) + (DTF_r * ty01_{f,a,r}) \quad \forall r \in rgn \quad (T1.7)$$

In the import tariff rate equation (T1.1) $tm0_{w,c,r}$ is the vector of import duties in the base solution, $dabtm_{w,c,r}$ is a vector of absolute changes in the vector of import duties taxes, $TMADJ_r$ is a region specific variable whose initial value is ONE, DTM_r is a region specific variable whose initial value is ZERO and $tm01_{w,c,r}$ is a vector of zeros and non-zeros. In the base solution the values of $tm01_{w,c,r}$ and $dabtm_{w,c,r}$ are all ZERO and $TMADJ_r$ and DTM_r are fixed as their initial values – this is a closure rule decision. Given this decision the model treats the tax rates as (fixed) parameters and the applied import tariffs are those from the base solution. The different methods of adjustment can be considered in turn.

1. If $TMADJ_r$ for one region is made a variable, which requires the fixing of another variable for that region, and all other initial conditions hold then the solution value for $TMADJ_r$ yields the optimum equiproportionate change in the import duty rates necessary to satisfy model constraints, e.g., if $TMADJ_r$ equals 1.1 then all applied (non-zero) import duty rates (that are found in the base data) for the specified region are increased by 10%.
2. If any element of $dabtm_{w,c,r}$ is not zero, and all the other initial conditions hold, then an absolute change in the initial import tariff rate for the relevant

commodity is imposed, e.g., if $tm0_{w,c,r}$ for one element of c is 0.1 (a 10% import duty) and $dabtm_{w,c,r}$ for that element is 0.05, then the *applied* import tariff rate is 0.15 (15%).

3. If $TMADJ_r$ for one region is made a variable, which requires the fixing of another variable for that region, any elements of $dabtm_{w,c,r}$ are non-zero and all other initial conditions hold, then the solution value for $TMADJ_r$ yields the optimum equiproportionate change in the *applied* import duty rates, i.e., $tm0_{w,c,r} + dabtm_{w,c,r}$ (and these applied rates can be different from those in the base in which $dabtm_{w,c,r}$ is set at zero).
4. If DTM_r for one region is made a variable, which requires the fixing of another variable for that region, AND at least one element of $tm01_{w,c,r}$ is ONE then the subset of elements of $tm0_{w,c,r}$ identified by $tm01_{w,c,r}$ are allowed to (additively) increase by an equiproportionate amount determined by the solution value for DTM_r and the initial values of the import duty rates, e.g., if food products are to be excluded from the change in tax rates then the elements of $tm01_{w,c,r}$ corresponding to the food commodities are (left as) zeros. Note how in this case it is necessary to both ‘free’ a variable and give values to a parameter for a solution to emerge. If the change in the applied tax rates is to be other than equiproportionate then values of $tm01_{w,c,r}$ other than one can be applied, e.g., if the changes in the import duties on food products are to be half those on other sectors then the elements of $tm01_{w,c,r}$ corresponding to the food commodities can be set to 0.5 and for the other commodities can be set to 1.

This combination of alternative adjustment methods covers the range of common tax rate adjustments used in the majority of applied applications while being flexible and easy to use. However experience has shown that when working with GTAP data it is very important to check the tax rates that are applied in the base solution; in some aggregations some of the applied tax rates only differ marginally from zero but are a mix of negatives and positives, in such cases it may be necessary to exogenously reset the tax rates to avoid apparently odd results. All the model parameters are exported to a GDX file and can be easily accessed using GDXViewer and/or SeeResults.¹⁵

¹⁵ See Section 6 on complementary programmes.

The other tax rate equations follow the same structure. Export taxes ($TE_{c,r}$) are defined in (T1.2), sales taxes ($TS_{c,r}$) are defined in (T1.3), production tax rates ($TX_{a,r}$) are defined in (T1.4), factor income tax rates ($TYF_{f,r}$) are defined in (T1.5), household income rates ($TYH_{h,r}$) are defined in (T1.6) and factor use tax rates ($TF_{f,r}$) are defined in (T1.7.)

Government Tax Revenues Equations

$$MTAX_r = \sum_w \sum_c (TM_{w,c,r} * PWM_{w,c,r} * ER_r * QMR_{w,c,r}) \quad \forall r \in rgn \quad (T2.1)$$

$$ETAX_r = \sum_w \sum_c (TE_{c,w,r} * PWE_{c,w,r} * ER_r * QER_{c,w,r}) \quad \forall r \in rgn \quad (T2.2)$$

$$STAX_r = \sum_c \left(TS_{c,r} * PQS_{c,r} * \left(QINTD_{c,r} + \sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} \right) \right) \quad \forall r \in rgn \quad (T2.3)$$

$$ITAX_r = \sum_a (TX_{a,r} * PX_{a,r} * QX_{a,r}) \quad \forall r \in rgn \quad (T2.4)$$

$$FYTAX_r = \sum_f (TYF_{f,r} * (YF_{f,r} - (deprec_{f,r} * YF_{f,r}))) \quad \forall r \in rgn \quad (T2.5)$$

$$HTAX_r = \sum_h (TYH_{h,r} * YH_{h,r}) \quad \forall r \in rgn \quad (T2.6)$$

$$FTAX_r = \sum_f \sum_a (TF_{f,a,r} * WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r}) \quad \forall r \in rgn \quad (T2.7)$$

The government tax revenue equations simply sum the revenues from each different tax instrument for each region. These are not strictly necessary for the model since the equations could be collapsed into a single government income equation (see G1 below). However it is useful to carry around these additional variables since they can then be used in model closures to accommodate specific government tax revenue objectives and they are useful sources of information when analysing simulation results.¹⁶ Import duty revenues ($MTAX_r$) are defined

¹⁶ It is a simple matter to generate such aggregates from model results as part of the reported output of the simulations. The trade off between the increase in model ‘size’ associated with extra variables and reduction in the size of the files that generate results is a matter of judgement; the general approach used

as total import duty revenue in region r (T2.1); Export tax revenues ($ETAX_r$) are defined as total export tax revenue in region r (T2.2); sales tax revenues ($STAX_r$) are defined as total sales tax revenue in region r (T2.3); production tax revenues ($ITAX_r$) are defined as total production tax revenue in region r (T2.4); factor income tax revenues ($FYTAX_r$) are defined as total factor income tax revenue in region r (T2.5); household income tax revenues ($HTAX_r$) are defined as total household income tax revenue in region r (T2.6) and factor use tax revenues ($FTAX_r$) are defined as total factor use tax revenue in region r (T2.7).

4.2.9 Government Block Equations

Government income (YG_r) is defined as the sum of government tax revenues (G1), where the tax revenues are treated as expenditures by the accounts paying the taxes and hence are defined in the tax block. While this approach adds equations it has the arguable advantage of being more transparent and easier to modify.

Other Government Equations

$$YG_r = MTAX_r + ETAX_r + STAX_r + ITAX_r + FYTAX_r + HTAX_r + FTAX_r \quad \forall r \in rgn \quad (G1)$$

$$QGD_{c,r} = qgdconst_{c,r} * QGDADJ_r \quad \forall r \in rgn \quad (G2)$$

$$EG_r = \sum_c PQD_{c,r} * QGD_{c,r} \quad \forall r \in rgn \quad (G3)$$

Government demand for commodities (G2) is assumed fixed in real terms, i.e., the volume is fixed, but can be scaled or allowed to vary using an adjustment factor ($QGDADJ_r$). The precise specification depends upon the choice of closure rule (see below). Thereafter Government consumption expenditure (EG_r) is defined as the sum of commodity consumption (G3). The advantage of separately expressing the volume and value of government expenditures is the increased flexibility it provides in the choice of closure rules for the government account. This arrangement allows adjustment of government demand either through the volume, or the expenditure or the value share of final demand (see below).

here is to include as variable such aggregates that may have substantive benefits in terms of transparency and simulation formulation.

4.2.10 Kapital Account Block Equations

Income to the capital (savings and investment) account, total savings, comes from household savings, depreciation allowances, government savings ($KAPGOV_r$) and the surplus on the capital account of the balance of payments ($KAPWOR_r$), and hence is simply defined as the sum of savings by domestic and ‘foreign’ agents (K1.2).

In this model the household savings rates are declared as variables ($SHH_{h,r}$) that define the proportions of income saved after the payment of income taxes. The savings rate equations (K1.1) used the same adjustment structure as used for the tax rate equations; hence $shh0_{h,r}$ are the base solution savings rates $dabshh_{h,r}$ are absolute changes in the base savings rates, $SADJ_r$ are multiplicative adjustment factors, $DSHH_r$ are additive adjustment factors and $shh01_{h,r}$ is a matrix of zero and non-zero values that determine for which households and regions the savings rates can adjust additively. Note that household savings are taken out of after-tax income.

Savings Block Equations

$$SHH_{h,r} = \left((shh0_{h,r} + dabshh_{h,r}) * SADJ_r \right) + (DSHH_r * shh01_{h,r}) \quad \forall r \in rgn \quad (K1.1)$$

$$\begin{aligned} TOTSAV_r = & \left(\sum_h (YH_{h,r} * (1 - TYH_{h,r})) * (SHH_{h,r}) \right) \\ & + \sum_f deprec_{f,r} * YF_{f,r} \\ & + KAPGOV_r + (KAPWOR_r * ER_r) \quad \forall r \in rgn \end{aligned} \quad (K1.2)$$

Government savings are calculated as residual (see the $KAPGOV_r$ equations, MC3.1, below). The surplus on the capital account ($KAPWOR_r$) is defined in terms of the foreign currency (see MC3.4 and MC3.3) and therefore the exchange rate appears in this equation (this is a matter of preference).

Investment demand is modeled in a similar way to government demand. Demand for commodities (K2.1) used in investment is assumed to be in fixed volumes ($qinvdconst_{c,r}$) multiplied by an investment-scaling variable ($IADJ_r$) that can accommodate changes in the exogenously determined level of investment and/or changes in the availability of funds for

investment. The second stage (K2.2) captures the price effect by identifying the total value of investment ($INVEST_r$). This arrangement allows adjustment of investment demand either through the volume, or the expenditure or the value share of final demand (see below).

Investment Block Equations

$$QINVD_{c,r} = IADJ_r * qinvdconst_{c,r} \quad \forall r \in rgn \quad (K2.1)$$

$$INVEST_r = \sum_c (PQD_{c,r} * QINVD_{c,r}) \quad \forall r \in rgn \quad (K2.2)$$

4.2.11 Market Clearing Block Equations

In the base solution to the model the market clearing, or equilibrium, conditions are relatively straightforward. Factor supplies must equal factor demands (MC1) and (composite) commodity supplies must equal (composite) commodity demands (MC2). It appears that there is no equilibrium condition for the supply of domestic output to the domestic market. In fact this is achieved through the commodity output equation (P1.9), which could have been treated as a market clearing equation.

Factor Account Equations

$$FS_{f,r} = \sum_a FD_{f,a,r} \quad \forall r \in rgn \quad (MC1.1)$$

Commodity Account Equations

$$QQ_{c,r} = QINTD_{c,r} + \sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} \quad (MC2.1)$$

$$\forall c \in (cd \cup cm), r \in rgn$$

The government account is cleared by defining government savings ($KAPGOV_r$) as the difference between government income and government expenditure on consumption and

transfers; hence government savings are explicitly treated as a residual, (MC3.1). The deficit/surplus on the current account is computed in two-stages. First the bilateral trade balances ($KAPREG_{w,r}$) are calculated as the difference in the values of imports and exports, for first the trade flows between regions other than GILD (MC3.2) and then for trade between ANARRES and all other regions (MC3.3) – the latter being the trade balances on margins trade. These transactions are valued in terms of the global numéraire. Then the overall balance of trade ($KAPWOR_r$) is computed for each region, (MC3.4). Finally, the commodity trade balance for GILD is defined (MC3.5), since this should be zero by definition a slack variable ($GILDSLACK$) is attached.

Investment and Savings Account Equations

$$KAPGOV_r = YG_r - EG_r \quad \forall r \in rgn \quad (MC3.1)$$

$$KAPREG_{w,r} = \left(\sum_c PWMFOB_{w,c,r} * QMR_{w,c,r} \right) - \left(\sum_c PWE_{c,w,r} * QER_{c,w,r} \right) \quad (MC3.2)$$

$\forall w \in wgn$

$$KAPREG_{w,r} = \left(\sum_c \sum_w PT_{c,r} * QT_{w,c,r} \right) - \left(\sum_c PWE_{c,w,r} * QER_{c,w,r} \right) \quad (MC3.3)$$

$\forall w \notin wgn$

$$KAPWOR_r = \sum_w KAPREG_{w,r} \quad \forall r \quad (MC3.4)$$

$$\sum_w (PWM_{w,c,r} * QMR_{w,c,r}) = \sum_w (PWE_{c,w,r} * QER_{c,w,r}) + GLOBESLACK \quad (MC3.5)$$

$\forall c \in ct2, r \notin rgn$

The commodity trade accounts define the market clearing conditions for bilateral trade. The *fob* prices for imports ($PWMFOB_{w,c,r}$) for all imports by destination and source must be equal to the *fob* prices for exports ($PWE_{c,w,r}$) by source and destination (MC4.1). In addition the quantities of imports ($QMR_{w,c,r}$) for all imports by destination and source must be equal to the quantities of exports ($QER_{c,w,r}$) by source and destination (MC4.2). These equations are not completely straightforward since it is necessary in their implementation to employ mappings between exporting and importing regions that require the ‘switching’ of labels on accounts within the equation.

Commodity Trade Account Equations

$$PWMFOB_{w,c,r} = PWE_{c,w,r} \quad \forall c \in cmr, map_r_w, map_w_r \quad (MC4.1)$$

$$QMR_{w,c,r} = QER_{c,w,r} \quad \forall c \in cmr, map_r_w, map_w_r \quad (MC4.2)$$

The trade consistency equations do not however deal with the requirements for market clearing with respect to the trade transactions undertaken by the GILD region. However similar conditions apply for the margins trade. These require that the total demand for each and every trade and transport service ($QT_{w,c,r}$) is exactly equal to the exports of that service by GILD ($QER_{c,w,"glo"}$) and the import prices of margin services ($PT_{c,r}$) must equal the export prices (fob) from GILD; these conditions are imposed by (MC5.1) and (MC5.2) respectively.

Margin Trade Account Equations

$$\sum_w QT_{w,c,r} = \sum_w QER_{c,w,"glo"} \quad \forall c \in ct2, r \in rgn \quad (MC5.1)$$

$$PT_{c,r} = PWE_{c,w,"glo"} \quad \forall c \in ct2, r \in rgn \quad (MC5.2)$$

The absorption closure equations exist to support closure rules choices relating to the distribution of final demand between the institutions within regions. The value of final demand by region ($VFDOMD_r$) is simply the sum of the value of final demand across the three sources of final demand – households, government and investment (MC6.1). The value share of final demand accounted for by investment ($INVESTSH_r$) is defined in (MC6.2), and the value share of final demand accounted for by government ($VGDSH_r$) is defined in (MC6.3). No similar expression exists for the households because household expenditure are defined after taxes and savings and it is difficult to justify setting the household shares relative to the value of total final demand since this would nullify the distributional consequences of a policy shock. If the expenditure shares by investment and government are fixed then the expenditure share of ALL households will by definition be fixed and $HEXP_r$ could be fixed for individual households if there was good reason.

Absorption Closure Equations

$$VFDOMD_r = \sum_c \left(PQD_{c,r} * \left(\sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} \right) \right) \quad (MC6.1)$$

$$\forall r \in rgn$$

$$INVESTSH_r * VFDOMD_r = INVEST_r \quad \forall r \in rgn \quad (MC6.2)$$

$$VGDSH_r * VFDOMD_r = \sum_c (PQD_{c,r} * QGD_{c,r}) \quad \forall r \in rgn \quad (MC6.3)$$

The final equations are for slacks. The satisfaction of Walras's law requires that if all other accounts balance then so must the final account; thus the difference between savings and investment for all regions that save should always be zero and hence the variable $WALRAS_r$ should equal zero in (MC7.1). That this variable equals zero is a good check on the correct specification of the model. Finally the global trade balance must by definition be zero and hence so must $KAPWORSYS$ (MC7.2)

Slack Equations

$$TOTSAV_r = INVEST_r + WALRAS_r \quad \forall r \in rgn \quad (MC7.1)$$

$$KAPWORSYS = \sum_r KAPWOR_r \quad \forall r \in rgn \quad (MC6.2)$$

5. Model Closure Conditions or Rules¹⁷

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However, the economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The

¹⁷ The term model 'closure' is used generically here to cover all permutations including variations in market clearing conditions and macroeconomic closure conditions.

closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of some of the options available with this formulation of the model by reference to the accounts to which the rules refer. However, as will become readily apparent there are many permutations available, and hence this discussion deals with the general principles rather than trying to define all possible permutations. The philosophy adopted in the implementation of this model is to define a (minimal) base closure for the replication of the base case and then impose closure changes within a loop in the experiment/simulation file. This approach allows substantial flexibility in the definition of policy simulations while simultaneously allowing sensitivity testing of the chosen model closures. For the base solution the choice of model closure is basically a neoclassical closure; the reason for choosing this closure is NOT a statement about its appropriateness, rather it is a closure that allows flexibility in model solution and hence simplifies the process of verifying that the model is correctly specified and calibrated.

5.1 Foreign Exchange Account Closure

For the world numéraire the exchange rate index for the reference regions (*EPRI*) is fixed (C1a), although a parameter, *numerchk*, is attached to allow for ease of checking the homogeneity of the model.¹⁸ At the same time the exchange rate for GILD is fixed as equal to the world numéraire (C1b).

The default closure for the non- GILD regions involves an assumption that the nominal exchange rate is flexible. Hence the trade balances ($KAPWOR_{rgn}$) for all non ANARRES regions are fixed (C1c) and the exchange rate is flexible. The most straightforward alternative is to fix the exchange rates for all non ANARRES regions (C1d) and unfix the trade balances.

¹⁸ In the base and model simulations, *numerchk* equals one. It is a parameter attached to the world numéraire and the regional numéraire (see C5a and C5b). To check for homogeneity, a value other than one should have no effect on real variables in the model.

However there are clearly a range of permutations whereby the exchange rates for some regions are flexible while for others they are fixed.¹⁹

Foreign Exchange Account Closure

$$EPRI = \overline{EPRI0} * numerchk \quad (C1a)$$

$$ER_{glo} = \overline{EPRI0} * numerchk \quad (C1b)$$

$$KAPWOR_{rgn} = \overline{KAPWOR_{rgn}} \quad (C1c)$$

$$ER_{rgn} = \overline{ER_{rgn}} \quad (C1d)$$

5.2 Capital Account Closure

This discussion of the closure rules for the capital account begins by abstracting from some of the complexities into interactions between different closure conditions. To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. The default closure for this account requires the assumption that savings rates are exogenously determined which involves fixing both the multiplicative savings rates adjusters ($SADJ_r$) (C2a) and the additive savings rates adjusters ($DSHH_r$) (C2b) and allowing the savings rates to be fixed – typically at their base rates. With such a savings-driven closure the value and volume of investment must be free to vary.

Choosing an investment-driven closure requires making two distinct choices; first, is the level investment to be determined in terms of volume, value or expenditure share of final demand, and second, will savings adjust in a multiplicative or additive mode. Clearly, again, it is possible that the choices will vary by region. If the investment volume adjuster ($IADJ_r$) is fixed (C2c), then the value of investment expenditure might change due to changes in the prices of investment commodities ($PQD_{c,r}$). If the value of investment ($INVEST_r$) is fixed (C2d) then both the value of savings must be free to adjust as must the actual volume of investment. The same applies if the shares of investment expenditures in final demand ($INVESTSH_r$) are fixed (C2e).

¹⁹ The option is provided in the programme to control choice of these, and several other, closures by selection of regions to assign to a set and its complement.

Capital Account Closure

$$SADJ_r = \overline{SADJ_r} \quad (C2a)$$

$$DSHH_r = \overline{DSHH_r} \quad (C2b)$$

$$IADJ_r = \overline{IADJ_r} \quad (C2c)$$

$$INVEST_r = \overline{INVEST_r} \quad (C2d)$$

$$INVESTSH_r = \overline{INVESTSH_r} \quad (C2e)$$

Fixing savings, and thus deeming the economy to be savings-driven, could be termed a Neo-Classical approach. Closing the economy by fixing investment however makes the model reflect a Keynesian investment-driven assumption for the operation of an economy.

However there are potentially important interaction effects. Note that there are other sources of potential savings for region – the government and the trade balances. The magnitudes of these other savings sources can also be controlled through the closure rules (see below). Consequently there will clearly be an important interdependence between the choices of closure rules for different accounts; the most obvious one to be aware of is the interaction between household savings rates and household income tax rates when tax rates are made flexible and the level of government savings/deficit is fixed.

5.3 Government Account Closure

The closure rules for the government account are slightly more complex because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income, i.e., tax rates, remittances etc., and expenditure are ‘fixed’, government savings must be free to adjust.

Thus in the base specification all the tax rates are fixed by declaring the tax rates as parameters and then fixing all the tax rate scaling factors (C3.1a – C3.1n). Consequently any changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (T2.1 to T2.6). Controlling the volumes of

commodity demand ($QGD_{c,r}$) in the base specification government expenditure is then achieved by fixing the volumes of commodity demand ($QGD_{c,r}$) through the government demand adjuster ($QGDADJ_r$) (T3.2a). Notice however that since prices are typically free to change it is necessary to ensure that both government expenditure (EG_r) in (C3.2b) and the share of government expenditure in final demand ($VGDSH_r$) in (C3.2c) must be flexible.

Tax Rate and Revenue Closure

$$TMADJ_r = \overline{TMADJ}_r \quad (C3.1a)$$

$$TEADJ_r = \overline{TEADJ}_r \quad (C3.1b)$$

$$TSADJ_r = \overline{TSADJ}_r \quad (C3.1c)$$

$$TXADJ_r = \overline{TXADJ}_r \quad (C3.1d)$$

$$TYFADJ_r = \overline{TYFADJ}_r \quad (C3.1e)$$

$$TYHADJ_r = \overline{TYHADJ}_r \quad (C3.1f)$$

$$TFADJ_r = \overline{TFADJ}_r \quad (C3.1g)$$

$$DTM_r = \overline{DTM}_r \quad (C3.1h)$$

$$DTE_r = \overline{DTE}_r \quad (C3.1i)$$

$$DTS_r = \overline{DTS}_r \quad (C3.1j)$$

$$DTX_r = \overline{DTX}_r \quad (C3.1k)$$

$$DTYF_r = \overline{DTYF}_r \quad (C3.1l)$$

$$DTYH_r = \overline{DTYH}_r \quad (C3.1m)$$

$$DTF_r = \overline{DTF}_r \quad (C3.1n)$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not directly control. Hence the equilibrating condition is that government savings, the internal balance ($KAPGOV_r$), is not fixed in (C3.2d).

The number of possible permutations for closing the government account for each region is consequently substantial. Practical experience indicates that great care is needed when adjusting the government closure rules to avoid both unbalancing the model and imposing closure rules that are contradictory.²⁰ A common change is to require that the government's savings/deficit is fixed; this requires fixing $KAPGOV_r$ in C3.2d, in which case either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

Government Expenditure and Savings Closure

$$QGDADJ_r = \overline{QGDADJ_r} \quad (C3.2a)$$

$$EG_r = \overline{EG_r} \quad (C3.2b)$$

$$VGDSH_r = \overline{VGDSH_r} \quad (C3.2c)$$

$$KAPGOV_r = \overline{KAPGOV_r} \quad (C3.2d)$$

In its simplest form this can be achieved by allowing one of the previously fixed tax rate adjusters (C3.1a to C3.1n) to vary. Thus if the sales tax adjuster ($TSADJ_r$) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables $TMADJ_r$ and $KAPGOV_r$ would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the import duties mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates it is necessary to change the

²⁰ The most common problems with contradictory government closure rules relate to the interactions between the government and capital (investment) accounts, in particular when a combination of flexible savings and tax rates producing mutually contradictory effects.

relevant tax parameters. Such changes would be implemented in policy experiment files by changing the values of the parameters that determine relative tax rates.

Note also that as with the investment account, there is a needed care over setting the constraints on government demand. If the government demand volume adjuster ($QGDADJ_r$) is fixed (C3.2a), then the value of government expenditure might change due to changes in the prices of commodities ($PQD_{c,r}$). If the value of government expenditure (EG_r) is fixed (C3.2b) then both government savings must be free to adjust as must the actual volume of expenditure, when tax rates are exogenous. The same applies if the shares of government expenditures in final demand ($VGDSH_r$) are fixed (C3.2e).

5.4 Technology and Efficiency

The default assumption in the model is that efficiency, commonly called technology, is fixed. Hence the shift parameters are assumed to be constant.

Technology Parameter Closure

$$ADX_{a,r} = \overline{ADX0}_{a,r} \quad (C4.3a)$$

$$ADVA_{a,r} = \overline{ADVA0}_{a,r} \quad (C4.4a)$$

$$ADFD_{f,a,r} = \overline{ADFD0}_{f,a,r} \quad (C4.5a)$$

Changes in efficiency can either be imposed exogenously or a technology variable can be freed up so as to satisfy some other condition, e.g., the required efficiency gain needed to satisfy a predetermined increase in GDP.

5.6 Numéraire

The model specification allows for a choice between two price normalisation equations, the consumer price index (CPI_r) and a producer price index (PPI_r). A *numéraire* is needed for each region, to serve as a base since the model is homogenous of degree zero in prices for each region, and hence only defines relative prices.

Numéraire Closure

$$CPI_r = \overline{CPI_r} * numerchk \quad (C5a)$$

$$PPI_r = \overline{PPI_r} * numerchk \quad (C5b)$$

5.7 Factor Market Closure

The factor market closure rules are more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

5.7.1 Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply of and total demand for factors equate, and since the total supplies of each factor are determined exogenously this condition is satisfied by the equilibrium condition (C6.1a). The demands for factor f by activity a in region r and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages ($WF_{f,r}$), by allowing for the possibility that the payments to notionally identical factors might vary across activities, due to activity specific ‘efficiency’ factors ($WFDIST_{f,a,r}$) through the variable that captures the ‘sectoral proportions for factor prices’. Since these proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, they are therefore assumed fixed (C6.1b). And finally bounds are placed upon the average factor prices, (C6.1c), so that meaningful results are produced.

Full Employment and Mobile Factors

$$FS_{f,r} = \overline{FS}_{f,r} \quad (C6.1a)$$

$$WFDIST_{f,a,r} = \overline{WFDIST}_{f,a,r} . \quad (C6.1b)$$

$$\begin{aligned} \text{Min } WF_{f,r} &= 0 \\ \text{Max } WF_{f,r} &= +\text{infinity} \end{aligned} \quad (C6.1c)$$

5.7.2 Factor Immobility and/or Unemployment Closures

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as fixed. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations (C6.2a) where *fact* indicates the specific factor and *activ* a specific activity. This block of equations includes all the variables that were declared for the model with reference to factors plus extra equations for $WFDIST_{fact,activ,r}$, i.e., $WFDIST_{fact,activ,r} = \overline{WFDIST}_{fact,activ,r}$, whose role will be defined below. The choice of which equations are binding and which are not imposed will determine the factor market closure conditions.

Factor Unemployment Closure

$$\begin{aligned}
FS_{fact,r} &= \overline{FS}_{fact,r} \\
WFDIST_{fact,a,r} &= \overline{WFDIST}_{fact,a,r} \\
\text{Min } WF_{fact,r} &= 0 \\
\text{Max } WF_{fact,r} &= +\text{infinity} \\
FD_{fact,a,r} &= \overline{FD}_{fact,a,r} \\
WF_{fact,r} &= \overline{WF}_{fact,r} \\
WFDIST_{fact,activ,r} &= \overline{WFDIST}_{fact,activ,r} \\
\text{Min } FS_{fact,r} &= 0 \\
\text{Max } FS_{fact,r} &= +\text{infinity}
\end{aligned}
\tag{C6.2a}$$

As can be seen the first four equations in the block (C6.2a) are the same as those in the ‘Full Factor Mobility and Employment Closure’; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the ‘Full Factor Mobility and Employment Closure’. Assume that this set of conditions represents the starting points, i.e., the first four equations are binding and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, thus (C6.3a) must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., (C6.3b) must now be relaxed.

Activity Specific and Immobile Factor

$$FD_{fact,a,r} = \overline{FD}_{fact,a,r} \quad (C6.3a)$$

$$WFDIST_{fact,a,r} = \overline{WFDIST}_{fact,a,r} \quad (C6.3b)$$

$$FS_{fact,r} = \overline{FS}_{fact,r} \quad (C6.3c)$$

$$WFDIST_{fact,activ,r} = \overline{WFDIST}_{fact,activ,r} \quad (C6.3d)$$

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition (C6.3a) is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e., (C6.3d), which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.²¹

Start again from the closure conditions for full factor mobility and employment and then assume that there is unemployment of one or more factors in the economy; typically this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing a condition that fixes the wage rate, (C6.4a) and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing (C6.4b).

It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions (C6.4c) can be imposed.²²

²¹ It can be important to ensure a sensible choice of reference activity. In particular this is important if a factor is not used, or little used, by the chosen activity.

²² If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

Unemployment of a Specific Factor

$$WF_{fact,r} = \overline{WF}_{fact,r} \quad (C6.4a)$$

$$FS_{fact,r} = \overline{FS}_{fact,r} \quad (C6.4b)$$

$$\begin{aligned} \text{Min } FS_{fact,r} &= 0 \\ \text{Max } FS_{fact,r} &= +\text{infinity} \end{aligned} \quad (C6.4c)$$

5.7.3 Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

If all factors used by an activity are fixed, this requires imposing the conditions C6.5a, where *activ* refers to the activity of concern. But the returns to these factors in these activities must now be allowed to vary, i.e., the conditions (C6.5b) must now be relaxed. In this case the condition fixing the total supply of the factor is not redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Fixed Factor Use

$$FD_{f,activ,r} = \overline{FD}_{f,activ,r} \quad (C6.5a)$$

$$WFDIST_{f,activ,r} = \overline{WFDIST}_{f,activ,r} \quad (C6.5b)$$

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to manage the model by gathering together factor market conditions that are inspired by activity characteristics after the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supersedes previous conditions.

6. Implementation and Complementary Programmes

The current version of the model is implemented as a mixed complementarity problem (MCP) in GAMS 250 21.5 using the Path solver. In a model with 15 sectors, 4 factors and 14 regions, which generates some 24,000 equations and variables, the initial, or replication, solve takes some 15 seconds on a well-equipped Pentium 4 2.6MHz PC, while the entire run for a series of 11 simple experiments run in a loop takes about 75 seconds, including writing out a comprehensive set of results to a series of GDX files.

However once a programme is established the time consuming activities are those associated with the generation of new aggregations and the analyses of results. In order to address these problems as pair of Excel based programmes, using VBA, and a series of GAMS programmes have been developed.

6.1 Database Aggregation

The Excel based programme for aggregating the database is called SAMGator (PROVIDE, 2004b). This is general aggregation programme that uses Excel to provide a front end that simplifies the creation of the set files required for the mapping aggregations. The user declares, with descriptors, the accounts required in the new aggregation and identifies those accounts in the database that will be aggregated to form the new accounts; this is all done

using point and click commands in an Excel worksheet. Once all the new accounts have been declared and assigned, and provided all the data in the original database are accounted for, a command triggers the creation of the mapping files. Finally the user triggers the running of the aggregation programme from within Excel. At this point SAMGator generates a GAMS programme file, *.gms, runs that programme and writes out the results to GDX and/or Excel.

6.2 Analyses of Results

The management and organisation of results from large CGE models can be both time consuming and confusing due to the large number of data points generated, e.g., the model with 15 sectors, 4 factors and 14 regions when run with 11 experiments generates more than 500,000 data points even when only running a limited set of analyses programmes. An Excel based programme for analysing, organising and summarising results from CGE models called SeeResults (PROVIDE, 2004a) has been developed for the interactive reading of GAMS model results that have been stored in GDX format.

The GDX format is technically superlative, with GDX files being both extremely fast and small. Once the result data are in GDX format, they can be viewed using GAMSIDE or GDXViewer, a specialised utility for viewing GDX files and both of these include basic facilities for changing data orientation and the GDXViewer also includes basic charting (plot) facilities. While these are useful, there are a number of limitations. Filtering ability is limited, which can make it difficult to compare results within a smaller subset of elements from a large set. Charting facilities in GDXViewer are limited and not designed for presentation quality output. It is not possible to manipulate data, change formatting, perform calculations or cut and paste data to other applications, although there are data output facilities. In short, these facilities are suitable for viewing raw data initially but are not a complete solution for the processing of model results.

Among proprietary programmes Excel is the logical choice; it has reasonably good charting facilities, is excellent for general data manipulation and provides the means to format data for presentation purposes. It can also provide limited database functionality such as reorientation and filtering via the pivot table feature. It is also useful as a means to distribute data, because it has a (very) wide user base. But it is often tedious getting large amounts of data into Excel and can produce extremely large files that are difficult to transfer. SeeResults automates much of this process; it avoids overly large Excel files by working interactively

with GDX files, that remain the files in which data are stored, and coordinates information that assists with the categorisation, classification and labeling of the results.

The guiding principle behind SeeResults is to make use of the functionality of GDX and Excel; thereby avoiding the need to develop additional standalone programme. Excel is conceptually a very powerful programming environment, allowing use of the spreadsheet itself as a data construct and combining the power of Visual Basic with the ability to use spreadsheet functions in Visual Basic code. A major usage of Excel functionality is the use of Pivot Tables for data reorientation and filtering in multidimensional data. Excel's pivot tables are ideally suited to the task of storing and presenting reasonable amounts of data and allowing the display format to be changed after data has been imported to Excel. It performs SeeResults technically demanding data handling tasks more efficiently than would have been possible using custom Visual Basic code.

7. Concluding Comments

This paper has described the second version of a new global CGE model. The model described here contains relatively simple behavioural relationships although it requires a programme that contains a number of technically sophisticated features. Moreover it is capable of being implemented with any aggregation of the GTAP database. This version of the model will be made generally available after further testing.

Appendices

A1. Alternative formulations for CET and Armington Functions

In addition to the primal form of the Armington/CES and CET functions reported above it is possible to express these functions in share form and with prices as the dependent variable. The choice of formulation can affect model performance due to the scaling effects, although the interpretation remains unchanged.

A1.1 CET Functions

Share Form of CET Function

$$QXC_{c,r} = QXC0_{c,r} * \left(\theta_{c,r}^e * \left(\frac{QE_{c,r}}{QE0_{c,r}} \right)^{\rho_{c,r}^t} + (1 - \theta_{c,r}^e) * \left(\frac{QD_{c,r}}{QD0_{c,r}} \right)^{\rho_{c,r}^t} \right)^{\frac{1}{\rho_{c,r}^t}} \quad (A1.1a)$$

$$\forall c \in (cd \cap ce), r \in rgn, c \notin cetn1$$

$$QE_{c,r} = QD_{c,r} \left[\frac{PE_{c,r}}{PD_{c,r}} * \frac{(1 - \theta_{c,r}^e)}{\theta_{c,r}^e} * \left(\frac{QE0_{c,r}}{QD0_{c,r}} \right)^{\rho_{c,r}^t} \right]^{\frac{-1}{(-\rho_{c,r}^t + 1)}} \quad (A1.1b)$$

$$\forall c \in (cd \cap ce), r \in rgn, c \notin cetn1$$

$$PER_{c,wm,r} = PE_{c,r} * \theta_{c,wm,r}^{ewm} * \left(\frac{QER_{c,wm,r}}{QE_{c,r}} \right)^{(\rho_{c,r}^e - 1)} * \left(\frac{QER0_{c,wm,r}}{QE0_{c,r}} \right)^{(-\rho_{c,r}^e)} \quad (A1.1c)$$

$$\forall c \in cer2, r \in rgn, c \notin cetn2$$

$$PER_{c,w,r} = PER_{c,wm,r} * \theta_{c,w,wm,r}^{ew} * \left(\frac{QER_{c,w,r}}{QER_{c,wm,r}} \right)^{(\rho_{c,wm,r}^{2e} - 1)} * \left(\frac{QER0_{c,w,r}}{QER0_{c,wm,r}} \right)^{(-\rho_{c,wm,r}^{2e})} \quad (A1.1d)$$

$$\forall c \in cer, r \in rgn, c \notin cetn2, map_wm_w$$

Primal Form of CET Function with Prices as Dependent Variables

$$PER_{c,wm,r} = PE_{c,r} * \left(\frac{QER_{c,wm,r}}{QE_{c,r}} \right)^{(\rho_{c,r}^e - 1)} * (\gamma_{c,wm,r}^r * at_{c,r}^r)^{\rho_{c,r}^e} \quad (A1.2a)$$

$$\forall c \in cer2, r \notin rgn, c \notin cetn2$$

$$PER_{c,w,r} = PER_{c,wm,r} * \left(\frac{QER_{c,w,r}}{QER_{c,wm,r}} \right)^{(\rho_{c,wm,r}^{2e} - 1)} * (\gamma_{c,w,r}^r * at_{c,wm,r}^{2r})^{\rho_{c,wm,r}^{2e}} \quad (A1.2b)$$

$$\forall c \in cer2, r \notin rgn, c \notin cetn2, map_wm_w$$

A1.2 Armington Functions

Share Form of Armington Function

$$QQ_{c,r} = QQ0_{c,r} * \left(\theta_{c,r}^m * \left(\frac{QM_{c,r}}{QM0_{c,r}} \right)^{-\rho_{c,r}^c} + (1 - \theta_{c,r}^m) * \left(\frac{QD_{c,r}}{QD0_{c,r}} \right)^{-\rho_{c,r}^c} \right)^{-1/\rho_{c,r}^m} \quad (A2.1a)$$

$$\forall c \in (cx \cap cm), r \in rgn$$

$$QM_{c,r} = QD_{c,r} \left[\frac{PM_{c,r}}{PD_{c,r}} * \frac{(1 - \theta_{c,r}^m)}{\theta_{c,r}^m} * \left(\frac{QM0_{c,r}}{QD0_{c,r}} \right)^{-\rho_{c,r}^m} \right]^{\frac{-1}{(\rho_{c,r}^m - 1)}} \quad (A2.1b)$$

$$\forall c \in (cx \cap cm), r \in rgn$$

$$PMR_{wm,c,r} = PM_{c,r} * \theta_{wm,c,r}^{mwm} * \left(\frac{QMR_{wm,c,r}}{QM_{c,r}} \right)^{-(\rho_{c,r}^m - 1)} * \left(\frac{QMR0_{wm,c,r}}{QM0_{c,r}} \right)^{\rho_{c,r}^m} \quad (A2.1c)$$

$$\forall c \in cmr2$$

$$PMR_{w,c,r} = PM_{c,r} * \theta_{w,c,wm,r}^{mw} * \left(\frac{QMR_{w,c,r}}{QMR_{wm,c,r}} \right)^{-(\rho_{wm,c,r}^{2m} - 1)} * \left(\frac{QMR0_{w,c,r}}{QMR0_{wm,c,r}} \right)^{\rho_{wm,c,r}^{2m}} \quad (A2.1d)$$

$$\forall c \in cmr2, map_wm_w$$

Primal Form of Armington Function with Prices as Dependent Variables

$$PMR_{wm,c,r} = PM_{c,r} * \left(\frac{QMR_{wm,c,r}}{QM_{c,r}} \right)^{-(\rho_{c,r}^m - 1)} * (\delta_{wm,c,r}^r * ac_{c,r}^r)^{-\rho_{c,r}^m} \quad (A2.2a)$$

$$\forall c \in cmr2$$

$$PMR_{w,c,r} = PMR_{wm,c,r} * \left(\frac{QMR_{w,c,r}}{QMR_{wm,c,r}} \right)^{-(\rho_{wm,c,r}^{2m} - 1)} * (\delta_{w,c,r}^r * ac_{wm,c,r}^{2r})^{-\rho_{wm,c,r}^{2m}} \quad (A1.2b)$$

$$\forall c \in cmr2, map_wm_w$$

A2. Notes on Nested CES/CET functions

A2.1 Nested CES Functions

At level one, there is a CES function over the domestic variety (QD) and aggregate imports (QM).

$$QQ = ac \left[\delta * QM^{-\rho_c} + (1 - \delta) * QD^{-\rho_c} \right]^{-1/\rho_c}$$

The standard reduced form of the first order condition (FOC) for an optimal combination is

$$\frac{MU_{QD}}{MU_{QM}} \frac{\partial QQ / \partial QD}{\partial QQ / \partial QM} = \frac{PD}{PM}$$

which on substitution produces

$$\frac{PD}{PM} = \frac{\delta}{(1 - \delta)} * \left\{ \frac{QM}{QD} \right\}^{(1 + \rho_c)} \quad (1B.3a)$$

or

$$\frac{QM}{QD} = \left[\frac{\delta}{(1 - \delta)} * \frac{PD}{PM} \right]^{\frac{1}{(1 + \rho_c)}} \quad (1B.3b)$$

Note how the choice of a linear homogenous functional form means that only relative input quantities and prices are defined.

Now solving for QM gives

$$QM = QD * \left[\frac{\delta}{(1 - \delta)} * \frac{PD}{PM} \right]^{\frac{1}{(1 + \rho_c)}} \quad (1B.4)$$

The constant in the CES function is AC, the coefficient is δ , and the exponent is ρ_c .

When there is a two level nest, the aggregate import, QM , is a CES function of imports from different regions, QMR_w ; imports from different regions have the same elasticity of substitution and the CES function is written as follows.

$$QM = acr * \left[\sum_{w=1}^W \delta R_w * QMR_w^{-\rho_M} \right]^{-1/\rho_M} \quad (A2.1)$$

This equation refers to each commodity and region, the commodity subscript, c , and the region subscript, r , are suppressed. In terms of notation, while the constant in the CES

function is ac at level one, it is acr at level two. Likewise, the coefficient is δ at level one and is δR at level two. There are imports from w regions, QMR_w . The exponent in the CES function, ρ_M , is derived from a constant elasticity of substitution across imports from all regions, i.e. across all QMR_w .

The first order conditions for the optimal choice of imports from each region are:

$$PMR_w = PM \cdot \frac{\partial QM}{\partial QMR_w} \quad (A2.2)$$

and

$$\frac{\partial QM}{\partial QMR_w} = QMR_w^{-\rho_M-1} * \delta R_w * acr \left[\sum_{w=1}^W \delta R_w * QMR_w^{-\rho_M} \right]^{\left(\frac{-1}{\rho_M}-1\right)}$$

Substituting (A2.1) into the expression yields,

$$\frac{\partial QM}{\partial QMR_w} = QMR_w^{-\rho_M-1} * \delta R_w * QM \left[\sum_{w=1}^W \delta R_w * QMR_w^{-\rho_M} \right]^{(-1)}$$

Note that (A2.1) can be re-written as:

$$\left(\frac{QM}{acr} \right)^{\rho_M} = \left[\sum_{w=1}^W \delta R_w * QMR_w^{-\rho_M} \right]^{(-1)}$$

and the first order condition can be written as:

$$PMR_w = PM * QMR_w^{-\rho_M-1} * \delta R_w * QM \left(\frac{QM}{acr} \right)^{\rho_M}$$

Rearranging yields:

$$\left(\frac{PMR_w}{PM} \right) = \left(\frac{QMR}{QM} \right)^{-\rho_M-1} \delta R_w * acr^{-\rho_M}$$

This expression can be re-arranged to have QMR_w on the left hand-side as follows:

$$QMR_w = QM \cdot \left[\frac{PMR_w * acr^{\rho_M}}{PM * \delta R_w} \right]^{-1/(\rho_M+1)} \quad (A2.3)$$

And there is a corresponding price definition:

$$PM \cdot QM = \sum_w PMR_w * QMR_w \quad (A2.4)$$

Suppose the regions can be grouped into aggregate categories (given the index w_m), such as developed and developing, and that we believe that imports from developed and

developing countries have different substitution elasticities. In the model, we represent this as an additional nest. Regions in the model, w , are assigned to the appropriate aggregate region, wm . Level two becomes a CES function over the aggregates, wm :

$$QM = ACR \cdot \left[\sum_{wm=1}^{WM} \delta R_{wm} \cdot QMR_{wm}^{-\rho_M} \right]^{-1/\rho_M} \quad (A2.5)$$

At level three, the imports from the regions wm , are a CES function over countries assigned to those regions:

$$QMR_{wm} = ACR2_{wm} \left[\sum_{w \subset wm} \delta R_w \cdot QMR_w^{-\rho_{Mwmr}} \right]^{-1/\rho_{Mwmr}} \quad (A2.6)$$

The variables QMR and PMR and the parameter δR refer to either the aggregate import region or the individual regions, depending on the index used.²³ As a result, some of the parameters in the CES equation for level three have an additional index to indicate which aggregate region they pertain to. In level three, the constant in the CES function, $ACR2$, and the exponent, $-\rho_M$, are indexed over the aggregate region. To indicate this in equation (A2.6) they have a subscript, wm . In CES of aggregate imports, the analogous parameters, ACR and $-\rho$, are not indexed by region (see equation A2.5).

The first order conditions for the optimal choice of imports from each aggregate region are:

$$PMR_{wm} = PM \cdot \frac{\partial QM}{\partial QMR_{wm}} \quad (A2.7)$$

and this can be rearranged as:

$$QMR_{wm} = QM \cdot \left[\frac{PMR_{wm} ACR^{\rho_M}}{PM \cdot \delta R_{wm}} \right]^{-1/(\rho_M+1)} \quad (A2.8)$$

Within each aggregate region, the optimal choice of imports from the countries in that aggregate region is given by:

$$PMR_w = PMR_{wm} \cdot \frac{\partial QMR_{wm}}{\partial QMR_w} \quad (A2.9)$$

and this can be rearranged as:

²³ In the GAMS code, the set ww contains all elements of the set w and the set wm . The set changes needed in GAMS are discussed in more detail later in this note.

$$QMR_w = QMR_{wm} \cdot \left[\frac{PMR_w ACR2_{wm}^{\rho_{Mwm}}}{PMR_{wm} \cdot \delta R_w} \right]^{-1/(\rho_{Mwm}+1)} \quad (A2.10)$$

Again, note that the constant in the CES function, $ACR2$, is indexed over wm , the aggregate region and the exponent, ρ_M , is indexed over wm .

The appropriate price definitions become:

$$PM \cdot QM = \sum_{wm} PMR_{wm} \cdot QMR_{wm} \quad (A2.11)$$

$$PMR_{wm} \cdot QMR_{wm} = \sum_{w \subset wm} PMR_w \cdot QMR_w \quad (A2.12)$$

There is no change to the price equation which links the domestic price of imports by region to the world price of imports by region, i.e. for the model with two level CES and for the model with a three level CES on the import side, the following equation holds:

$$PMR_w = PWM_w (1 + tm_w) \cdot ER \quad (A2.13)$$

Note that the region ANARRES imports margin services from all regions. Its import demand is specified as a CES function and it is included in the nested CES structure described above.

A2.2 Nested CET Functions

In a two level CET function, exports, QE , are a CET aggregation over exports by region, QER_w , as follows:

$$QE = atr * \left[\sum_{w=1}^W \gamma R_w * QER_w^{\rho_E} \right]^{1/\rho_E} \quad (A2.14)$$

The optimal choices of exports to each region are derived from first order conditions:

$$PER_w = PE * \frac{\partial QE}{\partial QER_w} \quad (A2.15)$$

This expression can be re-arranged to have QER_w on the left hand-side as follows:

$$QER_w = QE * \left[\frac{PER_w * atr^{-\rho_E}}{PE * \gamma R_w} \right]^{1/(\rho_E-1)} \quad (A2.16)$$

There is a corresponding price definition:

$$PE * QE = \sum_w PER_w * QER_w \quad (A2.17)$$

When there is a three level CET function, QE is a CET aggregation over aggregate regions, QER_{wm} , as follows:

$$QE = ATR \cdot \left[\sum_{wm=1}^{WM} \gamma R_{wm} \cdot QER_{wm}^{\rho_E} \right]^{1/\rho_E} \quad (A2.18)$$

Exports to the regions wm are a CET function over countries assigned to those regions:

$$QER_{wm} = ATR2_{wm} \left[\sum_{w \subset wm} \gamma R_w \cdot QER_w^{\rho_{Ewmr}} \right]^{1/\rho_{Ewmr}} \quad (A2.19)$$

The variables QER (exports to regions) and PER (the price of exports to regions) and the parameter $gammar$, refer to the aggregate imports from the set wm , as well as the imports from the individual regions, w , that make-up the aggregates. To calibrate the CET for level 2, QER_{wm} , PER_{wm} , and $gammar_{wm}$ are used. To calibrate the CET for level 3, QER_w , PER_w , and $gammar_w$ are used.

When there is a three level CET, the optimal choice of exports to each aggregate region is determined from first order conditions:

$$PER_{wm} = PE \cdot \frac{\partial QE}{\partial QER_{wm}} \quad (A2.20)$$

This expression can be re-arranged to have QER_{wm} on the left hand-side as follows:

$$QER_{wm} = QE \cdot \left[\frac{PER_{wm} ATR^{-\rho_E}}{PE \cdot \gamma R_{wm}} \right]^{1/(\rho_E-1)} \quad (A2.21)$$

Within each aggregate region, the optimal choice of exports to the countries in that aggregate region is given by:

$$PER_w = PER_{wm} \cdot \frac{\partial QER_{wm}}{\partial QER_w} \quad (A2.22)$$

This expression can be re-arranged to have QER_w on the left hand-side as follows:

$$QER_w = QER_{wm} \cdot \left[\frac{PER_w ATR2_{wm}^{-\rho_{Ewmr}}}{PER_{wm} \cdot \gamma R_w} \right]^{1/(\rho_{Ewmr}-1)} \quad (A2.23)$$

Again, note that the constant in the CET function, $ATR2$, is indexed over wm , the aggregate region and the exponent, ρ_E , is indexed over wm .

The appropriate price definitions become:

$$PE \cdot QE = \sum_{wm} PER_{wm} \cdot QER_{wm} \quad (A2.24)$$

$$PER_{wm} \cdot QER_{wm} = \sum_{w \subset wm} PER_w \cdot QER_w \quad (A2.25)$$

There is no change to the price equation which links the domestic price of imports by region to the world price of imports by region, i.e. for the model with two level CES and for the model with a three level CES on the import side, the following equation holds:

$$PER_w = PWE_w (1 - te_w) \cdot ER \quad (A2.26)$$

A2.3 Share Version of the CES:

When there is a two level CES, imports are an aggregate of imports by region using the following notation:

$$QM = QM0 \left[\sum_{w=1}^W \theta M_w \left(\frac{QMR_w}{QMR0_w} \right)^{-\rho_M} \right]^{\frac{-1}{\rho_M}} \quad (A2.27)$$

$$\text{And, } \theta M_w = \frac{PMR0_w \cdot QMR0_w}{\sum_w PMR0_w \cdot QMR0_w}$$

The derived demand for imports from each region can be written as:

$$PMR_w = PM \cdot \frac{\partial QM}{\partial QMR_w} \quad (A2.28)$$

$$\text{Where, } \frac{\partial QM}{\partial QMR_w} = QM0 \left[\sum_{w=1}^W \theta M_w \left(\frac{QMR_w}{QMR0_w} \right)^{-\rho_M} \right]^{\left(\frac{-1}{\rho_M}-1\right)} \theta M_w \cdot QMR0_w^{\rho_M} \cdot QMR_w^{(-\rho_M-1)}$$

Equation (A2.28) can be written with QMR_w , the quantity of imports from region w , on the left hand side.

To simplify the expression, first note that:

$$\left(\frac{QM}{QM0} \right)^{\rho_M} = \left[\sum_{w=1}^W \theta M_w \left(\frac{QMR_w}{QMR0_w} \right)^{-\rho_M} \right]^{-1} \quad (A2.29)$$

Substituting (A2.29) into equation (A2.28), one can derive the following expression for QMR_w :

$$QMR_w = QM \cdot \left[\left(\frac{1}{\theta M_w} \right) \left(\frac{PMR_w}{PM} \right) \left(\frac{QM0}{QMR0_w} \right)^{\rho_M} \right]^{-1/(\rho_M+1)} \quad (A2.30)$$

When there is a three level CES, imports are an aggregate of imports by aggregate region using the following notation:

$$QM = QM0 \left[\sum_{wm} \theta M_{wm} \left(\frac{QMR_{wm}}{QMR0_{wm}} \right)^{-\rho_M} \right]^{-1/\rho_M} \quad (A2.31)$$

$$\text{where, } \theta M_{wm} = \frac{PMR0_{wm} \cdot QMR0_{wm}}{\sum_{wm} PMR0_{wm} \cdot QMR0_{wm}}$$

Imports from each aggregate region are derived from first order conditions and can be written as follows:

$$QMR_{wm} = QM \cdot \left[\left(\frac{1}{\theta M_{wm}} \right) \left(\frac{PMR_{wm}}{PM} \right) \left(\frac{QM0}{QMR0_{wm}} \right)^{\rho_M} \right]^{-1/(\rho_M+1)} \quad (A2.32)$$

Within each aggregate region:

$$QMR_{wm} = QMR0_{wm} \left[\sum_{w \subset wm} \theta M_w \left(\frac{QMR_w}{QMR0_w} \right)^{-\rho_{M_{wm}}} \right]^{-1/\rho_{M_{wm}}} \quad (A2.33)$$

$$\text{Where, } \theta M_w = \frac{PMR0_w \cdot QMR0_w}{\sum_{w \subset wm} PMR0_w \cdot QMR0_w}$$

Imports from each region within each aggregate region are derived from first order conditions and can be written as follows:

$$QMR_w = QMR_{wm} \cdot \left[\left(\frac{1}{\theta M_w} \right) \left(\frac{PMR_w}{PMR_{wm}} \right) \left(\frac{QMR0_{wm}}{QMR0_w} \right)^{\rho_{M_{wm}}} \right]^{-1/(\rho_{M_{wm}}+1)} \quad (A2.34)$$

A2.4 Share Version of the CET:

When there is a two level CET, exports are an aggregate of exports by region using the following notation:

$$QE = QE0 \left[\sum_{w=1}^W \theta E_w \left(\frac{QER_w}{QER0_w} \right)^{\rho_E} \right]^{\frac{1}{\rho_E}} \quad (A2.35)$$

$$\text{Where, } \theta E_w = \frac{PER0_w \cdot QER0_w}{\sum_w PER0_w \cdot QER0_w}$$

The export supply to each region can be written as:

$$PER_w = PE \cdot \frac{\partial QE}{\partial QER_w} \quad (A2.36)$$

$$\text{Where, } \frac{\partial QE}{\partial QER_w} = QE0 \left[\sum_{w=1}^W \theta E_w \left(\frac{QER_w}{QER0_w} \right)^{\rho_E} \right]^{\left(\frac{1}{\rho_E}-1\right)} \theta E_w \cdot QER0_w^{-\rho_E} \cdot QER_w^{(\rho_E-1)}$$

Equation (A2.36) can be written with QER_w , the quantity of exports to region w, on the left hand side.

To simplify the expression, first note that:

$$\left(\frac{QE}{QE0} \right)^{-\rho_E} = \left[\sum_{w=1}^W \theta E_w \left(\frac{QER_w}{QER0_w} \right)^{\rho_E} \right]^{-1} \quad (A2.37)$$

Substituting (A2.37) into equation (A2.36), one can derive the following expression for QER_w :

$$QER_w = QE \cdot \left[\left(\frac{1}{\theta E_w} \right) \left(\frac{PER_w}{PE} \right) \left(\frac{QE0}{QER0_w} \right)^{-\rho_E} \right]^{\frac{1}{\rho_E-1}} \quad (A2.38)$$

When there is a three level CET, exports are an aggregate of exports to aggregate regions using the following notation:

$$QE = QE0 \left[\sum_{wm} \theta E_{wm} \left(\frac{QER_{wm}}{QER0_{wm}} \right)^{\rho_E} \right]^{\frac{1}{\rho_E}} \quad (A2.39)$$

$$\text{where, } \theta E_{wm} = \frac{PER0_{wm} \cdot QER0_{wm}}{\sum_{wm} PER0_{wm} \cdot QER0_{wm}}$$

Exports to each aggregate region are derived from first order conditions and can be written as follows:

$$QER_{wm} = QE \cdot \left[\left(\frac{1}{\theta E_{wm}} \right) \left(\frac{PER_{wm}}{PE} \right) \left(\frac{QEO}{QER0_{wm}} \right)^{-\rho_E} \right]^{\frac{1}{(\rho_E - 1)}} \quad (A2.40)$$

Within each aggregate region:

$$QER_{wm} = QER0_{wm} \left[\sum_{w \subset wm} \theta E_w \left(\frac{QER_w}{QER0_w} \right)^{\rho_{E_{wm}}} \right]^{\frac{1}{\rho_{E_{wm}}}} \quad (A2.41)$$

$$\text{Where, } \theta E_w = \frac{PER0_w \cdot QER0_w}{\sum_{w \subset wm} PER0_w \cdot QER0_w}$$

Exports to each region within each aggregate region are derived from first order conditions and can be written as follows:

$$QER_w = QER_{wm} \cdot \left[\left(\frac{1}{\theta E_w} \right) \left(\frac{PER_w}{PER_{wm}} \right) \left(\frac{QER0_{wm}}{QER0_w} \right)^{-\rho_{E_{wm}}} \right]^{\frac{1}{(\rho_{E_{wm}} - 1)}} \quad (A2.42)$$

A2.5 Set Definitions for Nested CES/CET Functions

To add another level CES in the GAMS code, add a master set which includes all regions (trade partners and ANARRES) and the aggregate regions:

```
ww(sac)          rest of world trade partners and aggregates
                  /w1, ... wn, wglo, w-rich, w-middle, w-poor, w-ANARRES/

wm(ww)           aggregate regions
                  /w-rich, w-middle, w-poor, w-ANARRES/

w(ww)            rest of world trade partners
                  /w1, ... wn, wglo /
```

There is a mapping set which assigns trade partners to aggregate regions:

```
Map_wm_w(wm, w)

/  w-poor          . (w1, w2)
   w-middle        . (w3, w4)
   w-rich          . (w5, ...wn)
   w-ANARRES       . (wglo)      /
```

Note that the GILD region, *wglo*, is the only region assigned to the aggregate *w- GILD*.

Also add the sets *cmr2_{c,r,wm}*, (and *cmrn2_{c,r,wm}*) the imported (not imported) commodities by aggregate region, *wm*, analogous to *cmr_{c,r,w}* and *cmrn_{c,r,w}* for the regions, *w*. The sets *cmr2* and *cmr2n* are assigned from the data. The set *cmr2* is used to condition the derived demand

for imports from the aggregate region; if there are no imports from the aggregate region in the base, the variable is fixed at zero using *cmrn2*.

For non-ANARRES regions, imports from ANARRES (margin services in this version of the model) are not substitutable with imports of margin services from other regions and so do not enter the CES specification of imports of margin services by trade partner. In a two-level CES function, the first order conditions for the optimal choice of imports by region are indexed over the set $cmr_{c,r,w}$ which does not include imports from the region *wglo* (and elements of the set $cmr(*)$ are assigned from data in SAMG). Instead, regions w_1, \dots, w_n import margin services from ANARRES (*wglo*) based on a fixed share per unit imported. This is an additional equation in the model.

In a three-level nested CES, the set $cmr2_{c,r,wm}$ does not include imports from the aggregate region *w-ANARRES* (and this is assigned from data in SAMG). The CES aggregate of imports from the aggregate regions is conditioned on $cmr2_{c,r,wm}$ and therefore is defined over *w-poor*, *w-middle*, and *w-rich* in this example.

In a three-level nested CES, the variables *QMR* (imports by region) and *PMR* (the price of imports by region) and the parameter *deltar*, are assigned over the set *ww* and so refer to the aggregate imports from the set *wm*, as well as the imports from the individual regions, *w*, that make-up the aggregates. To calibrate the CES for level 2, QMR_{wm} , PMR_{wm} , and $deltar_{wm}$ are used. To calibrate the CES for level 3, QMR_w , PMR_w , and $deltar_w$ are used.

All regions, except ANARRES, can export margin services to all trade partners w_1, \dots, w_n , and *wglo* according to a CET function. ANARRES's exports a homogenous good, margin services.

In a three-level nested CET, the set $cer2_{c,r,wm}$, which is assigned from data in SAMG, includes exports of margin services from all non-ANARRES regions to the aggregate region *w-ANARRES* (as well as to other aggregate regions). The CET aggregate of exports to the aggregate regions is conditioned on $cer2_{c,r,wm}$ and therefore is defined over *w-poor*, *w-middle*, *w-rich*, and *w-ANARRES* in this example.

In a three-level nested CET, the variables *QER* (exports to region) and *PER* (the price of exports to regions) and the parameter *gammar*, are assigned over the set *ww* and so refer to exports to the aggregate regions in the set *wm*, as well as exports to the individual regions, *w*,

that make-up the aggregates. To calibrate the CET for level 2, QER_{wm} , PER_{wm} , and $gammar_{wm}$ are used. To calibrate the CET for level 3, QER_w , PER_w , and $gammar_w$ are used.

A3. Notes on Homogenous Exports

The model has a two level CET. At level one, there is a CET over the domestic and exported aggregate, at level 2, there is a CET over exports to trade partners.

A3.1 Level one CET assignment:

When the export and the domestic good are not perfect substitutes, the following CET equation is used in the model:

$$QXC = AT \cdot [\gamma \cdot QE^{\rho_T} + (1 - \gamma) \cdot QD^{\rho_T}]^{1/\rho_T} \quad (A3.1)$$

Or in share form:

$$QXC = QXC0 \left[\theta_E \cdot \left(\frac{QE}{QE0} \right)^{\rho_T} + (1 - \theta_E) \cdot \left(\frac{QD}{QD0} \right)^{\rho_T} \right]^{1/\rho_T} \quad (A3.2)$$

There is an export supply function which is derived from the ratio of first order conditions choosing the optimal supply of exports (QE) and the good sold on the domestic market (QD). Using the share form of the CET, the export supply curve is:

$$QE = QD \left[\left(\frac{PE}{PD} \right) \left(\frac{1 - \theta_E}{\theta_E} \right) \left(\frac{QE0}{QD0} \right)^{\rho_T} \right]^{\left(\frac{-1}{(-\rho_T + 1)} \right)} \quad (A3.3)$$

Both equations (A3.2) and (A3.3) are included in the GAMS code.

When the export and the domestic variety are homogenous goods, the CET function is replaced by:

$$QXC = QE + QD \quad (A3.4)$$

(Note that this equation is also used when a country either has no domestic sales, $QD = 0$ or no export sales, $QE = 0$.)

When there are homogeneous goods, the export supply equation is replaced by a price equation:

$$PE = PD \quad (A3.5)$$

In the GAMS code, there is a set $cetn1(c,r)$ which identifies commodities and/or regions with homogeneous export and domestic goods at level one of the CET. This set is used to turn on

equations (A3.4) and (A3.5) above. For commodities and regions with non-homogenous goods, equations (A3.2) and (A3.3) are used and are conditioned on NOT cetn1(c,r).

A3.2 Level two CET assignment:

The export supply, QE , is a CET function over exports to each of the W region in the model, QER , indexed as $w = 1 \dots W$.

When exports to regions are not homogeneous, the relationship is:

$$QE = ATR \cdot \left[\sum_{w=1}^W \gamma_w QER_w^{\rho_E} \right]^{\frac{1}{\rho_E}} \quad (A3.6)$$

Or in share form:

$$QE = QE0 \left[\sum_{w=1}^W \theta_{Ew} \left(\frac{QER_w}{QER0_w} \right)^{\rho_E} \right]^{\frac{1}{\rho_E}} \quad (A3.7)$$

The GAMS code includes the first order conditions for the optimal choice of QER_w , using the share form of the CET function:

$$QER_w = QE \cdot \left[\left(\frac{1}{\theta_{Ew}} \right) \left(\frac{PER_w}{PE} \right) \left(\frac{QE0}{QER0_w} \right)^{-\rho_E} \right]^{\frac{1}{(\rho_E-1)}} \quad (A3.8)$$

Note that the GAMS code does not include equation (A3.6) or (A3.7). Instead, it includes each first order condition. This differs from level one, there the GAMS code includes the CET function and the ratio of the two first order conditions.

When the export and the domestic good at level one are homogenous, it is assumed that the exports across regions in level two are also homogeneous. The set cetn2(c,r) is used to assign commodities and/or regions that have a homogenous export good at level two. If a good is homogenous at level one, it is also homogeneous at level two and the assignment sets are linked. It is also possible that at level one, the export good and the domestic good are not homogenous, but the exports to region in level two are homogeneous. This case is also turned on by an assignment in set cetn2(c,r).

When exports to regions are homogeneous in level two, export supply equation (A3.8) is turned off and instead, the following price equation is used:

$$PER_w = PE \quad (A3.9)$$

Note that the following price definition holds for all commodities and regions, regardless of whether they are homogenous or not:

$$PE \cdot QE = \sum_w PER_w \cdot QER_w \quad (A3.10)$$

For the homogeneous exports to regions, (A3.9) and (A3.10)

$$QE = \sum_w QER_w \quad (A3.11)$$

A4. Equation and Variable Counts for the ANARRES CGE Model

Name	Equation	Number of Equations	Variable	Number of Variables	ANAR RES
EXPORTS BLOCK					
$PEDEF_{c,r}$	$PE_{c,r} * QE_{c,r} = \sum_w PER_{c,r,w} * QER_{c,r,w} \quad \forall c \in ce, r \in rgn$	$(ce*rgn)$	$PE_{c,r}$	$(ce*rgn)$	NO
$PERDEF2_{c,w,r}$	$PER_{c,r,w} = PWE_{c,r,w} * ER_r * (1 - TE_{c,r,w}) \quad \forall c \in cer$	$(cer*w*r)$	$PER_{c,r,w}$	$(cer*w*r)$	NO
$PERDEFHG_{c,w,r}$	$PER_{c,r,w} = PE_{c,r} \quad \forall c \in ct2, r \notin rgn, w \in wgn$	$(ct2*w*1)$		$(ct2*w*1)$	YES
$CET_{c,r}$	$QXC_{c,r} = at_{c,r} * \left(\gamma_{c,r} * QE_{c,r}^{\rho_{c,r}^t} + (1 - \gamma_{c,r}) * QD_{c,r}^{\rho_{c,r}^t} \right)^{1/\rho_{c,r}^t}$ $\forall c \in (cd \cap ce), r \in rgn$	$(c*rgn)$	$QD_{c,r}$	$(c*rgn)$	NO
$ESUPPLY_{c,r}$	$QE_{c,r} = QD_{c,r} \left[\frac{PE_{c,r} * (1 - \gamma_{c,r})}{PD_{c,r} \gamma_{c,r}} \right]^{\frac{1}{(\rho_{c,r}^t - 1)}} \quad \forall c \in (cd \cap ce), r \in rgn$	$(c*rgn)$	$QE_{c,r}$	$(c*rgn)$	NO
$CETALT_{c,r}$	$QXC_{c,r} = QD_{c,r} + QE_{c,r} \quad \forall (c \in (cd \cap cen), r \in rgn)$ OR $\forall (c \in (cdn \cap ce), r \in rgn)$				NO
$CETLEV2_{c,w,r}$	$QER_{c,w,r} = QE_{c,r} * \left(\frac{PER_{c,w,r}}{\left((PE_{c,r} * \gamma_{c,w,r}^r * at_{c,r}^r)^{\rho_{c,r}^e} \right)} \right)^{1/(\rho_{c,r}^e - 1)} \quad \forall c \in cer2, r \in rgn$	$(cer*rgn*w)$	$QER_{c,r,w}$	$(cer*rgn*w)$	NO
$CETREQG_{c,r}$	$QE_{c,r} = \sum_w QER_{c,r,w} \quad \forall c \in ct, r \notin rgn$	$(ct*1)$	$QE_{c,r}$	$(ct*1)$	YES
$TSHIP_{c,r}$	$QE_{c,r} = QM_{c,r} \quad \forall c \in ct2, r \notin rgn$	$(ct2*1)$	$QE_{c,r}$	$(ct2*1)$	YES

Name	Equation	Number of Equations	Variable	Number of Variables	ANARRES
IMPORTS BLOCK					
$PMLDEF_{c,r}$	$PML_{c,r} = \frac{\sum_{w \in cmr} (PMR_{w,c,r} * QMR_{w,c,r})}{QML_{c,r}} \quad \forall (c,r) \in cml$	$(cml*r)$	PML	$(cml*r)$	YES
$PMSDEF_{c,r}$	$PMS_{c,r} = \sum_w ioqmrqms_{w,c,r} * PMR_{w,c,r} \quad \forall (c,r) \in cms$	$(cms*r)$	PMS	$(cms*r)$	YES
$PMRDEF2_{w,c,r}$	$PMR_{w,c,r} = PWM_{w,c,r} * ER_r * (1 + TM_{w,c,r}) \quad \forall c \in cmr$	$(cmr*r*w)$	$PMR_{w,c,r}$	$(cmr*r*w)$	YES
$PMDEF_{c,r}$	$PM_{c,r} * QM_{c,r} = (PML_{c,r} * QML_{c,r}) + (PMS_{c,r} * QMS_{c,r}) \quad \forall (c,r) \in cm_{c,r}$	$(cm*r)$	$PM_{c,r}$	$(cm*r)$	YES
$PWMDEF_{w,c,r}$	$PWM_{w,c,r} = PWMFOB_{w,c,r} + \left(\sum_{cp \in c_{cp,r}} margcor_{w,cp,c,r} * PT_{cp,r} \right) \quad \forall c \in cmr$	$(cmr*r*w)$	$PWM_{c,r,w}$	$(cmr*r*w)$	YES

Name	Equation	Number of Equations	Variable	Number of Variables	ANARRES
IMPORTS BLOCK					
$ARMINGTON_{c,r}$	$QQ_{c,r} = ac_{c,r} \left(\delta_{c,r} * QM_{c,r}^{-\rho_{c,r}^c} + (1 - \delta_{c,r}) * QD_{c,r}^{-\rho_{c,r}^c} \right)^{-\left(\frac{1}{\rho_{c,r}^c}\right)}$ $\forall c \in (cx \cap cm), r \in rgn$	$(c*rgn)$	$QM_{c,rgn}$	$(c*rgn)$	NO
$COSTMIN_{c,r}$	$QM_{c,r} = QD_{c,r} * \left(\left(\frac{PD_{c,r}}{PM_{c,r}} \right) * \left(\frac{\delta_{c,r}}{(1 - \delta_{c,r})} \right) \right)^{\frac{1}{(1 + \rho_{c,r}^c)}}$ $\forall c \in (cx \cap cm), r \in rgn$	$(c*rgn)$	$PD_{c,rgn}$	$(c*rgn)$	NO
$ARMALT_{c,r}$	$QQ_{c,r} = QD_{c,r} + QM_{c,r}$ $\forall c \in (cx \cap cmn), r \in rgn$ $\text{OR } \forall c \in (cxn \cap cm), r \in rgn$				NO
$QMSEQ_{c,r}$	$QMS_{c,r} = ioqmsqm_{c,r} + QM_{c,r}$ $\forall c \in cms$	$(cms*r)$	$QMS_{c,r}$	$(cms*r)$	YES
$QMSEQ_{c,r}$	$QML_{c,r} = ioqmlqm_{c,r} + QM_{c,r}$ $\forall c \in cml$	$(cml*r)$	$QML_{c,r}$	$(cml*r)$	YES
$QMREQ_{w,c,r}$	$QMR_{w,c,r} = ioqmrqms_{w,c,r} + QMS_{c,r}$ $\forall c \in cmrs$				YES
$ARMLEV2_{w,c,r}$	$QMR_{w,c,r} = QML_{c,r} * \left(\frac{PMR_{w,c,r} * acr_{c,r}^{\rho_{c,r}^m}}{PML_{c,r} * \delta_{w,c,r}^r} \right)^{\left(\frac{-1}{(\rho_{c,r}^m + 1)}\right)}$ $\forall c \in cmrl$	$(w*c*r)$	$QMR_{w,c,r}$	$(w*c*r)$	YES
$QTEQ_{w,c,r}$	$QT_{w,c,r} = \sum_{cp} (QMR_{w,cp,r} * margcor_{w,cp,w,r})$ $\forall c \in ct2, r \in rgn$	$(ct2*rgn*w)$	$QT_{c,rgn,w}$	$(ct2*rgn*w)$	NO

Name	Equation	Number of Equations	Variable	Number of Variables	ANARRES
COMMODITY PRICE BLOCK					
			PD_c		
$PQSDEF_{c,r}$	$PQS_{c,r} = \frac{(PD_{c,r} * QD_{c,r}) + (PM_{c,r} * QM_{c,r})}{QQ_{c,r}} \quad \forall c \in (cd \cup cm), r \in rgn$	$(c*rgn)$	$PQS_{c,r}$	$(c*rgn)$	NO
$PQDDEF_{c,r}$	$PQD_{c,r} = PQS_{c,r} * (1 + TS_{c,r}) \quad \forall c \in (cd \cup cm), r \in rgn$	$(c*rgn)$	$PQD_{c,r}$	$(c*rgn)$	NO
$PXCDEF_{c,r}$	$PXC_{c,r} = \frac{(PD_{c,r} * QD_{c,r}) + (PE_{c \in ce, r} * QE_{c \in ce, r})}{QXC_c} \quad \forall c \in cx, r \in rgn$	$(cx*rgn)$	$PXC_{c,r}$	$(cx*rgn)$	NO
NUMERAIRE PRICE BLOCK					
$CPIDEF_r$	$CPI_r = \sum_c comtotsh_{c,r} * PQ_{c,r} \quad \forall r \in rgn$	rgn	CPI_r	rgn	NO
$PPIDEF_r$	$PPI_r = \sum_c vddtotsh_{c,r} * PD_{c,r} \quad \forall r \in rgn$	rgn	PPI_r	rgn	NO
$ERPIDEF$	$ERPI = \sum_{ref} tradtotsh_{ref} * ER_{ref}$	1	$ERPI$	1	NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables	S
PRODUCTION BLOCK – TOP LEVEL					
$PXDEF_{a,r}$	$PX_{a,r} = \sum_c ioqxcqx_{a,c,r} * PX_{c,r} \quad \forall r \in rgn$	$(a*rgn)$	$PX_{a,r}$	$(a*rgn)$	NO
$PVADEF_{a,r}$	$PX_{a,r} * (1 - TX_{a,r}) * QX_{a,r} = (PVA_{a,r} * QVA_{a,r}) + (PINT_{a,r} * QINT_{a,r}) \quad \forall r \in rgn$	$(a*rgn)$	$PVA_{a,r}$	$(a*rgn)$	NO
$PINTDEF_{a,r}$	$PINT_{a,r} = \sum_c ioqint_{c,a,r} * PQD_{c,r} \quad \forall r \in rgn$	$(a*rgn)$	$PINT_{a,r}$	$(a*rgn)$	NO
$ADXEQ_{a,r}$	$ADX_{a,r} = ((adx0_{a,r} + dabadx_{a,r}) * ADXADJ_r) + (DADX_r * adx01_{a,r})$	$(a*rgn)$	$ADX_{a,r}$	$(a*rgn)$	NO
$QXPRODFN_{a,r}$	$QX_{a,r} = ADX_{a,r} * \left[\delta_{a,r}^x * (QVA_{a,r})^{-\rho_{a,r}^x} + (1 - \delta_{a,r}^x) * (QINT_{a,r})^{-\rho_{a,r}^x} \right]^{-\frac{1}{\rho_{a,r}^x}} \quad \forall r \in rgn, a \in aqx$	$(aqx*rgn)$	$QX_{a,r}$	$(aqx*rgn)$	NO
$QXFOCF_{a,r}$	$QVA_{a,r} = QINT_{a,r} * \left(\left(\frac{PINT_{a,r}}{PVA_{a,r}} \right) * \left(\frac{\delta_{a,r}^x}{(1 - \delta_{a,r}^x)} \right) \right)^{\frac{1}{(1 + \rho_{a,r}^x)}} \quad \forall r \in rgn, a \in aqx$	$(aqx*rgn)$	$QINT_{a,r}$	$(aqx*rgn)$	NO
$QINTDEF_{a,r}$	$QINT_{a,r} = \sum_a ioqintqx_{a,r} * QX_{a,r} \quad \forall r \in rgn, a \in aqxn$	$(aqxn*rgn)$	$QX_{a,r}$	$(aqxn*rgn)$	NO
$QVADEF_{a,r}$	$QVA_{a,r} = \sum_a ioqvaqx_{a,r} * QX_{a,r} \quad \forall r \in rgn, a \in aqxn$	$(aqxn*rgn)$	$QINT_{a,r}$	$(aqxn*rgn)$	NO
$COMOUT_{c,r}$	$QXC_{c,r} = \sum_a ioqxcqx_{a,c,r} * QX_{a,r} \quad \forall r \in rgn$	$(c*rgn)$	$QXC_{c,r}$	$(c*rgn)$	NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables	S
PRODUCTION BLOCK – SECOND LEVEL					
$ADVAEQ_{a,r}$	$ADVA_{a,r} = \left((adva0_{a,r} + dabadva_{a,r}) * ADVAADJ_r \right) + (DADVA_r * adva01_{a,r})$	$(a*rgn)$	$ADVA_{a,r}$	$(a*rgn)$	NO
$QVAPRODFN_{a,r}$	$QVA_{a,r} = ADVA_{a,r} * \left(\sum_f \delta_{f,a,r}^{va} * (ADFD_{f,a,r} * FD_{f,a,r})^{\rho_{a,r}^{va}} \right)^{\left(\frac{-1}{\rho_{a,r}^{va}} \right)}$ $\forall r \in rgn, \delta_{f,a,r}^{va}$	$(a*rgn)$	$QVA_{a,r}$	$(a*rgn)$	NO
$QVAFOC_{f,a,r}$	$WF_{f,r} * WFDIST_{f,a,r} * (1 + TF_{f,a,r})$ $= PVA_{a,r} * QVA_{a,r} * \left[\sum_f \delta_{f,a,r}^{va} * (ADFD_{f,a,r} * FD_{f,a,r})^{-\rho_{a,r}^{va}} \right]^{(-1)}$ $* \delta_{f,a,r}^{va} * (ADFD_{f,a,r})^{-\rho_{a,r}^{va}} * (FD_{f,a,r})^{(-\rho_{a,r}^{va}-1)}$ $\forall r \in rgn, \delta_{f,a,r}^x$	$(f*a*rgn)$	$FD_{a,f,r}$	$(f*a*rgn)$	NO
$QINTDEQ_{c,r}$	$QINTD_{c,r} = \sum_a ioqint_{c,a,r} * QINT_{a,r}$ $\forall r \in rgn, cintd_{c,r}$	$(c*rgn)$	$QINTD_{c,r}$	$(c*rgn)$	NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables	S
FACTOR BLOCK					
$YFEQ_{f,r}$	$YF_{f,r} = \sum_a WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r} \quad \forall r \in rgn$	$(f*rgn)$	$YF_{f,r}$	$(f*rgn)$	NO
$YFDISTEQ_{f,r}$	$YFDIST_{f,r} = \left(YF_{f,r} - (deprec_{f,r} * YF_{f,r}) \right) * (1 - TYF_{f,r})$ $\forall r \in rgn$	$(f*rgn)$	$YFDIST_{f,r}$	$(f*rgn)$	NO
HOUSEHOLD BLOCK					
$YHEQ_{h,r}$	$YH_{h,r} = \sum_f hvash_{h,f,r} * YFDIST_{f,r} \quad \forall r \in rgn$	$(h*rgn)$	$YH_{h,r}$	$(h*rgn)$	NO
$HEXPEQ_{h,r}$	$HEXP_{h,r} = \left(YH_{h,r} * (1 - TYH_{h,r}) \right) * (1 - SHH_{h,r})$ $QCD_{c,h,r} * PQD_{c,r} = \left(PQD_{c,r} * qcdconst_{c,h,r} \right)$ $+ \beta_{c,h,r} * \left(HEXP_{h,r} - \sum_c PQD_{c,r} * qcdconst_{c,h,r} \right)$	$\forall r \in rgn$ $(h*rgn)$	$HEXP_r$	$(h*rgn)$	NO
$QCDEQ_{c,h,r}$		$(c*h*rgn)$	$QCD_{c,h,r}$	$(c*h*rgn)$	NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables	S
GOVERNMENT TAXES BLOCK					
$TEDEF_{c,w,r}$	$TE_{c,w,r} = \left((te0_{c,w,r} + dabte_{c,w,r}) * TEADJ_r \right) + (DTE_r * te01_{c,w,r})$ $\forall cer$	$(cer*r*w)$	$TE_{c,r,w}$	$(cer*r*w)$	NO
$TMDEF_{w,c,r}$	$TM_{w,c,r} = \left((tm0_{w,c,r} + dabtm_{w,c,r}) * TMADJ_r \right) + (DTM_r * tm01_{w,c,r})$ $\forall cmr$	$(cmr*r*w)$	$TM_{c,r,w}$	$(cmr*r*w)$	NO
$TSDEF_{c,r,w}$	$TS_{c,r} = \left((ts0_{c,r} + dabts_{c,r}) * TSADJ_r \right) + (DTS_r * ts01_{c,r})$ $\forall r \in rgn, c \in (cd \cup cm)$	$(c*r*w)$	$TS_{c,r,w}$	$(c*r*w)$	NO
$TXDEF_{a,r}$	$TX_{a,r} = \left((tx0_{a,r} + dabtx_{a,r}) * TXADJ_r \right) + (DTX_r * tx01_{a,r})$ $\forall r \in rgn$	$(a*rgn)$	$TX_{a,r}$	$(a*rgn)$	NO
$TYFDEF_{f,r}$	$TYF_{f,r} = \left((tyf0_{f,r} + dabtyf_{f,r}) * TYFADJ_r \right) + (DTYF_r * tyf01_{f,r})$ $\forall r \in rgn$	$(f*rgn)$	$TYF_{f,r}$	$(f*rgn)$	NO
$TYHDEF_{h,r}$	$TYH_{h,r} = \left((tyh0_{h,r} + dabtyh_{h,r}) * TYHADJ_r \right) + (DTYH_r * tyh01_{h,r})$ $\forall r \in rgn$	$(h*rgn)$	$TYH_{h,r}$	$(h*rgn)$	NO
$TFDEF_{f,a,r}$	$TF_{f,a,r} = \left((tf0_{f,a,r} + dabtf_{f,a,r}) * TFADJ_r \right) + (DTF_r * ty01_{f,a,r})$ $\forall r \in rgn$	$(f*a*rgn)$	$TF_{f,a,r}$	$(f*a*rgn)$	NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables
GOVERNMENT TAXES BLOCK				
$MTAXEQ_r$	$MTAX_r = \sum_w \sum_c (TM_{w,c,r} * PWM_{w,c,r} * ER_r * QMR_{w,c,r})$ $\forall r \in rgn$	rgn	$MTAX_r$	rgn NO
$ETAXEQ_r$	$ETAX_r = \sum_w \sum_c (TE_{c,w,r} * PWE_{c,w,r} * ER_r * QER_{c,w,r})$ $\forall r \in rgn$	rgn	$ETAX_r$	rgn NO
$STAXEQ_r$	$STAX_r = \sum_c \left(\frac{TS_{c,r} * PQ_{c,r}}{(QINTD_{c,r} + QCD_{c,r} + QGD_{c,r} + QINVD_{c,r})} \right)$ $\forall r \in rgn$	rgn	$STAX_r$	rgn NO
$ITAXEQ_r$	$ITAX_r = \sum_a (TX_{a,r} * PX_{a,r} * QX_{a,r})$ $\forall r \in rgn$	rgn	$INDTAX_r$	rgn NO
$FYTAXEQ_r$	$FYTAX_r = \sum_f (TYF_{f,r} * (YF_{f,r} - (deprec_{f,r} * YF_{f,r})))$ $\forall r \in rgn$	rgn	$FYTAX_r$	rgn NO
$HTAXEQ_r$	$HTAX_r = \sum_h (TYH_{h,r} * YH_{h,r})$ $\forall r \in rgn$	rgn	$HTAX_r$	rgn NO
$FTAXEQ_r$	$FTAX_r = \sum_f \sum_a (TF_{f,a,r} * WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r})$ $\forall r \in rgn$	rgn	$FTAX_r$	rgn NO

Name	Equation	Number of Equations	Variable	Number of ANARRES Variables	S
GOVERNMENT INCOME AND EXPENDITURE BLOCK					
$YGEQ_r$	$YG_r = MTAX_r + ETAX_r + STAX_r + ITAX_r + FYTAX_r + HTAX_r + FTAX_r \quad \forall r \in rgn$	rgn	YG_r	rgn	NO
$QGDEQ_{c,r}$	$QGD_{c,r} = qgdconst_{c,r} * QGDADJ_r \quad \forall r \in rgn$	$(c*rgn)$	$QGD_{c,r}$	$(c*rgn)$	NO
$EGEQ_r$	$EG_r = \sum_c PQD_{c,r} * QGD_{c,r} \quad \forall r \in rgn$	rgn	EG_r	rgn	NO
KAPITAL ACCOUNT BLOCK					
$SHHDEF_{h,r}$	$SHH_{h,r} = ((shh0_{h,r} + dabshh_{h,r}) * SADJ_r) + (DSHH_r * shh01_{h,r}) \quad \forall r \in rgn$	rgn	$SHH_{h,r}$	rgn	NO
$TOTSAVEQ_r$	$TOTSAV_r = \left(\sum_h (YH_{h,r} * (1 - TYH_{h,r})) * (SHH_{h,r}) \right) + \sum_f deprec_{f,r} * YF_{f,r} + KAPGOV_r + (KAPWOR_r * ER_r) \quad \forall r \in rgn$	rgn	$TOTSAV_r$	rgn	NO
$QINVDEQ_{c,r}$	$QINVD_{c,r} = IADJ_r * qinvdconst_{c,r} \quad \forall r \in rgn$	$(c*rgn)$	$QINVD_{c,r}$	$(c*rgn)$	NO
$INVESTEQ_r$	$INVEST_r = \sum_c (PQD_{c,r} * QINVD_{c,r}) \quad \forall r \in rgn$	rgn	$INVEST_r$	rgn	NO

Name	Equation	Number of Equations	Variable	Number of Variables	ANARRES
MARKET CLEARING BLOCK					
$FMEQUIL_{f,r}$	$FS_{f,r} = \sum_a FD_{f,a,r} \quad \forall r \in rgn$	$(f*rgn)$	$FS_{f,r}$	$(f*rgn)$	NO
$QEQUIL_{c,r}$	$QQ_{c,r} = QINTD_{c,r} + \sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r}$ $\forall c \in (cd \cup cm), r \in rgn$	$(c*rgn)$	$QQ_{c,r}$	$(c*rgn)$	NO
$KAPGOVEQ_r$	$KAPGOV_r = YG_r - EG_r \quad \forall r \in rgn$	rgn	$KAPGOV_r$	rgn	NO
$KAPREQUIL_{r,w}$	$KAPREG_{r,w} = \left(\sum_c PWMFOB_{w,c,r} * QMR_{w,c,r} \right) - \left(\sum_c PWE_{c,w,r} * QER_{c,w,r} \right)$ $\forall w \in wgn$	$(r*wgn)$	$KAPREG_{r,w}$	$(r*wgn)$	NO
$KAPREQUIL2_{r,w}$	$KAPREG_{w,r} = \left(\sum_c \sum_w PT_{c,r} * QT_{w,c,r} \right) - \left(\sum_c PWE_{c,w,r} * QER_{c,w,r} \right)$ $\forall w \notin wgn$	r	$KAPREG_{r,w}$	r	YES
$KAPEQUIL_r$	$KAPWOR_{r2} = \sum_w KAPREG_{w,r} \quad \forall r$	r	$KAPWOR_r$	r	YES

Name	Equation	Number of Equations	Variable	Number of Variables	ANARRES
MARKET CLEARING BLOCK					
$COMTRADE_{c,r}$	$\sum_w (PWM_{w,c,r} * QMR_{w,c,r}) = \sum_w (PWE_{c,w,r} * QER_{c,w,r}) + GLOBESLACK$ $\forall c \in ct2, r \notin rgn$	1	$ANARRESSLACK$	1	YES
$TRCONP_{w,c,r}$	$PWMFOB_{w,c,r} = PWE_{c,w,r} \quad \forall c \in cmr$	$(cmr * r * w)$	$PWMFOB_{c,r,w}$	$(cmr * r * w)$	YES
$TRCONQ_{w,c,r}$	$QMR_{w,c,r} = QER_{c,w,r} \quad \forall c \in cmr$	$(cmr * r * w)$	$QMR_{c,r,w}$	$(cmr * r * w)$	YES
$ANARRESQUIL_{c,r}$	$\sum_w QT_{w,c,r} = \sum_w QER_{c,w,"glo"} \quad \forall c \in ct2, r \in rgn$	$(ct2 * rgn)$		$(ct2 * rgn)$	YES
$PTDEF_{c,r}$	$PT_{c,r} = PWE_{c,w,"glo"} \quad \forall c \in ct2, r \in rgn$	$(ct2 * rgn)$	$PT_{c,r}$	(ct, rgn)	YES
$VFDOMDEQ_r$	$VFDOMD_r = \sum_c \left(PQD_{c,r} * \left(\sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} \right) \right)$ $\forall r \in rgn$	rgn	$VFDOMD_r$	rgn	NO
$INVESTSHEQ_r$	$INVESTSH_r * VFDOMD_r = INVEST_r \quad \forall r \in rgn$	rgn	$INVESTSH_r$	rgn	NO
$VGDSHEQ_r$	$VGDSH_r * VFDOMD_r = \sum_c (PQD_{c,r} * QGD_{c,r}) \quad \forall r \in rgn$	rgn	$VGDSH_r$	rgn	NO
$WALRASEQ_r$	$TOTSAV_r = INVEST_t + WALRAS_r \quad \forall r \in rgn$	rgn	$WALRAS_r$	rgn	NO
$SYSEQUIL$	$KAPWORSYS = \sum_{r2} KAPWOR_{r2}$	1	$KAPWORSYS$	1	NO

Name	Equation	Number of Equations	Variable	Number of Variables
MODEL CLOSURE				
	All ANARRES transactions are in world dollars (i.e. of the reference region)	\overline{ER}_{glo}		
	There is no current account equation for the ANARRES.	AND \overline{KAPWOR}_{glo}		
	Assign the reference region, eliminating one variable; add the constraint that the sum of KAPWOR across <i>rgn</i> must be zero	$\overline{KAPWORSYS}$	\overline{ER}_{ref}	
	For the non-reference regions, one can fix either the exchange rate or the current account balance	\overline{ER}_{nonref} or $\overline{KAPWOR}_{nonref}$		1
		\overline{SADJ} or \overline{IADJ} or \overline{INVEST}		1
	All but two of $\overline{TMADJ}, \overline{TSADJ}, \overline{TEADJ}, \overline{TXADJ}, \overline{TYHADJ}, \overline{QGDADJ}, \overline{KAPGOV}, \overline{EG}$			6
			\overline{FS}_f	3
		\overline{CPI} or \overline{PPI}		1
		$f + 12c + (4+f)a + 14$	$2f + 12c + (4+f)a + 23$	

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