ANARRES_CC: A SAM Based Global CGE Model

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Abstract

This paper provides a technical description of a version of a global computable general equilibrium (CGE) model that is calibrated using global data presented as a Social Accounting Matrix (SAM) with satellite accounts. This model was used in a study for UNFCCC; its use is not endorsed.

Keywords: Computable General Equilibrium; Social Accounting Matrix.

This document reports the ANARRES_CC model at the date listed above. There may be undocumented changes to the model from subsequent revisions.

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1: Introduction

This paper provides a technical description of a variant of a Social Accounting Matrix (SAM) based Global Computable General Equilibrium (CGE) model that can be calibrated using data derived from the Global Trade Analysis Project's (GTAP) database. The model is a member of a family of CGE models that model trade relationships using principles described in the 1-2-3 model (de Melo and Robinson, 1989; Devarajan, *et al*., 1990). More specifically this model is a direct descendant of an early US Department of Agriculture model (see Robinson *et al*., 1990; Kilkenny, 1991) and a NAFTA model (see Robinson *et al*., 1993). However numerous features of this model stem from other developments in CGE modelling over the last 25 years; some of these sources of inspiration are direct and easily identified, e.g., analysis used in the World Development Report, 1995 (Lewis, Robinson, and Wang), the IFPRI standard model (Lofgren *et al*., 2002), the PROVIDE Project model (McDonald, 2003), and the STAGE model (McDonald and Thierfelder, 2020): others are indirect and easily identified, e.g., the GTAP model (Hertel, 1997), while others are both direct and indirect but less easily identified; a substantial debt is owed to the community of CGE modellers. In addition, the model owes a lot to the development of the SAM approach to national accounting, e.g., Stone (1962a and b) and Pyatt (1991), and the SAM approach to modelling, e.g., Pyatt (1987), Drud *et al*., (1986), and the on-going development of the General Algebraic Modelling System (GAMS) software.

The underlying approach to multi-region modelling for this CGE model is the construction of a series of single country CGE models that are linked through their trading relationships. As is common with all known CGE models the price systems in the model are linear homogenous and hence the focus is upon movements in relative, rather than absolute, prices. Consequently, each region in the model has its own numéraire price, typically the consumer price index (CPI), and a nominal exchange rate, while the model as a whole requires a numéraire, which is an exchange rate index for a number of reference regions. As such this model contains a fundamentally different philosophical approach to global modelling to that found in the other global model.[1] Behind this difference lies a theoretical

1 For instance, the GTAP and GTAPnGAMS models, and, apparently, the LINKAGE model, do not contain nominal exchange rates. In fact, all these models contain (r + 1) numéraire, where *r* is the number of regions, although appearing to contain only a single global numéraire.

debate about how comparative static and finite horizon dynamic CGE models should value transfers associated with the capital account of the balance of payments (see Robinson, 2004).

A distinctive feature of the model is the use of a 'dummy' region, known as GILD, that allows for the recording of inter-regional transactions where either the source or destination are not identified. Examples of such transactions include trade and transportation margins and data on remittances. The GILD construct provides a general method for dealing with any transactions data where full bilateral data are missing.

The ANARRES model is a consolidation of many variants of the GLOBE 1 and 2 models developed since c 2002. ANARRES is a generic model that can encompass most, if not all, of the previous variants of the ANARRES/GLOBE model family; thus, ANARRES removes, for instance, the need for separate models for analyses of energy and inter-regional migration 'policies' while adding improved modelling of trade relationships, labour markets and household preferences.

This technical description encompasses both the comparative static version of ANARRES and the core model components required for the recursive dynamic variant, i.e., it only relates to the 'solution' phase. It does not include and details about the behavioural relationships included in the 'update' phase.[2]

The rest of this paper is organised as follows. Section 2 reviews the data used in the model; this section also provides a brief description of how the data were transformed from the GTAP database into a SAM. This is followed in section 3 by a descriptive overview of the model and then, in section 4, by a formal description of the model's equations. The description in section 4 is based upon a default setting for the model closure rules; one of the model's key features is the flexibility of the closure rules and consequently section 5 considers the alternatives built into the model's basic structure. All global CGE models are large and therefore present a series of potential implementation problems; section 6 briefly reviews some of the programmes that have been developed to support the basic model and provides some guidelines for use of this class of model. This is followed by some concluding comments that primarily focus upon planned model developments. Finally, there are a series

[2] Recursive dynamic CGE models can be thought of as being a series of comparative static 'solution' phases with 'update' phases that are used to calibrate successive 'solution' phase.

of technical appendices relating to aspects of model formulation, calibration of use – details about the content of these appendices is provided at the start of the appendices.

2: Model Data

The data used in the model were derived from the GTAP database (see Hertel, 1997) using a three-dimensional Social Accounting Matrix (SAM) method for organising the data. Details of the method used to generate a SAM representation are reported in McDonald and Thierfelder (2004a) while a variety of reduced form representations of the SAM and methods for augmenting the GTAP database are reported in McDonald and Thierfelder (2004b) and McDonald and Sonmez (2004) respectively. Detailed descriptions of the data are provided elsewhere so the discussion here is limited to the general principles.

2.1 Global Social Accounting Matrix

The Global SAM can be conceived of as a series of single region SAMs that are linked through the trade accounts; it is particularly valid in the context of the GTAP database to note that the ONLY way in which the regions are linked directly in the database is through commodity trade transactions although there are some indirect links through the demand and supply of trade and transport services. Specifically, the value of exports, valued free on board (*fob*) from source *s* to destination *d* must be exactly equal to the value of imports valued fob to destination *d* from source *s*, and since this holds for all commodity trade transactions the sum of the differences in the values of imports and exports by each region must equal zero. However, the resultant trade balances do not fully accord with national accounting conventions because other inter regional transactions are not recorded in the database (see McDonald and Sonmez, 2004). A description of the transactions recorded in a representative SAM for a typical region in the database is provided in Table 1.

A SAM is a transactions matrix; hence each cell in a SAM simply records the values of the transactions between the two agents identified by the row and column accounts. The selling agents are identified by the rows, i.e., the row entries record the incomes received by the identified agent, while the purchasing agents are identified by the columns, i.e., the column entries record the expenditures made by agents. As such a SAM is a compact single-entry representation of double entry bookkeeping that is 'complete' and 'consistent' and can be used to present the National Accounts of a country in a single two-dimensional matrix (see UN, 1993, for a detailed explanation of the relationship between conventional and SAM presentations of National Accounts). A SAM is 'complete' in the sense that the SAM should record ALL the transactions within the production boundary of the National Accounts, and 'consistent' in the sense that income transactions by every agent are exactly matched by expenditure transactions of other agents. A fundamental consequence of these conditions is that the row and column totals of the SAM for each region must be identical, and hence the SAM provides a complete characterisation of current account transactions of an economy as a circular (flow) system. In the context of a global SAM the 'complete' and 'consistent' conditions need extending to encompass transactions between regions; this simply requires that every import transaction by a region must have an identical counterpart export transaction by another region. This is enough to ensure that the resultant global SAM provides a 'consistent' characterisation of current account transactions of the global economy as a circular (flow) system.[3]

Given these definitions of a SAM the transactions recorded in a SAM are easily interpreted. In Table 1 the row entries for the commodity accounts are the values of commodity sales to the agents identified in the columns, i.e., intermediate inputs are purchased by activities (industries etc.,), final consumption is provided by households, the government and investment demand and export demand is provided by the all the other regions in the global SAM and the export of margin services. The commodity column entries deal with the supply side, i.e., they identify the accounts from which commodities are purchased so to satisfy demand. Specifically, commodities can be purchased from either domestic activities – the domestic supply matrix valued inclusive of domestic trade and transport margins – or they can be imported – valued exclusive of international trade and transport margins. In addition to payments to the producing agents – domestic or foreign – the commodity accounts need to make expenditures with respect to the trade and transport services needed to import the commodities and any commodity specific taxes.

[3] It follows from the definition of SAM that if a SAM is 'consistent' but 'incomplete' the recorded transactions must have been distorted to achieve 'consistency'.

Table 1 **Social Accounting Matrix for a Region in the Global Social Accounting Matrix**

	Commodities	**Activities**	**Factors**	**Households**	**Government**	**Capital**	**Margins**	**Rest of World**	**Totals**
Commodities	0	Combined Intermediate Use Matrix	0	Private Consumption	Government Consumption	Investment Consumption	Exports of Margins (*fob*)	Exports of Commodities (*fob*)	Total Demand for Commodities
Activities	Domestic Supply Matrix	0	0	0	0	0	0	0	Total Domestic Supply by Activity
Factors	0	Expenditure on Primary Inputs	0	0	0	0	0	0	Total Factor Income
Households	0	0	Distribution of Factor Incomes	0	0	0	0	0	Total Household Income
Government	Taxes on Commodities	Taxes on Production Taxes on Factor Use	Direct/Income Taxes	Direct/Income Taxes	0	0	0	0	Total Government Income
Capital	0	0	Depreciation Allowances	Household Savings	Government Savings	0	Balance on Margins Trade	Foreign Savings	Total Savings
Margins	Imports of Trade and Transport Margins	0	0	0	0	0	0	0	Total Income from Margin Imports
Rest of World	Imports of Commodities (*fob*)	0	0	0	0	0	0	0	Total Income from Imports
Totals	Total Supply of Commodities	Total Expenditure on Inputs by Activities	Total Factor Expenditure	Total Household Expenditure	Total Government Expenditure	Total Investment	Total Expenditure on Margin Exports	Total Expenditure on Exports	

The GTAP database provides complete coverage of bi lateral transactions in commodities – these are valued free on board (*fob*) - but only provides partial coverage of transactions in trade and transport margins. Specifically, the imports of trade and transport margins by each region are directly associated with the imports of specific commodities, hence for each commodity import valued *fob* the source and destination regions are identified and the value of each trade and transport margin service used is identified. The sum of the values of trade and transport services and the *fob* value of the commodity imports represent the carriage insurance and freight (*cif*) paid value of each imported commodity. But the source regions of the trade and transport services are NOT identified, and similarly the values of exports of trade and transport services by a region do NOT identify the destination regions. To overcome this lack of information an artificial region called GILD is included in the database. This region collects all the exports of trade and transport services by other regions as its imports and then exports these to other regions to satisfy their demand for the use of trade and transport services associated with commodity imports. By construction, the value of imports by GILD for each and every trade and transport margin service must exactly equal the value of exports for the corresponding trade and transport service. However, this does not mean that the trade balance between GILD and every region must exactly balance, rather it requires that the sum of GILD's trade balances with other regions is exactly equal to zero.

An important feature of the construction of a SAM can be deduced from the nature of the entries in the commodity account columns. The column and row totals must equate, and these transaction totals can be expressed as an implicit price times a quantity, and the quantity of a commodity supplied must be identical to the quantity of a commodity demanded. The column entries represent the expenditures incurred to supply a commodity to the economy and hence the implicit price must be exactly equal to the average cost incurred to supply a commodity. Moreover, since the row and column totals equate and the quantity represented by each corresponding entry must be same for the row and column total the implicit price for the row total must be identical to average cost incurred to supply the commodity. Hence the column entries identify the components that enter into the formation of the implicit prices in the rows, and therefore identify the price formation process for each price in the system. Typically, a SAM is defined such that the commodities in the rows are homogenous and that all agents purchase a commodity at the same price.

Total income to the activity accounts is identified by the row entries. In the simple representation of production in the GTAP database each activity makes a single commodity and each commodity is made by a single activity, which means that the domestic supply matrix is a diagonal (square) matrix. The expenditures on inputs used in production are recorded in the activity columns. Activities use intermediate inputs, which in this version of the database are record as composites of domestically produced and imported commodities, primary inputs and pay taxes on production and factor use. For each region the sum of the payments to primary inputs and on production and factor use taxes by activity is equal to the activity's contribution to the value-added definition of GDP while the sum over activities equals the region's value-added measure of GDP.

The remaining accounts relate to the institutions in the SAM. All factor incomes are distributed to the single private household after making allowance for depreciation of physical capital and the payment of direct (income) taxes on factor incomes. Incomes from factor sales are also the sole source of income to the household account. Three categories of expenditures by the household account are recorded: direct (income) taxes, savings and consumption. The government receives incomes from commodity taxes, production taxes and direct taxes on factor and household incomes and uses that income to pay for consumption and for savings. In the basic form of the database government savings are set to zero for all regions; this stems from the reduced form representation of intra institutional transactions provided by the GTAP database (see McDonald and Thierfelder, 2004b).[4] There are therefore five sources of savings in each region: depreciation, household/private savings, government savings, balances on trade in margin services and balances on trade in commodities, but only a single expenditure activity – investment (commodity) demand.

As is apparent from the description of the SAM for a representative region, the database is strong on inter regional transactions but relatively parsimonious on intra-regional transactions.

4 McDonald and Sonmez (2004) demonstrate that it is straightforward to overcome this limitation of the database. The model described in this paper operates whether the government savings are zero or non-zero.

2.2 Other GTAP Data

In addition to the transactions value data the GTAP database contains other data that can be used with this model, and/or variants of the model. The most obviously useful data are the import and primary factor elasticity data used in the GTAP model; the programme used to derive an aggregation of the SAM also contains a routine for aggregating these elasticities for use in this model. However, the GTAP elasticities are only a subset of the elasticities used in this model and it is therefore necessary to provide other elasticities even when using the GTAP elasticity data.

Other data of interest to modellers include estimates of energy usage and emissions and land use (carbon sinks). These data are used in this variant of the model.

2.3 Database Dimensions

The dimensions of the SAM are determined by the numbers of accounts within each aggregate group identified in Table 1, which are identify for versions 5.4, 8 and 10 of the GTAP database in Table 2. Given the large number of accounts in the SAMs for each region, and the relatively large number of regions, the total number of cells in the global SAM is very large, although only slightly over 10 percent of the cells contain non zero entries. Nevertheless, this still means that the GTAP 10 database contains c 18 million transaction values, which implies that there are some 36 million possible prices and quantities that can be deduced from the database. Even allowing for the implications of adopting the law of one price for transactions in the rows of each region's SAM and for other ways of reducing the numbers of independent prices and quantities that need to be estimated in a modelling environment, it is clear that the use of the GTAP database without aggregation is likely to generate extremely large models (in terms of the number of equations/variables). Consequently, except in exceptional circumstances all CGE models that use the GTAP data operate with aggregations of the database.

The dimensions of the disaggregated SAM used by ANARRES are reduced by truncating the GTAP transactions value data. For technical (software) reasons the GTAP database reports transaction values (TV) in millions of US dollars to 6 decimal places. Many of these TVs do not represent real transactions and moreover create problems for CGE models. The software to prepare GTAP data for use by ANARRES provides the user with an option on the degree to which the GTAP TVs are truncated.

Table 2 Dimensions of the Global Social Accounting Matrix

Account Groups	Sets	Total Number of Accounts		
		GTAP 5.4	**GTAP 8.0**	**GTAP 10**
Commodities	*C*	57	57	65
Activities	*A*	57	57	65
Factors	*F*	5	5	8
Taxes	(2 or 3*r)+(1*f)+3	164	398	434
Other Domestic Institutions	3	3	3	3
Margins	3*r	234	387	423
Trade	*R*	78	129	141
Total		598	1,036	1,139
Total Number of Cells in the Global SAM		27,893,112	138,445,184	182,922,261

3: Overview of the Model

3.1 Behavioural Relationships

The behavioural relationships in the model need to explain ALL the transactions reported in the SAM database, PLUS all transactions that could take place but were set equal to zero in the SAM. The within regional behavioural relationships in this variant of the ANARRES model are standard in various models; the novelty is the configuration of the model code so that the range of behavioural relationships governing production, trade and utility can be encompassed in a single model. In the ANARRES 1 and 2 models it was, relatively, easy to code more elaborate variants of the standard behavioural relationships, but the process could be time consuming and error prone.

In common with all CGE models, ANARRES assumes that all agents seek to optimise subject to the constraints they face. The key feature of these models is the presumption that agents are price responsive: if the relative price paid, by a purchasing agent, for an input increases so the agents will try to substitute away from that input, while if the relative price of a factor in an activity increases so the agent will seek to sell the factor to that activity.

The activities are assumed to maximise profits using technology characterised by Constant Elasticity of Substitution (CES) and/or Leontief production functions between aggregate 'primary' inputs and aggregate 'intermediate inputs', where aggregate 'primary' inputs can include selected, user defined, intermediate inputs. Non-selected intermediate inputs are aggregated with Leontief technology, with scope for substitution (CES) between subsets of intermediate inputs. Aggregate 'primary' inputs are defined by (multiple) nested CES production functions with the arguments defined over 'natural' primary inputs (labour, capital, and land) and factor commodities. The households maximise utility subject to preferences represented by nested CES and Stone-Geary utility function, i.e., a linear expenditure system, having first paid income taxes and having saved a fixed proportion of after-tax income.[5]

Emissions are modelled using coefficients that the quantities of greenhouse gases (GHG) associated with the use, in production and consumption, of a unit volume of each

5 With appropriate parameter specification the LES collapses to a Cobb-Douglas specification.

input associated with each GHG included in the database. For each GHG the model includes a tax rate that is levied on each volume unit of the input. Thus, as GHG tax rates increases so does the cost of the input to the agent that purchases the input, which induces optimising agents to substitute away from that input to another whose relative price has fallen.

The Armington 'insight' is used for trade. Domestic output is distributed between the domestic market and exports according to a *n*-stage, where *n* is greater than or equal to two, Constant Elasticity of Transformation (CET) functions. In the first stage a domestic producer allocates output to the domestic or export market according to the relative prices for the commodity on the domestic market and the composite export commodity. The composite export commodity is a CET aggregate of the exports to different groups of regions[6] where exports may be differentiated by the region receiving the exports. The distribution of the exports between regions being determined by the relative export prices to those regions. Consequently, domestic producers are responsive to prices in the different markets – the domestic market and all other regions in the model – and adjust their volumes of sales according relative prices. The elasticities of transformation are commodity and region specific. The CET functions across exports can be switched off so that export supplies are determined by import demands.[7]

Domestic demand is satisfied by composite commodities that are formed from domestic production sold domestically and composite imports. This process is modelled by a *n*-stage CES function, where *n* is greater than or equal to two. At stage of the nest each composite import commodity is a CES aggregate of imports from different regions within a group, with the quantities imported from different regions being responsive to relative prices. The composite imports from each group are then aggregated with another level of CES functions until there is a single composite import. The top stage defines a composite consumption commodity as a CES aggregate of a domestic commodity and a composite import commodity with the mix being determined by the relative prices. The elasticities of substitution are commodity and region and region group specific.[8] In addition, Leontief aggregation is available for the treatment of imports whose volumes are small and can, therefore have large terms of trade effects. The aggregate of these small trade volumes with other trade flows takes

6 Each group of regions can contain one or more regions, or all regions can be defined as within a single group.

7 Switching off the CET function allows the model to function in a similar manner to the GTAP model.

8 This is different to the GTAP model where the elasticities are only commodity specific.

places at the level of the aggregate import commodity. Hence the optimal ratios of imports to domestic commodities and exports to domestic commodities are determined by first order conditions based on relative prices. The price and quantity systems are described in greater detail below.

Most commodity and activity taxes are expressed as *ad valorem* tax rates, while income taxes are defined as fixed proportions of household incomes. Import duties and export taxes apply to imports and exports, while general sales taxes[9] are applied to all domestic absorption, i.e., imports are subject to sequential import duties and general sales taxes, and VAT is applied to household demand. Two import duties are identified; those charged *ad valorem* on import values and those charged on the volume of imports. Production taxes are levied on the value of output by each activity, while activities also pay taxes on the use of specific factors. Factor income taxes are charged on factor incomes after allowance for depreciation after which the residual income is distributed to households. Income taxes are taken out of household income and then the households are assumed to save a proportion of disposable income. This proportion is either fixed or variable according to the closure rule chosen for the capital account.

The treatment of taxes levied on selected commodities diverges from how these were accommodated in previous versions of ANARRES. The selected commodities are those for which the underlying GTAP database reports non-trivial differences in the commodity tax rate by different purchasing agents. These taxes are levied as *ad valorem* uplifts over and above the uplifts on basic prices due to other commodity taxes (GST and VAT); consequently, the purchaser prices for these commodities are purchasing agent specific.[10] These taxes can be levied on commodities that are included in the 'intermediate' or 'primary' input aggregates according to how the production function nests are defined.

Government expenditure consists of commodity (final) demand, which is assumed to be in fixed proportions in real/volume terms.[11] Hence government saving, or the internal balance, is defined as a residual in the default settings. However, the closure rules for the government

9 Typically, it is recommended that the GST rates are unchanged and treated as distortions in the GTAP database and hence in the ANARRES model.

10 The LOOP is preserved IF there is no scope for profitable arbitrage between different purchasing agents.

11 In some models it is assumed that the government demand can be represented using a utility function (CD, CES, LES, etc.). Such a representation presumes that the choice of consumption by the government obeys similar 'rules' as that by households! The implicit 'utility' functions for government are Leontief.

account allow for multiple permutations. In the base case it is assumed that the tax rates and volume of government demand are fixed and government savings are calculated as a residual. However, the tax rates can all be adjusted using various forms of scaling factors; hence for instance the value of government savings can be fixed and one of the tax scalars, multiplicative or additive, can be made variable thereby producing an estimate of the constrained optimal tax rate; the choice of instrument is region specific. If an analyst wishes to change the relative tax rates across commodities (for import duties, export taxes and sales taxes) or across activities (for production taxes) then the respective tax rate parameters can be altered via a second adjuster. Equally the volume of government consumption can be changed by adjusting the closure rule with respect the scaling adjuster attached to the volumes of government consumption. The patterns of government expenditure are altered by changing the parameters that controls the pattern of government expenditure (*qgdconst*).

Total savings come from the households, the internal balance on the government account and the external balance on the trade accounts. The external balance is defined as the difference between the value of total exports and total imports, i.e., it is defined as the trade balance,[12] converted into domestic currency units using the exchange rate. The model includes equations and variables that encompass international current account transactions. The base settings for the model assume that the exchange rates are flexible and hence that the external balances are fixed. Alternatively, the exchange rates can be fixed, and the external balances can be allowed to vary. Expenditures by the capital account consist solely of commodity demand for investment. The base settings for the model assume that the savings rates are fixed and that the volumes of investment adjust to clear the account, i.e., total expenditures on investment are equal to total savings. The patterns of investment volume are fixed, and hence the volume of each commodity changes equiproportionately according to the total values of investment expenditures, i.e., the implicit production functions for new capital goods are Leontief. It is possible to fix the volumes of real investment and then allow the savings rates, by households, to vary to maintain balances in the capital account, and it is possible to change the patterns of investment by changing the investment parameters (*qinvdconst*).

[12] The GTAP databases does not record other international current account transactions. An implication is that the GTAP model specification presumes all other international current account transactions are fixed in the numéraire currency units.

Table 3 **Behavioural Relationships for a Global CGE Model**

	Commodities	Activities	Factors	Households	Government	Capital	Margins	Rest of World	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Stone-Geary Utility Functions	Fixed Exogenously	Fixed Shares of Savings	n-Stage CET Functions	n-Stage CET Functions	Consumer Commodity Price
Activities	Total Supply from Domestic Production	0	0	0	0	0	0	0	Activity Prices
Factors	0	n-stage CES Production Functions	0	0	0	0	0	0	Factor Prices
Households	0	0	Fixed Shares of Factor Income	0	0	0	0	0	
Government	*Ad valorem* tax rates Specific Tax rates	*Ad valorem* tax rates on Output and Factor Use	Average tax rates	Average tax rates	0	0	0	0	
Capital	0	0	Shares of Factor Incomes	Shares of household income	Government Savings (Residual)	0	Current Account 'Deficit' on Margins Trade	Current Account 'Deficit	
Margins	Fixed Technical Coefficients	0	0	0	0	0	0	0	
Rest of World	n-Stage CES Functions	0	0	0	0	0	0	0	
Prices	Producer Prices Domestic and World Prices for Imports	Value Added Prices							

3.2 Price and Quantity Systems for a Representative Region

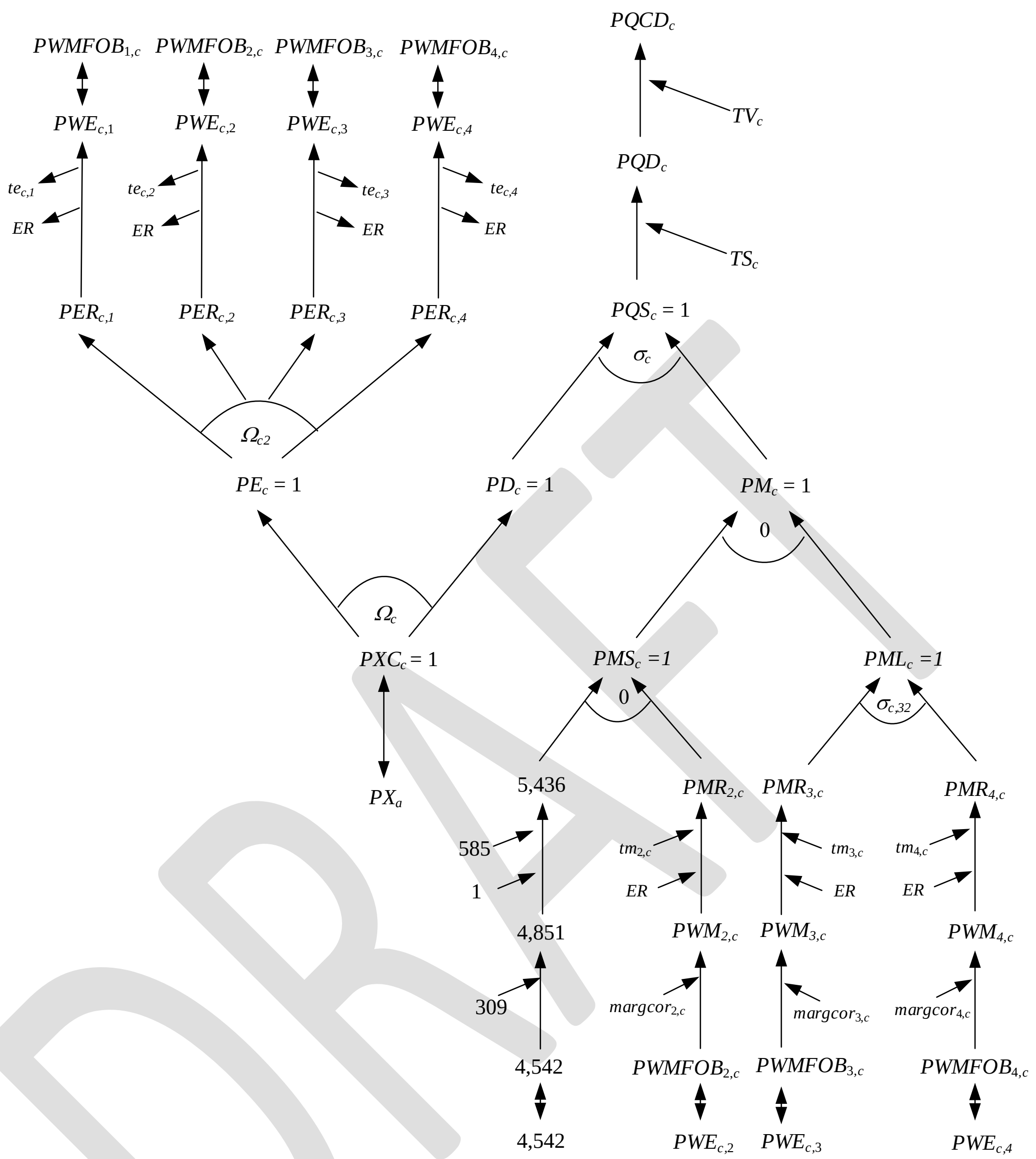
3.2.1 Price System

The price system is built up using accounting identities underpinning the SAM database, i.e., the principle that the components of the 'price definitions' for each region are the entries in the columns of the SAM. Hence there are a series of explicit accounting identities that define the relationships between the prices and thereby determine the processes used to calibrate the tax rates for the base solution. However, the model is set up using a series of linear homogeneous relationships and hence is only defined in terms of relative prices. Consequently, as part of the calibration process it is necessary set some of the prices equal to one (or any other number that suits the modeller). This model adopts the convention that prices are normalised at the level of the CES and CET aggregator functions *PQS*, the supply price of the domestic composite consumption commodity and *PXC*, the producer price of the composite domestic output. This is the nearest equivalent to basic prices in the GTAP database. The price system for a typical region in a 4-region global model is illustrated by Figure 1 – note that this representation abstracts from the GILD region.

The relationships between the various prices in the model are illustrated in Figure 1. The domestic consumer prices (*PQD*) are determined by the domestic prices of the domestically supplied commodities (*PD*) and the domestic prices of the composite imports (*PM*), and by the sales taxes (*TS*) that are levied on all domestic demand. The purchaser price paid by households on final consumption (*PQCD*) is further increased by the VAT rates (*TV*) that are only levied on household final demand: the purchaser price paid by households on final consumption will be further increased by household specific taxes on some commodities. The prices of the composite imports are determined as aggregates of the domestic prices paid for imports from all those regions that supply imports to this economy (*PMR*) under the maintained assumption that imports are differentiated by their source region. If the quantity imported from the source region is a 'large' share of the commodity imported, then the composite import price (*PML*) is a CES aggregate of the prices from the source regions. On the other hand if the quantity imported from the source region is a 'small' share of the commodity imported then the composite import price (*PMS*) is a Leontief aggregate of the prices from the source regions.[13] The user can adjust the definition of a 'small' source region when configuring the model; the definition of a 'large' source region is then defined as the complement. The region-specific import prices are expressed in terms of the domestic currency units after paying for trade and transport services and any import duties. Thus, a destination region is assumed to purchase a commodity in a source economy where the price is defined in "world dollars" at the basket exchange rate and is valued free on board (*fob*), i.e., *PWMFOB*. The carriage insurance and freight (*cif*) price (*PWM*) is then defined as the *fob* price plus trade and transport margin services (*margcor*) times the unit price of margin services (*PT*). The *cif* prices are related to the domestic price of imports by the addition of any import duties (*TM*) and then converted into domestic currency units using the nominal exchange rate (*ER*).

[13] The impact of adding an additional level of nesting is explored in McDonald and Thierfelder (2006).

Figure 1 **Commodity Price System for a Typical Region**



The prices (basic) for commodities sold by activities (*PXC*) are determined by the domestic prices (*PD*) and the composite export prices (*PE*). The composite export prices are CET aggregates of the export prices received by the source economy for exports to specific destinations (*PER*). The prices of the composite exports are determined as aggregates of the domestic prices paid for exports by all those regions that demand exports from this economy under the maintained assumption that exports are differentiated both by their destination region and the 'regional' group that the destination region is part of; hence there is a two-stage sub aggregation process whereby exports to like groups so regions are aggregated to form intermediate aggregates of exported commodities, which are then aggregated to form the composite export commodities. This allows for a degree of differentiation by both destination and commodity. The prices paid by the destination regions (*PWE*) are net of export taxes (*TE*) and are expressed in the currency units of the model's reference region by use of the nominal exchange. Notice how the export prices by region of destination (*PER*), and the intermediate aggregates, are all normalised on 1, but the seeming counterpart of normalising import prices by source region (*PMR*) are not normalised on 1. The link between the regions is therefore embedded in the identification of the quantities exchanged rather than the normalised prices and is a natural consequence of the normalisation process. The CET function can be switched off so that the domestic and export commodities are assumed to be perfect substitutes.[14]

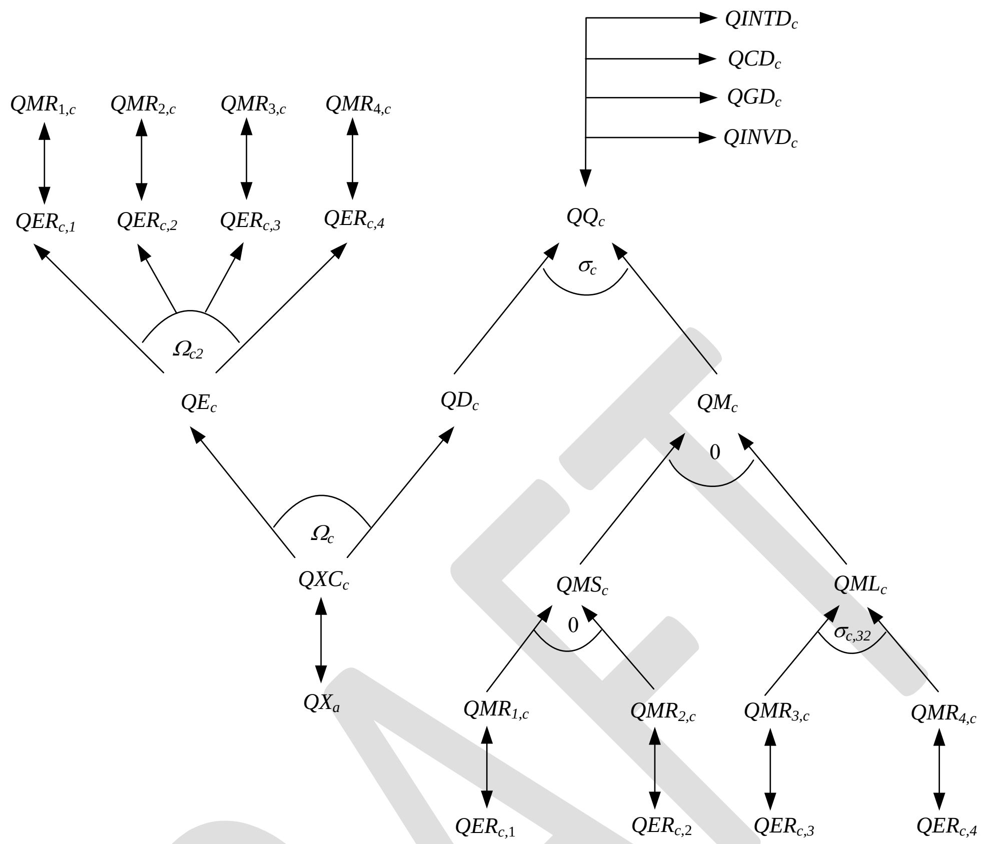
The price system also contains a series of equilibrium identities. Namely the *fob* export price (*PWE*) for region *x* on its exports to region *y* must be identical to the *fob* import price (*PWMFOB*) paid by region *y* on its imports from region *x*. These equilibrium identities are indicated by double headed arrows.

3.2.2 Quantity System

The quantity system for a representative region is somewhat simpler. The composite consumption commodity (*QQ*) is a mix of the domestically produced commodity (*QD*) and the composite import commodity (*QM*), where the domestic and imported commodities are imperfect substitutes, and the imported commodities are differentiated by their source region via a two-stage sub aggregation process whereby imports are differentiated by reference to their shares in the imports of that commodity by the destination region. The composite imported commodity is a Leontief aggregate of the composite imports from regions with 'small' (*QMS*) and 'large' (*QML*) import shares. *QMS* is a Leontief aggregate of the imports from source regions with 'small' import share while *QML* is a CES aggregate of imports from source regions with 'large' import shares. The equilibrium conditions require that the quantities imported from different regions (*QMR*) are identical to the quantities exported by other regions to the representative region (*QER*).

14 This is the assumption in the GTAP model and is an option in this model. However, before doing so it should be noted that the import CES ('Armington') may need increasing to avoid large terms of trade effects.

Figure 2 Quantity System for a Typical Region



The composite consumption commodity is then allocated between commodities used in the intermediate or value added components of the production (see below) (*QINTD*), private consumption demand (*QCD*), government demand (*QGD*) and investment demand (*QINVD*).

On the output side the model allows for multi-product activities. Domestic supply of each commodity (*QXC*) is defined as the aggregation, across activities, of the production of that commodity by activities (QXAC) and domestic output (*QX*) is defined as the aggregation, across commodities, of the production of commodities by each activity. Domestically produced commodities (*QXC*) are then allocated between the domestic market (*QD*) and composite export commodities (*QE*) under the maintained assumption of imperfect transformation. Exports are allocated between the different destination regions (*QER*) under the maintain assumption of imperfect transformation.

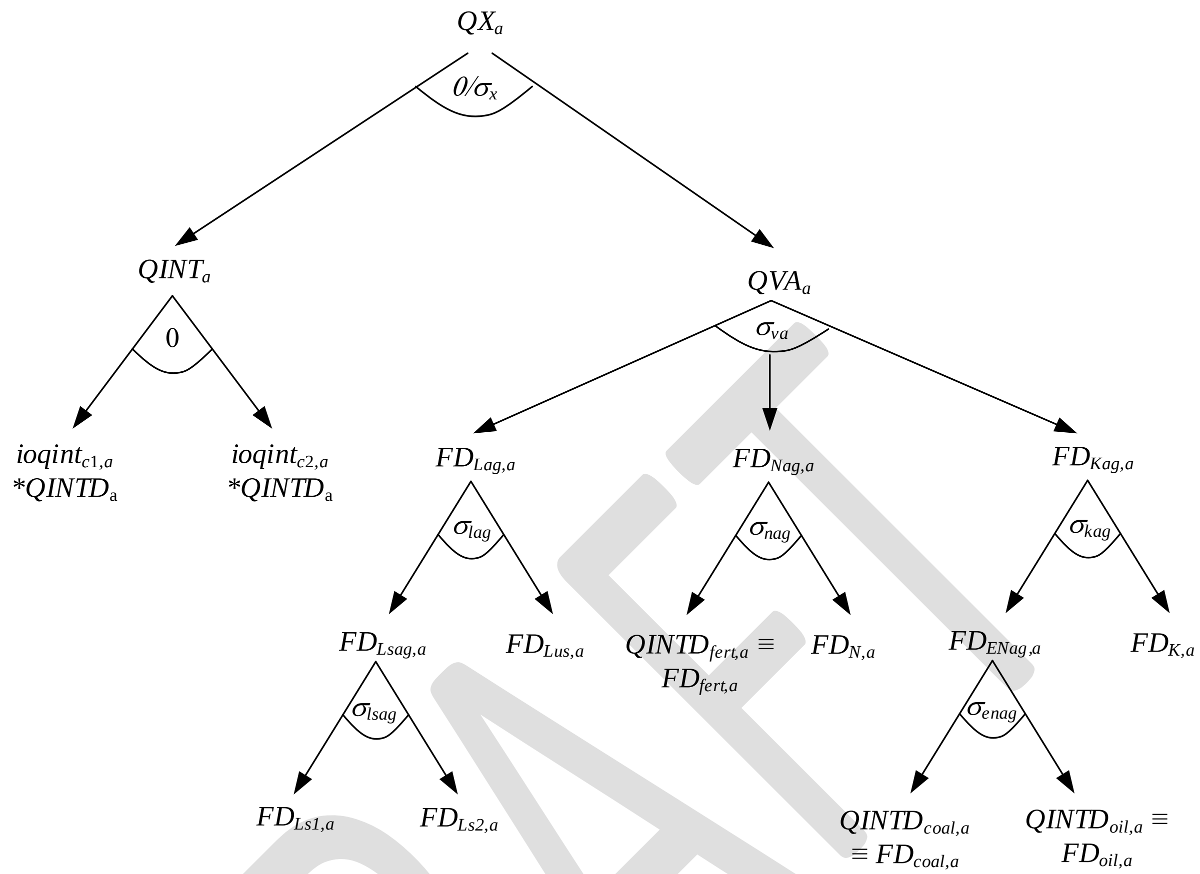
3.2.3 Production System

The production system is set up as a flexible *n*-level nest of CES production functions, with the option of Leontief functions at various levels: the system can be different for each activity, although the standard configuration presumes that the same nesting structure applies for each activity in all regions. The system is controlled by assigning the memberships of several sets when calibrating the model. This flexibility in the specification of the production system has advantages and disadvantages. On the plus side, it allows the use of one core set of model codes in various contexts, e.g., land fertiliser substitution in agriculture, modelling of water as a produced product and substitution between energy inputs in an energy model. There are negative considerations: first, the model code is more complex and less transparent; second, the set assignments are potentially error prone and ensuring the planned system is the implemented system is less easy; and third, a wider range of options may encourage over-elaboration and hence less clarity in the results.

It is important to note that in any nested system, with aggregates being created at the various levels of the system, that the aggregates have no 'real' world counterparts. The aggregates are constructs that exist solely to facilitate different degrees of substitutability between the various natural factors and, if in the specification, factor commodities in the system. No one owns these aggregates and the associated prices are only required to provide solutions to the first-order conditions. Moreover, since these aggregates are not 'real' and are not owned they cannot pay taxes, and hence their prices cannot be tax ridden, except to the extent that they encompass taxes paid on natural factors and factor commodities.

Moreover, the plethora of different production systems means that the expositional simplicity associated with a fix production system is lost. Hence, the description here is generic. The system described is for one activity: the system may or may not be same for all activities in a region. The system assumes that the activity uses 5 commodity inputs and five natural/primary factors; three of the commodity inputs will be included in the value-added nest and therefore will be known as factor commodities (*fc*), so the system will effectively have 8 factors (*ff*) and 2 intermediate (commodity) inputs.

Figure 3 Production Quantity System for a Typical Region



Starting with the quantity system illustrated in Figure 3. At the top level aggregate intermediate inputs (*QINT*) are combined with aggregate factor inputs (*QVA*) to produce the output of an activity (*QX*). This top level production function can take either CES or Leontief form, with CES being the default and the elasticities being activity **and** region specific.[15] Aggregate intermediate inputs are a Leontief aggregation of the individual commodities used as intermediate inputs, i.e., those commodities that are not factor commodities, where the input-output coefficients (*ioqint*) are defined in terms of input quantities relative to the aggregate intermediate input.[16] The value added production function is a CES function over **aggregate** capital (*Kag*), **aggregate** land (*Nag*) and **aggregate** labour (*Lag*), with the elasticities being activity **and** region specific: these aggregates are created at the third level.

15 The model allows the user to specify the share of intermediate input cost in total cost below which the Leontief alternative is automatically selected. The user also has the option to make activity and region-specific decisions about the selection of CES or Leontief forms.

16 A more flexible specification, with substitution possibilities between intermediate inputs, exists but is not included in this version of ANARRES.

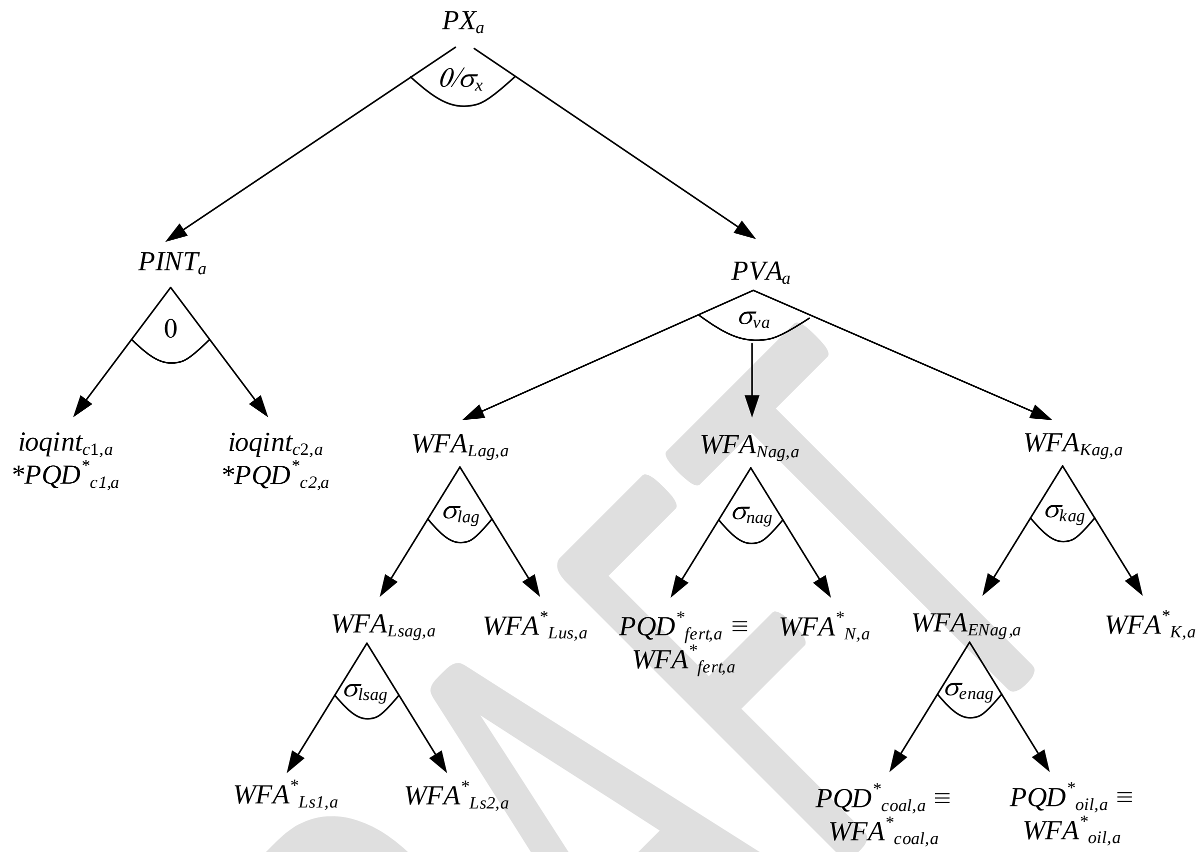
Aggregate land is defined as a CES aggregate of the natural factor land (*N*)[17] and the commodity factor fertiliser (*fert*), which is assumed to be a land saving input. The nests for aggregate labour and capital have two levels of nest. Aggregate labour (*Lag*) is defined as a CES aggregate of the natural factor unskilled labour (*Lus*) and aggregate skilled labour (*LSag*), and aggregate skilled labour is defined as a CES aggregate of two types of skilled labour (*Ls1* and *Ls2*); the elasticities are activity, level **and** region specific. Finally, aggregate capital (*Kag*) is defined as a CES aggregate of the natural factor capital (*K*) and aggregate energy (*ENag*), and aggregate energy is defined as a CES aggregate of two types of fossil fuels (*coal* and *oil*); the elasticities are activity, level **and** region specific.[18]

In the price system for production, the price of aggregate intermediate commodities is defined as the input (coefficient) weighted average of the intermediate commodities prices (*PQD**) where the * indicates that the price is the common purchaser (*PQD*) plus any agent commodity specific taxes. The price of value-added (*PVA*) is determined by the share weighted average of the prices of the three (labour, capital and land) aggregate inputs (WFA); note that these aggregate inputs are not subject to any taxes (hence no *). The price of aggregate land is determined by the by the share weighted average of the prices of the factor commodity (*fert*) and the natural factor land, where *PQD** is the price of the factor commodity, which is composed of the common purchaser price (*PQD*) plus any agent commodity specific taxes, as indicated by the *. The price of aggregate capital is determined by the share weighted average of the prices of the natural factor capital (*WFA*) and the price of aggregate energy input (WFA_{ENag}), which is a share weighted average of the prices of the factor commodities coal and oil capital. Again, and asterisks indicate that a price includes any agent specific taxes. In a similar manner, the price of aggregate labour is determined by the share weighted average of the prices of the natural factor unskilled labour (*WFA*) and the price of aggregate skilled labour (*WFA*), which is a share weighted average of the prices of the two different types of skilled labour (*Ls1* and *Ls2*). actor commodities coal and oil capital.

17 ‘Natural’ land may be made up of various ‘types’ of land, e.g., Agro-ecological zones.

18 In practical applications it would be expected that the aggregate energy nest is more complex, e.g., aggregate energy is formed from electricity and non-electricity inputs with non-electricity inputs being formed from fossil fuels.

Figure 4 A Production Price System for a Typical Region



A critical feature of the price system is the distinction between the prices paid by activities and prices received by the natural factors and commodity factors. The price paid by the activities, indicated by an *, are the relevant prices for the selection of commodity input mixes, i.e., the prices that enter the respective first-order conditions. Whereas the prices received are those that are important to decision making by the agents that own the factors.

The price of activity output (*PX*) a share weighted average of the prices of aggregate intermediates (*PINT*) and aggregated natural factors and commodity factors (*PVA*), uplifted by any the production tax rates (*TX*). The activity prices are a one to one mapping of the commodity prices received by activities (*PXC*); this is a consequence of the supply matrix being a square diagonal matrix.

3.2.4 Commodity and Activity Output System

The model features multi-product activities and commodities produced by multiple activities[19]. The composite supplies of each commodity (*QXC*) are aggregates of the commodity outputs by each activity (*QXAC*). The default assumption is that when a commodity is produced by multiple activities it is differentiated by reference to the activity that produces the commodity; this is achieved by defining total production of a commodity as a CES aggregate of the quantities produced by each activity. An alternative specification for commodity aggregation is provided where commodities produced by different activities are modelled as perfect substitutes to accommodate circumstances where perfect substitution may be a more appropriate assumption given the characteristics of either or both activity and commodity accounts.

Finally, it is necessary to determine the quantities of each commodity produced by each activity. There are two basic assumptions included in the model: first that secondary commodities are produced with pure by-product technologies, i.e., in a fixed ratio to the principal product, and second that activities can adjust their output mix in response to changes in the prices of the commodities they produce employing CET technology.

3.2.4 Utility System

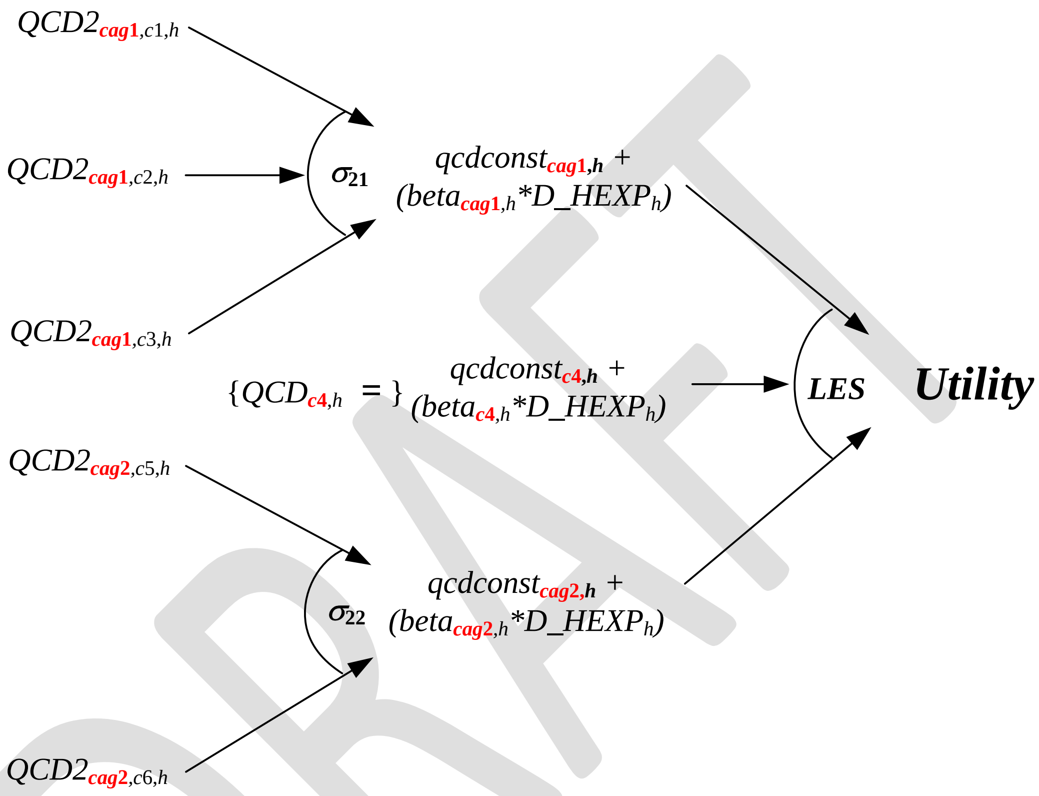
The nested (CES-LES) utility functions have a linear expenditure system (LES) defined over a mix of natural and aggregate commodities. This is illustrated in quantity terms in Figure 5 where the subscript '*cag#*' indicates an aggregate commodity and the subscript '*c#*' a natural commodity. The underlying logic is that each household demands subsistence quantities of certain aggregate commodities, e.g., food, energy, etc., but not necessarily of all natural commodities, e.g., meat, gas, etc. Thus, the LES utility functions for each household are defined over a mix of aggregate and natural commodities demands for which there are subsistence quantities (*qcdconst*) and marginal budget shares (*beta*) of discretionary household consumption expenditures (*D_HEXP*).

The aggregate commodities are CES aggregates of various natural commodities that are demanded to generate the aggregate. Since each household, *h*, has different preferences, as

[19] While the standard GTAP database has a 1:1 mapping between activities and commodities, this feature allows to include additional data, e.g., detail in the energy market when employing GTAP Power database.

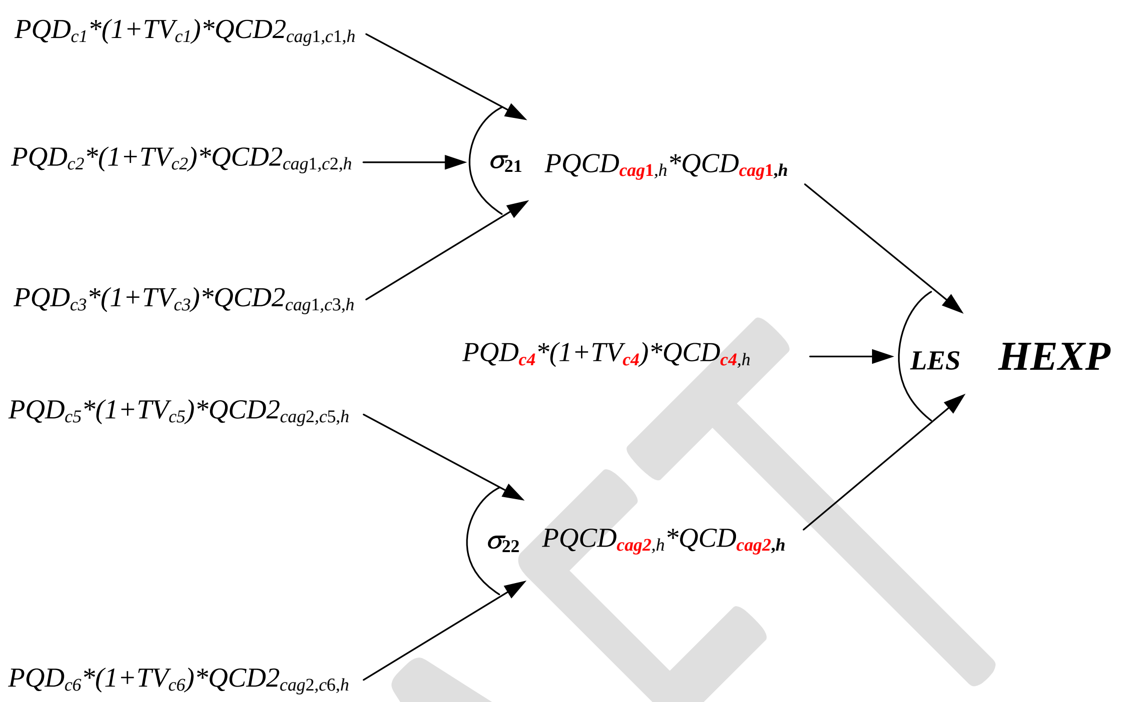
disclosed by the data, the quantities of each commodity, *c*, used to generate an aggregate, *cag*, the demand for each commodity (*QCD*2) has three arguments. As illustrated in Figure 5 the system is general in the sense that any number of commodities can be used to generate each aggregate and there can be any mix of aggregates and natural commodities in the LES.

Figure 5 Utility Functions in Quantities



The Law of One Price (LOOP) must however be retained. Thus, despite the demand for commodities by each household depending on *c* and *h* the prices paid are only determined by the commodity *c*. However, since the mix of commodities in each aggregate commodity varies by household because the quantities of each natural commodity, the weights, are different for each household. Consequently, the aggregate prices (*PQHD*) are indexed on both the aggregate commodity, *cag*, and the household, *h*. This is illustrated in Figure 6 where the components of the transaction values are identified.

Figure 6 Utility Functions in Transaction Values



3.2.5 Emissions System

The model includes relationships that record the emissions of Greenhouse Gases (GHG) and prices for GHGs. The current version of the model includes relationships for carbon dioxide (CO2), nitrous oxide (N2O), methane (CH$) and the carbon dioxide equivalent emissions of N2O and CH4: it can be easily extended to include other GHGs if the requisite data are available. The rates of emissions associated with each GHG are coefficients that identify the quantity of each GHG emitted for the quantity unit of each (commodity) input used by each agent (activities and households) that results in emissions. The GHG coefficients are therefore GHG, input, agent and region specific; the coefficients are parameters.

The prices for GHG gases are defined as taxes. The current formulation allows for separate taxes on nitrous oxide and methane and a compound tax instrument for carbon dioxide and cardon dioxide equivalents for nitrous oxide and methane. The tax instruments for emission follow the same conventions as other tax instruments with the addition of a mechanism that allows for uniform global rates/prices.

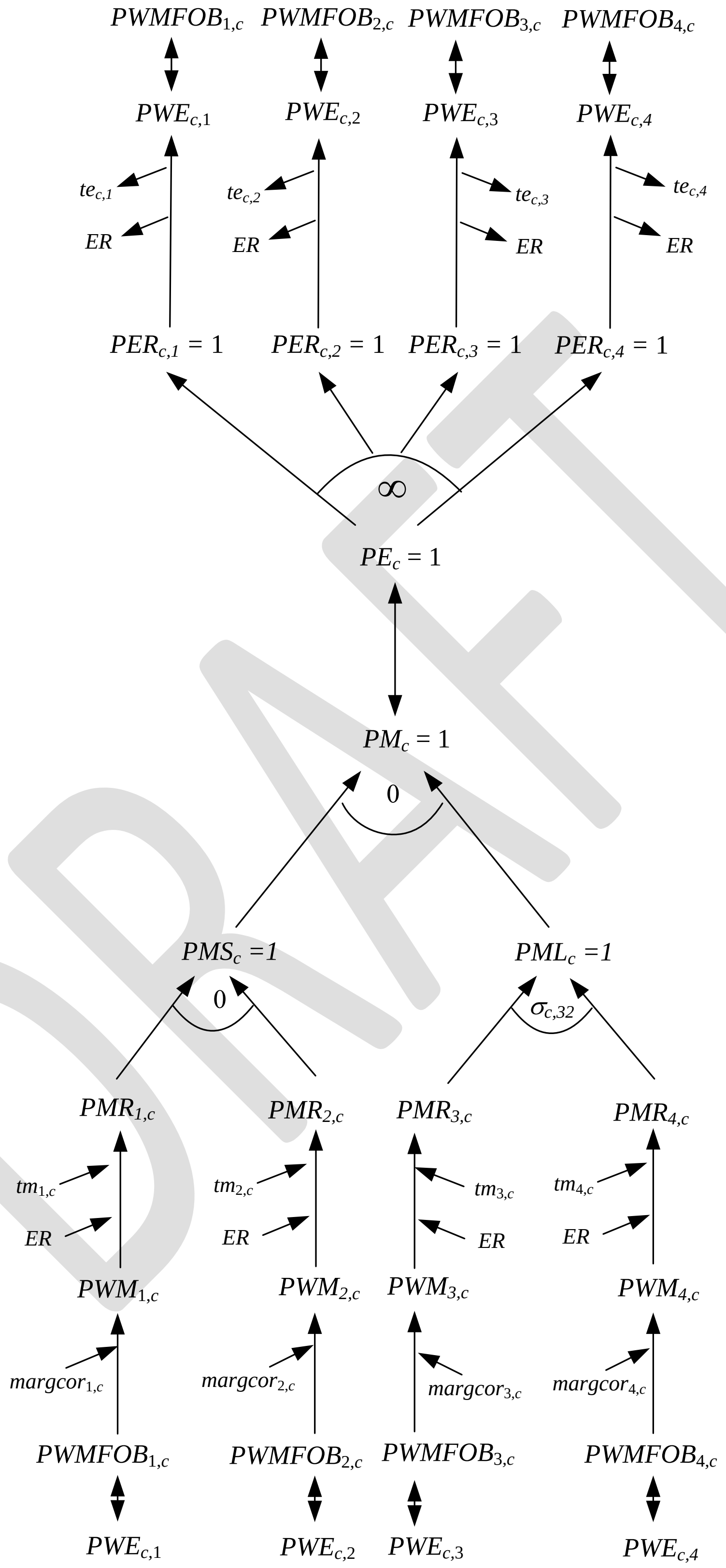
3.3 The GILD Region

An important feature of the model is the use of the concept of a region known as GILD. While the GTAP database contains complete bilateral information relating to the trade in commodities, i.e., in all cases transactions are identified according to their region of origin and their region of destination, this is not the case for trade in margins services associated with the transportation of commodities. Rather the GTAP database identifies the demand, in value terms, for margin services associated with imports by all regions from all other regions but does not identify the region that supplies the margin services associated with any specific transaction. Consequently, the data on the demand side for margin services are relatively detailed but the supply side are not. Indeed, the only supply side data are the total value of exports of margin services by each region. The GILD construct allows the model to get around this shortage of information, while simultaneously providing a general method for dealing with any other transactions data where full bilateral data are missing.

The price system for the GILD region is illustrated in Figure 5. On the import side GILD operates like all other regions. The commodities used in trade and transport services are assumed to be differentiated by source region and the proportion of imports accounted for by the source region. Thus, a two-level Leontief and CES aggregation nest is used. It is assumed that imports of trade and transport services can *potentially* incur trade and transport margins (*margcor*) and face tariffs (*TM*); in fact the database does not include any transport margins or tariff data for margin services in relation to the destination region, although they can, and do, incur export taxes levied by the exporting region.

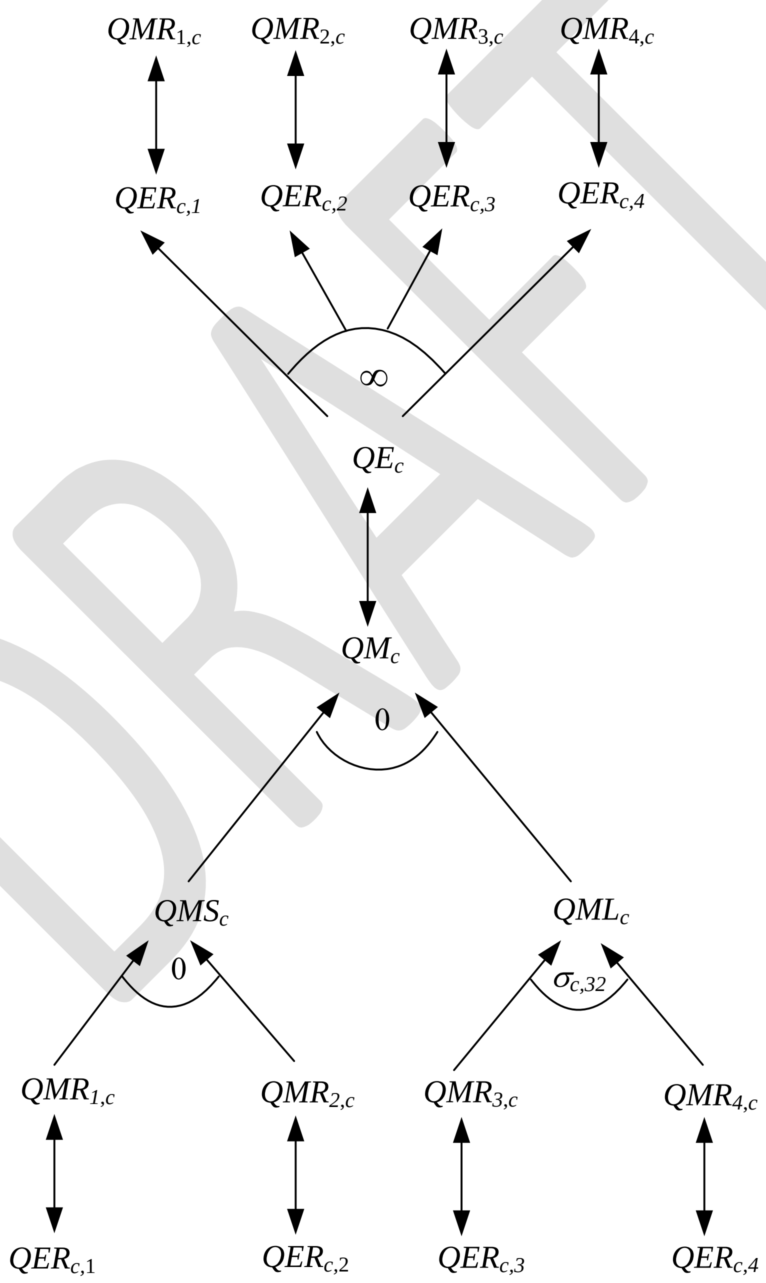
The export side is slightly different. In effect, the GILD region is operating as a method for pooling differentiated commodities used in trade and transport services and the only differences in the use of trade and transport services associated with any specific import are the quantities of each type of trade service used and the mix of types of trade services. Underlying this is the implicit assumption that each type of trade service used is homogenous and should be sold therefore at the same price. Hence the export price system for GILD needs to be arranged so that GILD exports at a single price, i.e., there should be an infinite elasticity of substitution between the exports to different regions of each type of trade service. Therefore, the average export price (*PE*) should equal the price paid by each destination region (*PER*), which should equal the export price in world currency units (*PWE*) and will be common across all destinations (*PT*).

Figure 7 **Price System for the GILD Region**



The linked quantity system contains the same asymmetry in the treatment of imports and exports by GILD (see Figure 7). The imports of trade and transport commodities are assumed to be differentiated by region and the proportion of imports accounted for by the source region, hence the elasticity of substitution is greater than or equal to zero but less than infinity, while the exports of trade and transport commodities are assumed to be homogenous and hence the elasticities of transformation are infinite.

Figure 6 Quantity System for the GILD Region



One consequence of using a GILD region for trade and transport services is that GILD runs trade balances with all other regions. These trade balances relate to the differences in the values of trade and transport commodities imported from GILD and the value of trade and transport commodities exported to GILD; however, the sum of GILD 's trade balances with other regions must be zero since GILD is an artificial construct rather than a real region. But the demand for trade and transport services by any region is determined by technology, i.e., the coefficients *margcor*, and the volume of imports demanded by the destination region. This means that the prices of trade and transport commodities only have an indirect effect upon their demand – the only place these prices enter the import decision as a variable is as a partial determinant of the difference between the *fob* and *cif* valuations of other imported commodities. Consequently, the primary market clearing mechanism for the GILD region comes through the quantity of trade and transport commodities it chooses to import.

The GILD concept has other potential uses in the model. All transactions between regions for which there is an absence of full bilateral information can be routed through the GILD region. While this is not a 'first best' solution, it does provide a 'second best' method by which augmented versions of the GTAP database can be used to enrich the analyses of international trade in a global model prior to availability of full bilateral transactions data (see McDonald and Sonmez (2006) for and application).

4: Formal Description of the Model

This formal description of the model proceeds in five stages with three of them in this section and the fourth, relating to model closure rules, being detailed in the next section. The fifth stage is in series of Appendices the that provide additional details. Appendix A1 briefly outline the genealogy of the ANARRES model. Alternative structures for the production system are given in Appendices A2 and A3. Appendix A4 provides a tabulated listing of the equations and dependent variables in the model together with counts for the equations and variables.

This section begins with a description the sets used in the model. This is followed by algebraic statements of each equation (block) in the model, organised by theme, together with descriptions of each equation that identifies the intentions behind the formulation and their roles in the model. The macroeconomic closure conditions and the labour market clearing conditions, which are important components of the model, are detailed and discussed in section 5.

4.1 Model Sets

GAMS is a set-based programming language that allows all behavioural relationship to be defined over sets, which means each equation block only needs to be specified once. The natural choices for this model are a set for all the transactions by each region's SAM (*sac*) and a set for each region (*r*). Subsets for each region that group commodities, activities, factors, import duties, export taxes, trade margins, trade and finally some individual accounts relating to domestic institutions. The outer set for any region is defined as

$$sac = \{c, a, f, h, tmr, tmrs, ter, tff, g, i, owatpmarg, w, total\}$$

and the following are the basic sets for each region in this model

$$cs(sac) = \{\text{natural and aggregate commodities}\}$$
$$c(cc) = \{\text{natural commodities}\}$$
$$a(sac) = \{\text{activities}\}$$
$$ff(sac) = \{\text{natural and aggregate factors}\}$$
$$f(ff) = \{\text{natural factors}\}$$
$$h(sac) = \{\text{households}\}$$
$$a\_h(sac) = \{\text{activities and households}\}$$
$$tmr(sac) = \{\textit{ad valorem}\text{ import duties}\}$$
$$tmrs(sac) = \{\textit{specific}\text{ import duties}\}$$
$$ter(sac) = \{\text{export taxes}\}$$
$$tsd(sac) = \{\text{esales tax adjustments for domestic demand}\}$$
$$tff(sac) = \{\text{factor use taxes}\}$$
$$g(sac) = \{\text{saltax, vattax, prodtax, facttax, dirtax, govt}\}$$
$$i(sac) = \{\text{kap}\}$$
$$owatpm\arg(sac) = \{\text{trade and transport margins}\}$$
$$w(sac) = \{\text{rest of the world - trade partners and aggregates}\}$$

Various subsets of *a* and *c* are declared and then assigned on the basis of certain characteristics of the data set used to calibrate the specific implementation of the model, so-called dynamic sets. The subsets of *a* used in the model are

$$acx(a,r) = \{\text{activities purchased domestically}\}$$
$$acxn(a,r) = \{\text{activities NOT purchased domestically}\}$$
$$aqx(\text{a,r}) = \{\text{Activities with CES function at Level 1 of nest}\}$$
$$aqxn(a,r) = \{\text{Activities with Leontief function at Level 1 of nest}\}$$
$$aleon(a) = \{\text{activities with Leontief top level prodn function}\}$$

while the subsets of *c* used in this model are

$$ct(c,r) = \{\text{trade margin commodities}\}$$
$$ctn(c,r) = \{\text{non-trade margin commodities}\}$$
$$ct2(c,r) = \{\text{trade margin commodities used for Globe}\}$$
$$ctn2(c,r) = \{\text{non-trade margin commodities used for Globe}\}$$
$$ce(c,r) = \{\text{export commodities}\}$$
$$cen(c,r) = \{\text{non-export commodities}\}$$
$$cer(c,r,w) = \{\text{export commodities by region}\}$$
$$cern(c,r,w) = \{\text{non-export commodities by region}\}$$
$$cm(c,r) = \{\text{imported commodities}\}$$
$$cmn(c,r) = \{\text{non-imported commodities}\}$$
$$cmr(w,c,r) = \{\text{imported commodities by region}\}$$
$$cmrn(w,c,r) = \{\text{non-imported commodities by region}\}$$
$$cmrs(w,c,r) = \{\text{small shares imported commodities by aggregate region}\}$$
$$cmrsn(w,c,r) = \{\text{non-small shares imported commodities by aggregate region}\}$$
$$cms(c,r) = \{\text{commodities small shares}\}$$
$$cmrl(w,c,r) = \{\text{large shares imported commodities by aggregate region}\}$$
$$cmrln(w,c,r) = \{\text{non-large shares imported commodities by aggregate region}\}$$
$$cml(c,r) = \{\text{commodities large shares}\}$$
$$cmrn2(c,r,w) = \{\text{non-imported commodities by aggregate region}\}$$
$$cles(cc) = \{\text{natural and aggregate commodities in the LES utility function}\}$$
$$cces(cc) = \{\text{natural commodities in the CES utility function}\}$$
$$cag(cc) = \{\text{aggregate commodities from the CES utility function}\}$$
$$cx(c,r) = \{\text{commodities produced domestically}\}$$
$$cxn(c,r) = \{\text{commodities NOT produced domestically AND imported}\}$$
$$cd(c,r) = \{\text{commodities produced AND demanded domestically}\}$$
$$cdn(c,r) = \{\text{commodities NOT produced AND demanded domestically}\}$$
$$cintd(c,r) = \{\text{commodities WITH intermediate demand by region}\}$$
$$cintdn(c,r) = \{\text{commodities WITHOUT intermediate demand by region}\}$$
$$cfa(c,a) = \{\text{commodity factors by commodity and activity}\}$$
$$cfan(c,a) = \{\text{NOT commodity factors by commodity and activity}\}$$

and a subset of w is needed to allow for GILD

$$wgn(w) = \{\text{Rest of world without Globe}\}.$$

It is also necessary to define a set of regions, r, for which there are two subsets

$$rgn(r) = \{\text{all regions excluding Globe}\}$$
$$ref(r) = \{\text{reference regions for global numeraire}\}$$
$$rleon(r) = \{\text{regions with Leontief top level prodn function}\}$$

A macro SAM that facilitates checking various aspects of model calibration and operation is used in the model and this needs another set, *ss*,

$$ss = \left\{ \begin{matrix} commdty, activity, valuad, hholds, \\ tmtax, tetax, tftax, tsddtax, govtn, \\ kapital, margs, world, totals \end{matrix} \right\}.$$

The model also makes use of a series of mapping sets are used to link sets. These can be divided into three groups; those that must be assigned ‘manually’, those that can be assigned (dynamically) from existing sets or subsets and those that control the operation of the production and utility systems. The ‘manually’ assigned ‘mapping’ sets are

$$map\_c\_w\_marg(c, w, owatpmarg) = \{\text{Trade margin mapping of owatpmarg to ct2 and w}\}$$
$$map\_marg\_w(owatpmarg, w) = \{\text{Trade margin mapping of w to owatpmarg}\}$$
$$map\_aagg\_a(aagg, a) = \{\text{Mapping from activities to aggregate activities}\}$$
$$map\_cagg\_c(cagg, c) = \{\text{Mapping from commodities to aggregate commodities}\}$$
$$map\_wagg\_w(wagg, w) = \{\text{Mapping from regions to aggregate regions}\}$$
$$map\_ragg\_r(ragg, r) = \{\text{Mapping from regions to aggregate regions}\}$$

Care is needed when using dynamically assigned sets. It is essential that the order of the two sets mapped to each other are identical since GAMS makes the assignments from the order of the elements in each set. The dynamically assigned ‘mapping’ sets are

$$map\_w\_tmr(w,tmr) = \{Ad\ \ valorem \text{ tariff mapping}\}$$
$$map\_tmr\_w(tmr,w) = \{Ad\ \ valorem \text{ tariff mapping reverse}\}$$
$$map\_w\_tmrs(w,tmrs) = \{Specific \text{ tariff mapping}\}$$
$$map\_tmrs\_w(tmrs,w) = \{Specific \text{ tariff mapping reverse}\}$$
$$map\_w\_ter(w,ter) = \{\text{Export tax mapping}\}$$
$$map\_ter\_w(ter,w) = \{\text{Export tax mapping reverse}\}$$
$$map\_c\_tsd(c,tsd) = \{\text{Commodities to sales tax adjustments}\}$$
$$map\_tsd\_c(tsd,c) = \{\text{Sales tax adjustments to commodities}\}$$
$$map\_f\_tff(f,tff) = \{\text{Factor taxes to factors}\}$$
$$map\_tff\_f(tff,ff) = \{\text{Factor taxes to factors reverse}\}$$
$$map\_cf\_fc(c,ff) = \{\text{From commodity factors to factor commodities}\}$$
$$map\_fc\_cf(ff,c) = \{\text{From factor commodities to commodity factors}\}$$
$$map\_r\_w(r,w) = \{\text{Region to trade partner mapping}\}$$
$$map\_w\_r(w,r) = \{\text{Region to trade partner mapping}\}$$

The sets required to control the production and utility systems need to be assigned manually; the difference between these assignments and those for other manually assigned sets is that the process is not (largely) mechanical but requires careful consideration of the economic interpretation. The maps to control the production and utility systems are

$$map\_va\_ff(ff,a) = \{\text{Mapping for arguments in Value added level nest}\}$$
$$map\_fagg\_ff(ff,ff,a) = \{\text{Mapping for aggregate arguments below value added level}\}$$
$$map\_agg(ff,ff,a) = \{\text{Mapping for aggregating factor transactions}\}$$
$$map\_cag\_cc(cc,cc) = \{\text{Mapping for aarguments in CES level nest}\}$$

Finally, various other sets are declared to facilitate model operation. These are

$$sacn(sac) = \{\text{SAM accounts without totals}\}$$
$$ssn(ss) = \{\text{Macro SAM accounts without totals}\}$$
$$cons = \{\text{set for programme control parameters}\}$$

4.1.1 Singleton Sets and Reserved Names

The model uses a few names that are reserved; these are

govt	Government
i_s	Investment-Savings
dstoc	Stock changes
total	Account totals

dirtax	Direct taxes
saltax	Sales taxes
vattax	Value added taxes
prodtax	production taxes

The model avoids needing to use these reserved names by using GAMS's singleton set feature. The assignment of elements to singleton sets is carried out in the file anar_single_set.inc; this file ONLY needs changing if the user chooses to change the reserved names.

4.1.2 Conventions

The equations for the model are set out in 11 'blocks' each of which can contain a number of sub blocks. The equations are grouped under the following headings:

1. TRADE BLOCK
    a. Exports Block
    b. Imports Block
    c. Trade margin quantities
2. COMMODITY PRICE BLOCK
3. NUMERAIRE PRICE BLOCK
4. PRODUCTION BLOCK
    a. Production
        i. Level 1
        ii. Level 2
        iii. Levels 3 to n
    b. Intermediate Input Demand
    c. Commodity Output
    d. Activity Output
5. FACTOR BLOCK
6. HOUSEHOLD BLOCK

   a. Household Income
   b. Household Expenditure
7. EMISSIONS BLOCK
8. GOVERNMENT BLOCK
   a. Government Tax Rates
   b. Government Tax Revenues
   c. Government Income
   d. Government Expenditure Block
9. KAPITAL BLOCK
   a. Savings Block
   b. Investment Block
10. MARKET CLEARING BLOCK
    a. Factor Accounts
    b. Commodity Accounts
    c. Commodity Trade Accounts
    d. Margin Trade Accounts
    e. Absorption
11. MACROECONOMIC AGGREGATES BLOCK
12. SLACKS BLOCK
13. CLOSURE and MARKET CLEARING CONDITIONS
    a. Foreign Exchange Market Closure
    b. Investment-Savings Closure
    c. Govt Closure Rules
    d. Factor Market Closure
    e. Miscellaneous Fixed Variables

This grouping is carried throughout the model code, i.e., it is followed for the parameter declaration and calibration, variable declaration and variable initialization sections. This modularization of the code is adopted for ease of reading and altering the model rather than being a requirement of the model.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modelling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- The standard prefixes for variable names are: *P* for price variables, *Q* for quantity variables, *W* for factor prices, *F* for factor quantities, *E* for expenditure variables, *Y* for income variables, and *V* for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix and are used to initialise variables and analyse results.
- A series of variables are declared that allow for the equiproportionate multiplicative adjustment of groups of variables. These variables are named using the convention ***ADJ*, where ** is the variable series they adjust.
- A series of variables are declared that allow for the additive adjustment of groups of variables. These variables are named using the convention *D***, where ** is the variable series they adjust.
- All parameters are in lower case, except those paired to variables that are used to initialise variables.
- Parameter names have a two- or five-character suffix which distinguishes their definition, e.g., ***sh* is a share parameter, ***av* is an average and ***const* is a constant parameter.
- For the CES functions all the share parameters are declared with the form *delta***, all the shift/efficiency parameters are declared with the form *ac***, and all the elasticity parameters are declared with the form *rho***, where ** identifies the function in which the parameter operates.
- For the CET functions all the share parameters are declared with the form *gamma***, all the shift/efficiency parameters are declared with the form *at***, and all the elasticity parameters are declared with the form *rho***, where ** identifies the function in which the parameter operates.
- All coefficients in the model are declared with the form *io*****, where **** consists of two parts that identify the two variables related by the coefficient.
- The index ordering follows the specification in the SAM: row, column, and then r to indicate the region. For example, exports from region *r* to region *w* would be $QER_{c,w,r}$ because region *r*'s export data in its SAM is found in the commodity row (*c*) and the trade partner column (*w*). Likewise, imports in region *r* from region *w*

are designated, $QMR_{w,c,r}$ because region *r*'s import data in its SAM is found in the trade partner row (w) and the commodity column (c).

- All sets have another name, or alias, given by the set name followed by "p". For example, the set of commodities may be called *c* or *cp*. Where more than one alias is required the set name is followed by "pp", etc.

4.2 Equations for the Model

The model equations are reported and described by blocks/groups below and then they are summarised in table A4 in the appendix.

4.2.1 Exports Block Equations

The treatment of exports is complicated by the incorporation of the facility to treat export commodities as imperfect and by the need to accommodate the special case of exports (of trade and transport services) that are homogenous from GILD. The presumption of imperfect substitution is the default presumption in this model; reasons for this decision being its symmetry with the Armington assumption on the imports side, the amelioration of the terms of trade effects associated with the Armington assumption and a belief that in general there is differentiation between commodities supplied to domestic and export markets. However, there are proponents of the arguments for treating exports as perfect substitutes and there are clearly cases where such an assumption may be appropriate, e.g., supplies of unprocessed mineral products.[20] A formulation of the model allows the CET functions to be switched off at either or both levels of the export supply nest for specific commodities and/or for specific regions.

The domestic prices of commodity exports, *c*, by destination, *w*, and source, *r*, region (*PER*) are defined as the product of world prices of exports (*PWE*) – also defined by commodity and destination and source region, the source region's exchange rate (*ER*) and one

[20] The GTAP model assumes perfect substitution and historically it has been argued that perfect substitution is appropriate for Australia. It has been argued that the use of CET functions for exports can be avoided by increasing the degree of disaggregation, however, as a rule, this is not appropriate for models calibrated to the GTAP database because the commodity accounts are fixed. This argument also lacks symmetry since the same argument could be adopted with respect to imports and the Armington assumption. There are however reasons to be cautious about the use of CET function (see McDonald, 2011).

minus the export tax rate (*TE*)[21] (E1). The definition of the domestic prices of commodity exports (*PER*) also includes an option for levying a tax (*TE_C*) on the greenhouse gases (GHG) embed in each unit of the exported commodity (*IO_EMIT_EMBED*) The possibility of non-traded commodities means that the equations for the domestic prices of exports (and imports) are only implemented for those commodities that are traded; this requires the use of a dynamic set, *cer*, which is defined by those commodities that are exported in the base data. Also notice that the concept of a single world price for a commodity is a valid concept since, by definition, world prices are endogenous and therefore ALL regions are treated as 'large' producers of a commodity. Hence an extreme version of the small country assumption is not supported but the shares of total exports of a commodity accounted for by each region will be a determinant of each region's impact on the average world price.

Export Block Equations 1

$$PER_{c,w,r} = \left\{ PWE_{c,w,r} * ER_r * \left(1 - TE_{c,w,r}\right) \right\} - \left\{ TE\_C_{c,w,r} * IO\_EMIT\_EMBED_{c,r} \right\} \quad \forall c \in cer \qquad \text{(E1)}$$

$$PE_{c,r} * QE_{c,r} = \sum_{w} PER_{c,w,r} * QER_{c,w,r} \qquad \forall c \in ce, r \in rgn \qquad \text{(E2)}$$

$$PER_{c,w,r} = PE_{c,r} \qquad \forall c \in ct2, r \notin rgn, w \in wgn \qquad \text{(E3)}$$

$$QXC_{c,r} = at_{c,r} \cdot \left(\gamma_{c,r} * QE_{c,r}^{\rho^t_{c,r}} + \left(1 - \gamma_{c,r}\right) * QD_{c,r}^{\rho^t_{c,r}} \right)^{1/\rho^t_{c,r}} \qquad \forall c \in \left(cd \cap ce\right), r \in rgn \qquad \text{(E4)}$$

$$QE_{c,r} = QD_{c,r} \left[\frac{PE_{c,r}}{PD_{c,r}} * \frac{\left(1 - \gamma_{c,r}\right)}{\gamma_{c,r}} \right]^{\frac{1}{\left(\rho^t_{c,r} - 1\right)}} \qquad \forall c \in \left(cd \cap ce\right), r \in rgn \qquad \text{(E5)}$$

$$QXC_{c,r} = QD_{c,r} + QE_{c,r} \qquad \forall \left(c \in \left(cd \cap cen\right), r \in rgn \right) \text{ OR } \forall \left(c \in \left(cdn \cap ce\right), r \in rgn \right) \qquad \text{(E6)}$$

The prices of the composite export commodities can then be expressed as simple volume weighted averages of the export prices by region, where $PE_{c,r}$ and $QE_{c,r}$ the price and

[21] In the GLOBE model ALL taxes are expressed as having positive values; any tax rate with a negative value is a subsidy rate.

quantity of the composite export commodity *c* from region *r*, and the weights are the volume shares of exports and are variable. This comes from the fact that a CET function is liner homogenous and hence Eulers theorem can be applied. Notice however that (E2) is only implemented of the set *rgn*, i.e., the region GILD is excluded. Rather, the composite export price for trade margin commodities from GILD is defined in X3, which indicates that it is assumed that the trade margin commodities exported by GILD are perfect substitutes for each other, i.e., the same price is paid for each trade margin commodity by ALL purchasing regions. The implicit presumption is that GILD produces margin commodities from purchases of specific commodities from multiple regions.

Domestic commodity outputs (*QXC*) are either exported (*QE*) as composite commodities or supplied to the domestic market (*QD*). The allocation of output between the domestic and export markets is determined by output transformation functions, Constant Elasticity of Transformation (CET) functions, (E4) with the optimum combinations of *QE* and *QD* determined by first-order conditions (E5). In this version of the model primal forms of the CET are used with associated first-order conditions. However, some commodities are non-traded and therefore X4 and X5 are implemented if and only if the commodity is traded. This means that domestic commodity outputs are undefined for non-traded commodities, but, by definition, the quantity supplied to the domestic market is the amount produced, and it is necessary to cover the possibility that a commodity may be produced domestically and exported but not consumed domestically. These two sets of possibilities are covered by X6.

Export Block Equations 2

$$QER_{c,w,r} = QE_{c,r} * \left(\frac{PER_{c,w,r}}{\left(\left(PE_{c,r} * \gamma^{r}_{c,w,r} * at^{r}_{c,r} \right)^{\rho^{e}_{c,r}} \right)} \right)^{\left(1 / \left(\rho^{e}_{c,r} - 1 \right) \right)} \quad \forall c \in cer2, r \in rgn \qquad \text{(E7)}$$

$$QE_{c,r} = \sum_{w} QER_{c,w,r} \quad \forall c \in ct, r \notin rgn . \qquad \text{(E8)}$$

$$QE_{c,r} = QM_{c,r} \quad \forall c \in ct2, r \notin rgn . \qquad \text{(E9)}$$

These quantity equations deal however only with the composite export commodities, i.e., hypothetical commodities whose roles in the model are to act as neutral intermediaries that enter into the first-order conditions that determine the optimal mix between domestic use and exports of domestic commodity production (E5). In the model the composite export commodities are themselves CET aggregates of commodity exports to different regions (*QER*), and the appropriate first order condition is given by (E7). Initially the formulation of E7 is not intuitive but, as demonstrated in Appendix A1, the formulation is a straightforward manipulation of a more conventional representation; this form is used because it improves model performance. Note however that (E7) does not define the exports of trade margin commodities BY GILD; this is because these commodities are assumed to be perfect substitutes and therefore simple addition is adequate, this is done by X8.

Finally, there is a need for equilibrium conditions for trade by GILD. Since GILD is an artificial construct whose sole role in the model is to gather exports, whose destinations are unknown and supply imports whose sources are unknown, and vice versa, it must always balance its trade within each period. Thus, the volume of exports of trade margin commodities by GILD must be exactly equal to the volume of imports of trade margin commodities, see X9.

4.2.2 Imports Block Equations

The prices of imported commodities are made up of several components. The export price in foreign currency units – valued free on board (*fob*) (*PWMFOB*) – plus the cost of trade and transport services, which gives the import price carriage insurance and freight (*cif*) paid (*PWM*), plus any import duties; all of which are then converted into domestic currency units (*PMR*). Clearly the import price value *fob* (*PWMFOB*) is identical to the export price valued *fob* (*PWE*) – this condition is imposed in the market clearing block (see below) – and hence the *cif* price is defined in M3, where *margcor* is the quantity of trade and transport services required to import a unit of the imported commodity and *PT* is the price of trade and transport services. Embedded in the definition of the coefficient *margcor* is the explicit assumption that transporting a commodity from a specific source to a specific destination requires the use of a specific quantity of services – the actual cost of these services can vary according to changes in the prices of the trade and transport services, or the quantity of services required to transport a particular commodity.

The domestic prices of imports from a region (*PMR*) are then defined as the product of world prices of imports (*PWM*) – after payment for carriage, insurance and freight (*cif*) - the exchange rate (*ER*) and one plus the *ad valorem* import tariff rate (*TM*) plus any (quantity) specific import duty (*TMS*). The possibility of non-traded commodities means that the equations for the domestic prices of imports are only implemented for those commodities that are traded; this requires the use of a dynamic set, *cmr*, which is defined by those commodities that are imported by a region from another region in the base data.

Import Block Equations 1

$$PML_{c,r} = \sum_{w \in cmrl_{w,c,r}} \left(PMR_{w,c,r} * QMR_{w,c,r}\right) \Big/ QML_{c,r} \qquad \forall (c,r) \in cml \qquad \text{(M1)}$$

$$PMS_{c,r} = \sum_{w} ioqmrqms_{w,c,r} * PMR_{w,c,r} \qquad \forall (c,r) \in cms \qquad \text{(M2)}$$

$$PWM_{w,c,r} = PWMFOB_{w,c,r} + \left(\sum_{cp \in ct_{cp,r}} margcor_{w,cp,c,r} * PT_{cp,r} \right) \qquad \forall c \in cmr \qquad \text{(M3)}$$

$$PMR_{w,c,r} = PWM_{w,c,r} * ER_{r} * \left(1 + TM_{w,c,r}\right) + TMS_{w,c,r} \qquad \forall c \in cmr\,. \qquad \text{(M4)}$$

$$PM_{c,r} * QM_{c,r} = \left(PML_{c,r} * QML_{c,r}\right) + \left(PMS_{c,r} * QMS_{c,r}\right) \qquad \forall (c,r) \in cm_{c,r} \qquad \text{(M5)}$$

The model treats imports that account for ‘small’ shares of imports of a commodity by a region differently from those that account for ‘large’ shares of imports of a commodity by a region.[22] This is because the operations of CES, and CET, functions depend not only on the elasticity of substitution but also on the shares of the arguments: commodities with ‘small’ trade shares can have a disproportionately large impact on the terms of trade in such aggregation functions. For commodities with ‘small’ trade shares it is assumed that they account for fixed proportions ($ioqmrqms_{w,c,r}$) of the total volume of imports of a commodity:

22 The definition of ‘small’, and hence by complement ‘large’, is selected by the user when calibrating the model.

the justification for this assumption rest upon a (vaguely defined) specific factor specification. The composite price of 'small' share imports (*PMS*) is therefore a quantity share weighted aggregate of the landed prices (M2). The composite price of 'large' share imports (*PML*) is a simple volume weighted averages of the of the import prices by region: this derives from the fact that a CES function is liner homogenous and hence Eulers theorem can be applied (M1).

Import Block Equations 2

$$QQ_{c,r} = ac_{c,r}\left(\delta_{c,r} * QM_{c,r}^{-\rho^c_{c,r}} + \left(1-\delta_{c,r}\right) * QD_{c,r}^{-\rho^c_{c,r}}\right)^{-\left(\frac{1}{\rho^c_{c,r}}\right)} \quad \forall c \in \left(cx \cap cm\right), r \in rgn \qquad \text{(M6)}$$

$$QM_{c,r} = QD_{c,r} * \left(\left(\frac{PD_{c,r}}{PM_{c,r}}\right) * \left(\frac{\delta_{c,r}}{\left(1-\delta_{c,r}\right)}\right)\right)^{\frac{1}{\left(1+\rho^c_{c,r}\right)}} \quad \forall c \in \left(cx \cap cm\right), r \in rgn \qquad \text{(M7)}$$

$$QQ_{c,r} = QD_{c,r} + QM_{c,r} \quad \forall c \in \left(cx \cap cmn\right), r \in rgn \ \text{ OR } \forall c \in \left(cxn \cap cm\right), r \in rgn \qquad \text{(M8)}$$

$$QMS_{c,r} = ioqmsqm_{c,r} + QM_{c,r} \quad \forall c \in cms \qquad \text{(M9)}$$

$$QML_{c,r} = ioqmlqm_{c,r} + QM_{c,r} \quad \forall c \in cml \qquad \text{(M10)}$$

The prices of the composite import commodities can also be expressed as a simple volume weighted averages of the of the import prices by region, (M5) where $PM_{c,r}$ and $QM_{c,r}$ are the price and quantity of the composite import commodity *c* by region *r*, and the weights are the volume shares of imports and are variable. Notice however that (M5) is only controlled by the set *cm*, in contrast to (X2) – the composite export price – which was also controlled by the set *rgn*, i.e., the region GILD was excluded. This reflects the fact that the region GILD does import commodities using the same trading assumption as other regions but only exports homogenous trade and transport services, which explains the need for the equation (X3).

The composite supply of the imported commodity (*QM*) is a Leontief aggregate of imports with ‘small’ (*QMS*) and ‘large’ (*QML*) shares using appropriately defined input-output coefficients (M9 and M10). Similarly, the quantities imported of the ‘small’ share commodities by source region are defined by fixed (input-output) coefficients (M11). The use of a two stage Leontief nest in this case ensures no substitution possibilities between the aggregate imports *QMS* and *QML*; if some substitution possibilities are required the nesting system used for production can be adapted.

The composite imports of commodities with ‘large’ shares are defined as CES aggregates of the imports from different regions (*QMR*) (M12). The first order conditions come from the price definition terms for composite imports, *PML* (M1) and are only implemented for those cases where there were import transactions in the base period – this is controlled by the set *cmr*. Initially the formulation of M12 is not intuitive but, as demonstrated in Appendix A1, the formulation is a straightforward manipulation of a more conventional representation; this form, which is effectively a set of first-order conditions, is used because it reduces the number of equations in the model.

Import Block Equations 3

$$QMR_{w,c,r} = ioqmrqms_{w,c,r} + QMS_{c,r} \qquad \forall c \in cmrs \qquad \text{(M11)}$$

$$QMR_{wagm,c,r} = QML_{c,r} * \left(\frac{PMR_{wagm,c,r} * acrag_{c,r}^{\rho_{c,r}^{mag}}}{PML_{c,r} * \delta_{wagm,c,r}^{r}} \right)^{\left(-1 / (\rho_{c,r}^{mag}+1) \right)} \qquad \text{(M12)}$$

$$\forall c \in cml, wagm \in cmr2$$

$$QMR_{w,c,r} = QMR_{wagm,c,r} * \left(\frac{PMR_{w,c,r} * acr_{wagm,c,r}^{\rho_{wagm,c,r}^{m}}}{PMR_{wagm,c,r} * \delta_{w,c,r}^{r}} \right)^{\left(-1 / (\rho_{wagm,c,r}^{m}+1) \right)} \qquad \text{(M13)}$$

$$\forall c \in cmrl, w \in cmr, map\_wagm\_m(wagm, w)$$

$$QT_{w,c,r} = \sum_{cp} \left(QMR_{w,cp,r} * margcor_{w,c,cp,r} \right) \qquad \forall c \in ct2, r \in rgn \qquad \text{(M14)}$$

The composite (consumption) commodities are then a mixture of composite imports (*QM*) and domestic demand from domestic production (*QD*). The mixtures between the domestic and import supplies are determined by the substitution functions, Constant Elasticity of Substitution (CES) functions, (M6) with the optimal combinations of *QM* and *QD* being determined by first-order conditions, (M7). But, some commodities are non-traded and therefore M6 and M7 are implemented only if the commodity is traded, which leaves *QQ* undefined for non-traded commodities. By definition if there are no imports the quantity demanded by the domestic market is the amount produced, and if there is no domestic production the amount demanded is the amount imported. These two sets of possibilities are both covered by M8.

Also associated with any imported commodity is a specific quantity of trade and transport services. These services are assumed to be required in fixed quantities per unit of import by a specific region from another specific region, (M13) where the *margcor* are the trade and transport coefficients associated with a unit (quantity) import by region *r* from region *w*. This is only implemented for trade and transport commodities (*ct*2) and for regions that 'actually' import goods (*rgn*).

4.2.3 Commodity Price Block Equations

The composite price equations (CP1, CP2, CP3, CP4, CP5 and CP6) are derived from the first order conditions for tangencies to consumption and production possibility frontiers. By exploiting Euler's theorem for linearly homogeneous functions the composite prices can be expressed as expenditure identities rather than dual price equations for export transformation and import aggregation, such that $PQS_{c,r}$ (the basic price) is the weighted average of the producer price of a commodity, when $PD_{c,r}$ is the producer price of domestically produced commodities and $PM_{c,r}$ the domestic price of the composite imported commodity, (CP1) where $QD_{c,r}$ the quantity of the domestic commodity demanded by domestic consumers, $QM_{c,r}$ the quantity of composite imports and $QQ_{c,r}$ the quantity of the composite commodity. Notice how the commodity quantities are the weights.

The basic price of the composite commodity (CP1) does not include any taxes, which create price wedges between the purchaser prices of a commodity ($PQHD_{c,h,r}$) and the producer prices ($PQS_{c,r}$). In this model a series of intermediate prices are declared and assigned, each of these prices includes a single price instrument as mark-ups on a previous price. This is not necessary for the operation of the model, but it simplifies the subsequent analyses of the results from simulation exercises.

Hence the commodity prices inclusive of value added taxes (VAT) are $PQCD_{c,r}$ (CP3), where VAT rates[23] are applied as *ad valorem*, mark-ups over the basic prices plus the sales taxes $PQD_{c,r}$ (CP2). Note how these prices transparently follow the LOOP.

Commodity Price Block Equations

$$PQS_{c,r} = \frac{\left(PD_{c,r} * QD_{c,r}\right) + \left(PM_{c,r} * QM_{c,r}\right)}{QQ_{c,r}} \qquad \forall c \in \left(cd \cup cm\right), r \in rgn \qquad \text{(CP1)}$$

$$PQD_{c,r} = PQS_{c,r} * \left(1 + TS_{c,r}\right) \qquad \forall c \in \left(cd \cup cm\right), r \in rgn \qquad \text{(CP2)}$$

$$PQCD_{c,r} = PQD_{c,r} * \left(1 + TV_{c,r}\right) \qquad \forall c \in \left(cd \cup cm\right), r \in rgn \qquad \text{(CP3)}$$

$$PQ\_TSDD\_D_{c,h,r} = PQCD_{c,r} * \left(1 + TSDD_{c,h,r}\right) \qquad \forall c \in \left(cd \cup cm\right), r \in rgn \qquad \text{(CP4)}$$

$$PQHD_{c,h,r} = PQ\_TSDD\_D_{c,h,r} + \left[TEMIT_{c,h,r}\right] \qquad \forall c \in \left(cd \cup cm\right), r \in rgn \qquad \text{(CP5)}$$

$$PXC_{c,r} = \frac{\left(PD_{c,r} * QD_{c,r}\right) + \left(PE_{c \in ce,r} * QE_{c \in ce,r}\right)}{QXC_{c,r}} \qquad \forall c \in cx, r \in rgn \qquad \text{(CP6)}$$

The database includes a series of additional sales taxes that are purchasing agent specific. The prices $PQ\_TSDD\_D_{c,h,r}$ includes these taxes ($TSDD_{c,h,r}$) as *ad valorem* mark-ups over the price inclusive of VAT (*PQCD*) (CP4). Note how this price is household specific, which reflects the fact the additional sales taxes are agent specific. The final purchaser prices for households ($PQHD_{c,h,r}$) are inclusive of emission taxes ($TCARB_{c,h,r}$, $TNOX_{c,h,r}$, $TMETH_{c,h,r}$) which are defined as quantity taxes levied on the quantity of the commodity consumed not its value (CP5). The nested utility functions for households also require a price definition for aggregate commodities; this is defined in the household block equations below.

The composite output price for a commodity, $PXC_{c,r}$, is also derived by exploiting Euler's theorem for linearly homogeneous functions, and is given by (CP6) where $PD_{c,r}$ is the

[23] Estimated value added taxes are derived as differences between the average commodity taxes levied on commodities purchased by non-household agents and the taxes levied on households.

domestic producer price for the output of commodities supplied to the domestic market, $QD_{c,r}$ is the supply of output to the domestic market, $QE_{c,r}$ is the quantity exported by activities, and $QXC_{c,r}$ is the quantity of domestic production by commodity.

4.2.4 Numéraire Price Block

The ANARRES model has (r+1) explicit numéraire; one for each region and one for exchange rates.[24]

It is necessary to define a price numéraire for each region; for this model two alternative region numéraire are defined to allow the modeller some discretion in the choice of numéraire.[25] The consumer price indices (CPI_r) are defined as base weighted sum of the commodity prices, where the weights are the value shares of each commodity in final demand ($vqcdsh_{c,r}$), (N1). An advantage of with using the CPI as the numéraire is that the transactions within a region are expressed in real terms without the need to adjust values for price changes.

While the domestic producer price indices (PPI_r) are defined as the weighted sums of the commodity prices received by producers on the domestic market, where the weights are the value shares of each commodity supplied by domestic producers to the domestic market ($vqdsh_{c,r}$), (N2). This provides a convenient alternative price normalisation term.

24 All global CGE models have (r+1) numéraire, although the r numéraires are usually implicit, e.g., the GTAP model assumes fixed nominal exchange rates.

25 The price index not used as the numéraire provides useful additional information when interpreting the results. Other choices of numéraire are available, e.g., indices of import prices, and may be appropriate in some instances.

Numéraire Price Block Equations

$$CPI_r = \sum_c vqcdsh_{c,r} * PQD_{c,r} \qquad \forall r \in rgn \qquad \text{(N1)}$$

$$PPI_r = \sum_c vqdsh_{c,r} * PD_{c,r} \qquad \forall r \in rgn \qquad \text{(N2)}$$

$$ERPI = \sum_{ref} vqesh_{ref} * ER_{ref} \qquad \text{(N3)}$$

The exchange rate numéraire (N3) is defined as an exchange rate index for the set *ref*, which allows the user to define the reference region/regions for the exchange rate. This numéraire can be defined as the exchange rate of a single region, e.g., the USA, or a weighted aggregate of the exchange rates for several regions, e.g., the OECD regions. EU, etc., was fixed and served as the numéraire. Experience suggests that model performance is (marginally) improved by including several regions in *ref*; typically composed of regions that are relatively large, but users are free to use other alternatives.[26] The single region alternative is chosen if the membership of *ref* is limited to a single region.

Fixed country trade balances must be specified in "real" terms defined by the global numéraire. So, if the US exchange rate as fixed to one, the global numéraire is defined as US dollars, and all trade balances can be seen as "real" variables defined in terms of the value of US exports. If the weighted exchange rate for a group of regions is chosen as global numéraire, trade balances can be seen as "claims" against a weighted average of exports by the group of regions.

Notice how both the consumer and producer price indices are implemented only for those regions that have consumption and production activities. Hence the GILD region does not have its own price indices; rather the price indices for GILD are those of the reference region(s) in the model.

[26] A guiding principle for selecting the membership of *ref* is to choose a region or group of regions that accounts for a large share of global trade and output.

4.2.5 Production Block Equations

The output price by activity ($PX_{,a,r}$) is defined by the shares of commodity outputs produced by each activity, (X1.1) where, for this case, the weights ($ioqxcqx_{a,c,r}$) are equal to one where the commodities and activities match and zero otherwise, i.e., there is a one to one mapping between the commodity and activity accounts. The weights are derived from the information in the supply or make matrix.[27]

The value of output by activity is defined as the activity price ($PX_{a,r}$) less production taxes ($TX_{a,r}$) times the volume of output ($QX_{a,r}$). This revenue must be divided between payments to primary inputs – the price of value added ($PVA_{a,r}$) times the quantity of value added ($QVA_{a,r}$) – and intermediate inputs – the price of aggregate intermediate inputs ($PINT_{a,r}$) times the volume of aggregate intermediate inputs ($QINT_{a,r}$) (X1.2). Given the assumption that intermediate inputs are used in fixed (volume) proportions, the price of aggregate intermediate inputs ($PINT_{a,r}$) is defined as the weighted average price of the intermediate inputs where the weights are the (normalised) input-output coefficients (X1.3).

The default top level production function (X1.5), is a CES aggregation of aggregate primary and intermediate inputs, where the first order conditions for profit maximization (X1.6) determine the optimal ratio of the inputs. The efficiency factor ($ADX_{a,r}$) and the factor shares parameters ($\delta^{x}_{a,r}$) are calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ($\rho^{x}_{a,r}$), are exogenously imposed. Note in this case the efficiency factor is declared as variable and is determined by (X1.4), where $adxb_{a,r}$ is the vector of efficiency factors in the base solution, $dabadx_{a,r}$ is a vector of absolute changes in the vector of efficiency factors, $ADXADJ_r$ is a variable whose initial value is ONE, $DADX_r$ is a variable whose initial value is ZERO and $adx01_c$ is a vector of zeros and non-zeros.[28] In the base solution the values of $adx0_{a,r}$ and $dabadx_{a,r}$ are all ZERO and $ADXADJ_r$ and $DADX_r$ are fixed as their initial values – a closure rule decision –then the applied efficiency factors are those from the base solution. This formulation allows flexibility in the formulation of the

27 When using GTAP data, $ioqxcqx_{a,c,r}$ is always a diagonal matrix. However, using this specification for the activity price (PX) makes it easier to extend the model to include activities that produce multiple commodities and commodities that are produced by more than one activity.

28 Typically, the values are either one or zero, i.e., the adjustment factor is switched on or off. Non-zero values other than one switch on the adjustment factor and allow a more complex set of adjustments although it is important to be careful about the rationale for such a set of adjustments.

efficiency parameter that is especially useful in the contexts of a dynamic model or simulations that examine the implications of productivity growth or evaluate the productivity growth equivalent some policy change. The structure of this equation (X1.4) is identical to that used for the tax rate equations; a description of how these equations operate is provided when describing the tax rate equations (see below).

The production function (X1.5) is only implemented for members of the set *aqx*; for its complement, *aqxn*, the CES function is replaced by Leontief functions. The Leontief functions require that aggregate intermediate inputs (X1.7) and aggregate values added (X1.8) are fixed proportions of the volumes of output. If there are no intermediate inputs used by an activity the top-level function is automatically Leontief. In the default settings the Leontief assumption is also imposed automatically if the costs of total intermediate inputs accounts for 10 percent or less of total inputs; the user can change the share below which the Leontief assumption is imposed, by activity and region, when calibrating the model.

Finally, it is necessary to define the relationship between activity and commodity outputs, which is the counterpart to the price equation linking commodity and activity prices (X1.1). This is defined as a simple linear relationship whereby the commodity output is defined as the sum of the quantities of each commodity produced by each activity, (X1.9). But given the standard GTAP data, whereby each activity only produces a single commodity and *ioqxqxc* is an identity matrix, (X1.9) simply maps activities to commodities and *vice versa*.

Production Block Equations – Top Level

$$PX_{a,r} = \sum_{c} IOQXACQXV_{a,c,r} * PXC_{c,r} \qquad \forall r \in rgn \qquad (X1.1)$$

$$PX_{a,r} * (1 - TX_{a,r}) * QX_{a,r} = (PVA_{a,r} * QVA_{a,r}) + (PINT_{a,r} * QINT_{a,r}) \qquad \forall r \in rgn \qquad (X1.2)$$

$$PINT_{a,r} = \sum_{c} ioqint_{c,a,r} * PQD_{c,r} \qquad \forall r \in rgn \qquad (X1.3)$$

$$ADX_{a,r} = ((adxb_{a,r} + dabadx_{a,r}) * ADXADJ_{r}) + (DADX_{r} * adx01_{a,r}) \qquad (X1.4)$$

$$QX_{a,r} = ADX_{a,r} * \left[\delta^{x}_{a,r} * (QVA_{a,r})^{-\rho^{x}_{a,r}} + (1 - \delta^{x}_{a,r}) * (QINT_{a,r})^{-\rho^{x}_{a,r}} \right]^{-1/\rho^{x}_{a,r}} \qquad \forall r \in rgn, a \in aqx \qquad (X1.5)$$

$$QVA_{a,r} = QINT_{a,r} * \left(\left(\frac{PINT_{a,r}}{PVA_{a,r}} \right) * \left(\frac{\delta^{x}_{a,r}}{(1 - \delta^{x}_{a,r})} \right) \right)^{\left(1/(1+\rho^{x}_{a,r}) \right)} \qquad \forall r \in rgn, a \in aqx \qquad (X1.6)$$

$$QINT_{a,r} = \sum_{a} ioqintqx_{a,r} * QX_{a,r} \qquad \forall r \in rgn, a \in aqxn \qquad (X1.7)$$

$$QVA_{a,r} = \sum_{a} ioqvaqx_{a,r} * QX_{a,r} \qquad \forall r \in rgn, a \in aqxn \qquad (X1.8)$$

$$QXC_{c,r} = \sum_{a} ioqxcqx_{a,c,r} * QX_{a,r} \qquad \forall r \in rgn \qquad (X1.9)$$

Subsequent levels in the production systems use natural (*f*), factor commodities (*fc*), i.e., commodities included in the value added nests, and aggregate (*fag*) factors of production (*FD*) that are defined as subsets of the set of 'all' factors (*ff*).[29] All factors of production are paid factor, activity and region specific factor rates ($WFA_{ff,a,r}$) that are defined as the average wage rate for natural and aggregate factors (WF_{fcn}) (X2.1) or for factor commodities the purchaser price ($PQD_{c,r}$) (X2.2) multiplied by activity specific productivity/adjustment factors ($WFDIST_{ff,a,r}$).

[29] Since only natural factors, *f*, receive actual income the use of the set *ff* in the production modules allows the set *f* the control the distribution of factor incomes. Similarly, it should be noted that only natural and commodity factors should be subjected to factor use taxes (*TF*).

Production Block Equations – Level 2

$$WFA_{fcn,a,r} = WF_{fcn,r} * WFDIST_{fcn,a,r} \qquad \forall r \in rgn,\ ff \in fcn \qquad (X2.1)$$

$$WFA_{fc,a,r} = \sum_{cf\$map\_fc\_cf} PQD_{cf,r} * WFDIST_{fc,a,r} \qquad \forall r \in rgn,\ ff \in fc \qquad (X2.2)$$

$$QVA_{a,r} = ADVA_{a,r} * \left(\sum_{ff\$map\_va\_ff} \delta^{va}_{ff,a,r} * \left(ADFD_{ff,a,r} * FD_{ff,a,r} \right)^{-\rho^{va}_{a,r}} \right)^{\left(-1 / \rho^{va}_{a,r} \right)} \qquad (X2.3)$$

$$\forall r \in rgn, \rho^{va}_{a,r} > 0, \delta^{va}_{ff,a,r} > 0$$

$$\begin{aligned} WFADIST_{ff,a,r} &* \left(1 + TF_{ff,a,r}\right) \\ &= PVA_{a,r} * QVA_{a,r} * \left[\sum_{ff} \delta^{va}_{ff,a,r} * \left(ADFD_{ff,a,r} * FD_{ff,a,r} \right)^{-\rho^{va}_{a,r}} \right]^{(-1)} \\ &\quad * \delta^{va}_{ff,a,r} * \left(ADFD_{ff,a,r} \right)^{-\rho^{va}_{a,r}} * \left(FD_{ff,a,r} \right)^{\left(-\rho^{va}_{a,r} - 1\right)} \end{aligned}$$

$$\forall r \in rgn, map\_va\_ff > 0, \delta^{va}_{ff,a,r} > 0$$

$$(X2.4)$$

$$ADVA_{a,r} = \left(\left(advab_{a,r} + dabadva_{a,r} \right) * ADVAADJ_{r} \right) + \left(DADVA_{r} * adva01_{a,r} \right)$$

$$(X2.5)$$

$$ADFD_{f,a,r} = \begin{pmatrix} \left(adfdb_{f,a,r} + dabadfd_{f,a,r} \right) \\ * ADFDfADJ_{f} * ADFDaADJ_{a} * ADFDrADJ_{r} \end{pmatrix} \qquad (X2.6)$$

$$QINTD_{c,a,r} = \sum_{a} ioqint_{c,a,r} * QINT_{a,r} \qquad \forall r \in rgn, cfan_{c,a} \qquad (X2.7)$$

$$QINTD_{c,a,r} = \sum_{fc\$map\_cf\_fc} FD_{fc,a,r} \qquad \forall r \in rgn, cfa_{c,a} \qquad (X2.8)$$

The second level defines the production function for value added (*QVA*) that is generated from user defined combinations of natural, commodity and aggregate factors; this selection is controlled by the set $map\_va\_ff_{ff,a}$ that defines which factors from the set *ff* are included in the *QVA* production function for each activity *a*.[30] The production functions are

30 In the GTAP database there are eight natural factors: 5 types of labour, capital, land and natural resources. Typically, labour and capital are used by all activities, but land and natural resources are segmented, i.e., no activity employs both land and natural resources. It is recommended that land and natural resources are

CES aggregation functions over the user defined factors that are demanded by each activity ($FD_{ff,a,r}$), with efficiency factors ($ADVA_{a,r}$) and the factor shares ($\delta^{va}_{ff,a,r}$) calibrated from the transactions data and the exogenous elasticities of substitution, from which the substitution parameters are derived ($\rho^{va}_{a,r}$) (X2.3). Note how the production function for *QVA* is specified to use the subset of factors defined by the set *map_va_ff*. The associated first-order conditions for optimal factor combinations are derived from equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity, (X2.4); while defined over *ff* its operation is controlled by *map_va_ff*. The wage rate for a each factor used by each activity is $WFA_{ff,a,r}$; this allows for non-homogenous factors where the differentiation is defined by the activity that employs the factor. However the actual returns to a factor must be adjusted to allow for taxes on factor use ($TF_{ff,a,r}$).

The efficiency factors are declared as variables (X2.4), where $advab_{a,r}$ is the vector of efficiency factors in the base solution, $dabadva_{a,r}$ is a vector of absolute changes in the vector of efficiency factors, $ADVAADJ_r$ is a variable whose initial value is ONE, $DADVA_r$ is a variable whose initial value is ZERO and $adva01_{a,r}$ is a vector of zeros and non-zeros.[31] In the base solution the values of $advab_{a,r}$ and $dabadva_{a,r}$ are all ZERO and $ADVAADJ_r$ and $DADVA_r$ are fixed as their initial values – a closure rule decision – then the applied efficiency factors are those from the base solution. A similar specification is adopted for factor specific efficiency factors, i.e., factor that can alter/adjust the stock-flow relationship between factor quantities and factor services, although it differs in the adjustment mechanism (X2.6). Specifically, only a multiplicative variant is defined and then three adjustment varioables are defined that allow for factor specific ($ADFDfADJ_f$), activity specific ($ADFDaADJ_a$) and region specific ($ADFDrADJ_r$) adjustments.

Since production uses intermediate inputs, it is also necessary to specify the demand for intermediate inputs ($QINTD_{c,a,r}$); intermediate inputs are recorded as commodity, activity and regions specific, which allows flexibility in defining production nests. This must be done separately for commodity factors (X2.8), for which demand is defined by the paired factor

NEVER aggregated AND that aggregates are not formed across activities that use land and natural resources in the disaggregated data.

31 Typically, the values are either one or zero, i.e., the adjustment factor is switched on or off. Non-zero values other than one switch on the adjustment factor and allow a more complex set of adjustments although it is important to be careful about the rationale for such a set of adjustments.

quantity ($FD_{fc,a,r}$) from the value added nest, and non-commodity factors (*fcn*), which are defined from the Leontief intermediate input aggregation (X2.7).

All remaining levels in the production nests are defined by one compound production function (X3.1) and its associated first-order conditions (X3.2); the equations for factor price definitions are part of the general specifications for factor prices (X2.1 and P2.2).

Production Block Equations – Levels 3 to n

$$FD_{ff,a,r} = ADFAG_{ff,a,r} * \left(\sum_{ffp\$map\_fagg\_ff} \delta^{fd}_{ff,ffp,a,r} * \left(FD_{ffp,a,r} \right)^{\rho^{fd}_{ff,a,r}} \right)^{\left(-1/\rho^{fd}_{ff,a,r} \right)} \quad \text{(X3.1)}$$

$$\forall r \in rgn, \delta^{fd}_{ff,ffp,a,r}, f \sum_{ffp} map\_fagg\_ff_{ff,ffp,a} > 0$$

$$WFADIST_{ffp,a,r} * \left(1 + TF_{ffp,a,r} + TCARB_{ffp,a,r}\right)$$
$$= WFA_{ff,a,r} * \left(1 + TF_{ff,a,r} + TCARB_{ffp,a,r}\right) * FD_{ff,a,r}$$
$$* \left[\sum_{ffpp\$map\_fagg\_ff} \delta^{fd}_{ff,ffpp,a,r} * FD_{ffpp,a,r}^{-\rho^{fd}_{ff,a,r}} \right]^{(-1)} * \delta^{fd}_{ff,ffp,a,r} * FD_{ffp,a,r}^{\left(-\rho^{fd}_{ff,a,r} - 1\right)}$$
$$\forall r \in rgn, \delta^{fd}_{ff,ffpp,a,r} > 0, map\_fagg\_ff_{ff,ffp,a} > 0$$

(X3.2)

The production function for aggregate factors ($FD_{ff,a,r}$) are CES aggregates over user defined combinations of natural, commodity and aggregate factors; the selection is controlled by the set $map\_fag\_ff_{ff,a}$ that defines which factors, natural, commodity or aggregate) from the set *ff* are included in each aggregate factor $FD_{fag,a,r}$. the *QVA* production function for each activity *a*. The production functions for each activity ($FD_{ff,a,r}$), with efficiency factors ($ADFAG_{ff,a,r}$), use the factor shares ($\delta^{fd}_{ff,ffp,a,r}$) calibrated from the transactions data and the exogenous elasticities of substitution, from which the substitution parameters are derived ($\rho^{fd}_{ff,a,r}$) (X2.3). The associated first-order conditions for optimal factor combinations are derived from equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity, (X3.2); while defined over *ff* its operation is controlled by *map_fagg_ff*. The wage rate for each factor used by each activity is

$WFA_{ff,a,r}$; this allows for non-homogenous factors where the differentiation is defined by the activity that employs the factor. Note that the actual returns to a factor must be adjusted to allow for taxes on factor use ($TF_{ff,a,r}$) and emission taxes related to commodity factors in production ($TCARB_{ff,a,r}$[32]), but that only natural and commodity factors pay these taxes.

This formulation requires a complex set of nested production functions controlled by the sets *map_va_ff* and *map_fagg_ff*. In this formulation of the model, it is presumed that mapping sets are identical for all regions, but even so it can be tricky to ensure that the intended nesting structure is implemented. The mapping sets are assigned in an Excel workbook where changes to the nesting structure can be implemented; but note that it is easy to make mistakes when changing the nesting structure. Extending the sets that control the nested functions so that they are region specific is theoretically straightforward, but it is practically tricky to implement.

The composite supplies of each commodity (*QXC*) are aggregates of the commodity outputs by each activity (*QXAC*). The default assumption is that when a commodity is produced by multiple activities it is differentiated by reference to the activity that produces the commodity; this is achieved by defining total production of a commodity as a CES aggregate of the quantities produced by each activity (X4.1). This provides a practical/modelling solution for two typical situations; first, where there are quality differences between two commodities that are notionally the same, e.g., modern digital vv disposable cameras, and second, where the mix of commodities within an aggregate differ between activities, e.g., a cereal grain aggregate made up of wheat and maize (corn) where different activities produce wheat and maize in different ratios. This assumption of imperfect substitution is implemented by a CES aggregator function with $adxc_c$ as the shift parameter, $\delta_{a,c}^{xc}$ as the share parameter and ρ_c^{xc} as the elasticity parameter.

The matching first order condition for the optimal combination of commodity outputs is therefore given by (X4.2), where *PXAC* are the prices of each commodity produced by each activity. Note how, as with the case of the value-added production function two formulations are given for the first-order conditions and the second version is the default version used in the model. Further note that the efficiency/shift factor is in this case declared as a parameter;

[32] Note that TCARB is in the model code defined over *cf,a,r* – and *cf* is mapped to commodity factors controlled by *map_fc_cf(ff,cf)* to be employed in equation X3.2.

this reflects the expectation that there will be no endogenously determined changes in these shift factors.

Production Block Equations: Commodity Outputs

$$QXC_{c,r} = adxc_{c,r} * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c,r} * QXAC_{a,c,r}^{-\rho^{xc}_{c}} \right]^{-1/\rho^{xc}_{c}} \qquad \forall cx_{c,r} \text{ and } cxac_{c,r}. \qquad (X4.1)$$

$$PXAC_{a,c,r} = PXC_{c,r} * adxc_{c,r} * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c,r} * QXAC_{a,c,r}^{-\rho^{xc}_{c}} \right]^{-\left(\frac{1+\rho^{xc}_{c,r}}{\rho^{xc}_{c,r}}\right)} * \delta^{xc}_{a,c,r} * QXAC_{a,c,r}^{\left(-\rho^{xc}_{c,r}-1\right)}$$

$$= PXC_{c,r} * QXC_{c,r} * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c,r} * QXAC_{a,c,r}^{-\rho^{xc}_{c,r}} \right]^{-\left(\frac{1+\rho^{xc}_{c,r}}{\rho^{xc}_{c,r}}\right)} * \delta^{xc}_{a,c,r} * QXAC_{a,c,r}^{\left(-\rho^{xc}_{c,r}-1\right)}$$

$$\forall \delta^{xc}_{a,c,r} \text{ and } cxac_{c,r} \qquad (X4.2)$$

$$PXAC.FX_{a,c,r} = 0.0 \qquad \not\forall SAM_{a,c,r} \qquad (X4.2b)$$

$$QXC_{c,r} = \sum_{a} QXAC_{a,c,r} \qquad \forall cx_{c,r} \text{ and } cxacn_{c,r}. \qquad (X4.3)$$

$$PXAC_{a,c,r} = PXC_{c,r} \qquad \forall \delta^{xc}_{a,c,r} \text{ and } cxacn_{c,r}. \qquad (X4.4)$$

$$QXAC_{a,c,r} = IOQXACQX_{a,c,r} * QX_{a,r} \qquad \forall ioqxacqx_{a,c,r} \textbf{ AND } acetn_{a,r}. \qquad (X4.5)$$

$$QXAC_{a,c,r} = QX_{a,r} * \left(\frac{PXAC_{a,c,r}}{\left(PX_{a,r} * gamma^{i}_{a,c,r} * at^{i}_{a,r}{}^{\rho^{i}_{a,r}}\right)} \right)^{\left(1/\left(\rho^{i}_{a,r}-1\right)\right)} \qquad (X4.6)$$

$$\forall IOQXACQX_{a,c,r} \textbf{ and } acet_{a,r}$$

$$QXAC.FX_{a,c,r} = 0.0 \qquad \forall SAM_{a,c,r} = 0.0 \qquad (X4.7)$$

However, there are circumstances where perfect substitution may be a more appropriate assumption given the characteristics of either or both activity and commodity accounts. Thus, an alternative specification for commodity aggregation is proved where commodities produced by different activities are modelled as perfect substitutes, (X4.3), and the matching price condition therefore requires that *PXAC* is equal to *PXC* for relevant commodity activity combinations (X4.4). The choice of aggregation function is controlled by the membership of the set *cxac*, with the membership of *cxacn* being the complement of *cxac*. It is assumed that the memberships of these sets are identical for all regions; extending the set definitions so that they are region specific is straightforward, but the practical benefits are questionable.

Finally, it is necessary to determine the quantities of each commodity produced by each activity. There are two basic assumptions included in the model: first that secondary commodities are produced with pure by-product technologies, i.e., in a fixed ratio to the principal product, and second that activities can adjust their output mix in response to changes in the prices of the commodities they produce. The function for by-product assumption is that fixed shares of products (*IOQXACQX*) are produced by each activity according to its level of total output (*QX*); although the shares are defined as variable the user determines which rows of the matrix *IOQXACQX* are fixed when configuring the model by defining membership of the set *acet* (X4.5). To implement the alternative assumption, it is only necessary to specify the first order condition for a CET function; this is reported in equation (X4.6). However, it is also now necessary to include a market clearing condition for production; this is reported in the market clearing section below (see equation MC2.1).

4.2.6 Emissions Equations

The emissions associated with the use of energy commodities by each agent are recorded for activities (EM1.1) and households (EM1.2) ($EMIT_{ghg,c,a\_h,r}$). The formulation is general in that the set *ghg* (Greenhouse Gases) can include all, or a subset, of the emissions recorded in the database.[33] These are defined as the product of the quantity of an energy commodity consumed and the coefficients that define emissions (*io_emit*) per unit of energy commodity consumed; hence the maintained assumption is that without a change in technology, by activities, or preferences, by households, the emissions per unit of input and consumption are constant. Note that in the model the quantities of energy commodities consumed are ‘value’ quantities: the coefficients are calibrated as emission per unit of ‘value’ quantity, which needs to be noted when defining simulations.

33 As GTAP 10 the emissions reported are carbon dioxide (CO_2), nitrous oxide (N_2O), methane (CH4) and fluorinated gases. For nitrous oxide and methane CO2 equivalents are also reported.

$$EMIT_{ghg,c,a,r} = QINTD_{c,a,r} * io\_emit_{ghg,c,a,r} \quad \text{(EM1.1)}$$

$$EMIT_{em,c,h,r} = QCD_{c,h,r} * io\_emit_{ghg,c,h,r} \quad \text{(EM1.2)}$$

$$EMIT\_R_{ghg,r} = \sum_{c,a\_h} EMIT_{ghg,c,a\_h,r} \quad \text{(EM1.3)}$$

$$EMIT\_G_{ghg} = \sum_{r} EMIT\_R_{ghg,r} \quad \text{(EM1.4)}$$

$$EMIT\_RA_{ghg,a\_h,r} = \sum_{c} EMIT_{ghg,c,a\_h,r} \quad \text{(EM1.5)}$$

$$EMIT\_GDP_{ghg,r} = \frac{EMIT\_R_{ghg,r}}{GDP_r} \quad \text{(EM1.6)}$$

$$EMIT\_POP_{ghg,r} = \frac{EMIT\_R_{ghg,r}}{POP_r} \quad \text{(EM1.7)}$$

$$EMIT\_EMBED_{ghg,c,r} = \sum_{w} IO\_EMIT\_EMBED_{ghg,c,r} * QER_{c,w,r} \quad \text{(EM1.8)}$$

$$IO\_EMIT\_EMBED_{ghg,c,r} = \frac{\left\{ \sum_{a\$ioqxacqxv} IOQXACQXV_{a,c,r} * \left[\sum_{ghg,cp} EMIT_{ghg,cp,a,r} \right] \right\}}{QXC_{c,r}} \quad \text{(EM1.9)}$$

The total quantity of emissions by region ($EMIT\_R_{ghg,r}$) are a summation of each emissions included in the set *ghg* by each agent (EM1.3), and the total global emissions, for each greenhouse gas, ($EMIT\_G_{ghg}$), are calculated in EM1.4).

Equations are included in the model for the total emissions of each Greenhouse gas per unit of GDP – *EMIT_GDP* - (EMI.6) and per head of population - *EMIT_POP* - (EMI.7). These, arguably, summary variables are included in the model because there are circumstances where the formulation of simulations would use these are target variables; otherwise, they could be reported as derived/calculated results.

Concerns have been expressed about the distinction between emissions generated by countries as distinct from emissions associated with the consumption of commodities, produced domestically and imported, by countries. The variable $EMIT\_EMBED_{ghg,c,r}$ (EMI.8) calculates the emissions embedded in exports of commodities, *c*, from region, *r*, to region w, where the input-output coefficient, *IO_EMIT_EMBED*, calculates the amount of emissions associated with the production of commodity in region *r* (EMI.9). The variable *EMIT_EMBED* provides the information necessary to determine estimates of the emissions embedded in imported commodities based on the emissions associated with the production of those commodities in the source regions.

The taxes on emissions, i.e., the prices of emissions, are recorded as components of the government account.

4.2.6 Factor Block Equations

The total income received by each factor account (YF_f) is defined as the summation of the earnings of that natural factor across all activities (F1.1).[34] However, only a proportion of total factor income is available for distribution to the domestic institutional accounts ($YFDIST_{f,r}$). First, allowance must be made for depreciation, which it is assumed takes place at fixed rates ($deprec_{f,r}$) relative to factor incomes, and then second, allowance must be made for the payment of factor income taxes ($TYF_{f,r}$)[35] (F1.2).

34 Note that only natural factors, *f*, receive actual income.

35 In the GTAP database direct taxes paid by domestic institutions, households, NPISH and incorporated business enterprises, usually known as enterprises, are recorded as direct taxes paid by factors (see the GTAP database documentation for the details). In the ANARRES database these direct taxes are assigned to the household account; thus, the implicit assumption is that the household account encompasses households, NPISH and enterprises. The tax on factor incomes is however kept as an instrument because some direct taxes are levied on factors, e.g., social security contributions.

Factor Block Equations

$$YF_{f,r} = \sum_{a}\left[WFA_{f,a,r} * FD_{f,a,r}\right] \forall r \in rgn \tag{F1.1}$$

$$YFDIST_{f,r} = \left(YF_{f,r} - \left(deprec_{f,r} * YF_{f,r}\right)\right) * \left(1 - TYF_{f,r}\right) \quad \forall r \in rgn \tag{F1.2}$$

$$YFIN_{f,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{YFOUT_{w,f,r}}{ER_r} \right) * ER_r \qquad \forall r \in rgn, w \in wgn \tag{F3}$$

$$YFDIST_{f,r} = \left(\begin{array}{l} \left(YF_{f,r} * \left(1 - deprec_{f,r}\right)\right) \\ + \sum_{w} YFIN_{f,w,r} - \sum_{w} YFOUT_{w,f,r} \end{array} \right) * \left(1 - TYF_{f,r}\right) \qquad \forall r \in rgn \tag{F4}$$

4.2.7 Household Block Equations

Households acquire income from the sale of factor services and government transfers. Therefore household income ($YH_{h,r}$) is defined as the sum of factor incomes available for distribution, i.e., after depreciation, in fixed parameters ($hvash_{h,f,r}$) that are defined as the value shares of each factor demanded in the economy supplied by each household. The model allows for the possibility of multiple households by indexing households on the set *h*. If the model is calibrated for multiple households the user needs to augment the database, whereas in the 'standard' case of one household all the shares equal one. The government transfers are exogenously defined ($hogovconst_{c,h,r}$) in real terms, hence the *CPI*; the variables *HGADJ* allow the transfers to be scaled endogenously.

Household consumption demand is derived in two stages. In the first stage (H2) household consumption expenditures ($HEXP_{h,r}$) are defined as household incomes after the payment of direct taxes and savings and inter household transfers. Note how the saving rates are defined as proportions of after-tax incomes that are saved; this is important for the calibration of the income tax and savings parameters.

The household utility functions are assumed to be systems of nested CES and Stone-Geary, i.e., a linear expenditure system (LES), utility functions. The LES utility functions are defined over the set *cles*, which includes aggregate commodities from the CES utility function with ‘natural’ commodities, *c*, that are defined as not being included in any aggregate commodity, *ccesn*. The CES function aggregates natural commodities, *cces*, into aggregate commodities, *cag*: the ‘natural’ commodities in each aggregate are user defined by assignment to the set $map\_cag\_cc_{cc,cc}$.

Household Block Equations

$$YH_{h,r} = \sum_{f} hvash_{h,f,r} * YFDIST_{f,r} + \left[\left(HGADJ_{r} * hogovconst_{h,r} \right) * CPI_{r} \right] \qquad \forall r \in rgn \tag{H1}$$

$$HEXP_{h,r} = \left[YH_{h,r} * \left(1 - TYH_{h,r} \right) \right] - HHSAV_{h,r} \qquad \forall r \in rgn \tag{H2}$$

$$QCD_{cc,h,r} * PQHD_{cc,h,r} = \left(PQHD_{cc,h,r} * qcdconst_{cc,h,r} \right) + \beta_{cc,h,r} * \left(HEXP_{h,r} - \sum_{c} PQHD_{cc,h,r} * qcdconst_{cc,h,r} \right) \qquad \forall r \in rgn, cc \in cles \tag{H3}$$

$$QCD2_{cc,ccp,h,r} = QCD_{cc,h,r} * \frac{\left(PQHD_{c,hr} * accd_{cc,h,r}{}^{rhocd_{cc,h,r}} \right)}{\left(PQHD_{c,hr} * deltacd_{cc,ccp,h,r} \right)^{1/rhocd_{cc,h,r}}} \qquad \forall r \in rgn, cc \in cag, ccp \in cces \tag{H4}$$

$$PQHD_{ccag,h,r} * QCD_{ccag,h,r} = \sum_{ccp\$map\_ccag\_cc} PQHD_{ccp,h,r} * QCD_{ccp,h,r} \qquad \forall cc \in ccag, r \in rgn \tag{H5}$$

A LES segments household consumption demand into two components, ‘subsistence’ demand ($qcdconst_{c,h,r}$) and ‘discretionary’ demand, and the equations (H3) capture both elements. Discretionary demand is defined as the marginal budget shares (β) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand. The quantities of each commodity demanded by the household are then defined by the shares of household consumption expenditure. The Stone-Geary function collapses to a Cobb-Douglas utility function if all the income elasticities of demand are set equal to one and the Frisch parameter, the elasticity of the marginal utility of income, is also set to one. One advantage of the Cobb-Douglas specification is that it results in the changes in the values for household consumption expenditures ($HEXP_{h,r}$) being equal to the changes in an equivalent variation measure of household welfare.

In many models the utility functions are only defined over 'natural' commodities. This can be an issue that depends on the choice of utility function. Assume the demand system has many 'natural' commodities: if the function is CES then it may encounter performance issues due to 'small' shares, while if the system is LES the subsistence commodities may be unrealistically differentiated, e.g., subsistence quantities of commodities that are close substitutes. A two-level system may mitigate both issues. The CES level can provide aggregates for which subsistence quantities are more coherent, e.g., grains, meat products, etc., while reducing the impact of 'small' shares on the performance of the utility functions.

Given that the LES function defines the quantities of the aggregates demanded, the CES function can be implemented using the first-order conditions for the optimal mix of 'natural' commodities in each aggregate commodity (H4), where $deltacd_{cag,cces,h,r}$ and $rhocd_{cag,h,r}$ are the share and elasticity parameters.

Note how the prices for final demand commodities ($PQHD_{c,r}$) are specified as inclusive of ALL taxes levied on household demand. and are then uplifted by agent specific *ad valorem* taxes ($TSDD_{c,h,r}$).

4.2.8 Government Tax Block Equations

There are eleven 'conventional' tax instruments and three plus three taxes on emissions. Ten of the 'conventional' tax rates are defined as simple *ad valorem* rates dependent upon the values of imports, exports, sales, household demand, production and the levels of factor and household and income. In this version of the model one of the tax rates is defined by reference to the quantity of imports - a specific tax rate – but other tax rates could easily be expressed using bases other than values. The tax on factor use by activities is complicated by the fact that this tax can be applied to commodities designated as factor-commodities and included in the valued added production system.

'Conventional' Tax Instruments

All tax rates are variables in this model and for each tax instrument a series of factors are declared to facilitate policy experiments. The tax rates in the base solution are defined as parameters, e.g., $tmb_{w,c,r}$ are the import tariff rates by commodity *c* imported from region w in region r in the base solution, and the tax rate equations then allow for varying the tax rates in four different ways. Two of the methods use variables that can be solved for optimum values in the model according to the choice of closure rule and two methods allow for deterministic adjustments to the structure of the tax rates. The operations of these methods are discussed in detail only for the equations for import duties while the other equations are simply reported.

In the import tariff rate equation (T1.1) $tmb_{w,c,r}$ is the vector of import duties in the base solution, $dabtm_{w,c,r,}$ is a vector of absolute changes in the vector of import duties taxes, $TMADJ_r$ is a region specific variable whose initial value is ONE, DTM_r is a region specific variable whose initial value is ZERO and $tm01_{w,c,r}$ is a vector of zeros and non-zeros. In the base solution the values of $tm01_{w,c,r}$ and $dabtm_{w,c,r}$ are all ZERO and $TMADJ_r$ and DTM_r are fixed as their initial values – this is a closure rule decision. Given this decision the model treats the tax rates as (fixed) parameters and the applied import tariffs are those from the base solution. The different methods of adjustment can be considered in turn.

Government Tax Rates Equations

$$TM_{w,c,r} = \left(\left(tmb_{w,c,r} + dabtm_{w,c,r} \right) * TMADJ_r \right) + \left(DTM_r * tm01_{w,c,r} \right) \qquad \forall c \in cmr \qquad (T1.1)$$

$$TMS_{w,c,r} = \left(\left(tmsb_{w,c,r} + dabtms_{w,c,r} \right) * TMSADJ_r \right) + \left(DTMS_r * tms01_{w,c,r} \right) \qquad \forall cmr \qquad (T1.2)$$

$$TE_{c,w,r} = \left(\left(teb_{c,w,r} + dabte_{c,w,r} \right) * TEADJ_r \right) + \left(DTE_r * te01_{c,w,r} \right) \qquad \forall c \in cer \qquad (T1.3)$$

$$TS_{c,r} = \left(\left(tsb_{c,r} + dabts_{c,r} \right) * TSADJ_r \right) + \left(DTS_r * ts01_{c,r} \right) \qquad \forall r \in rgn, c \in \left(cd \cup cm \right) \qquad (T1.4)$$

$$TV_{c,r} = \left(\left(tvb_{c,r} + dabtv_{c,r} \right) * TVADJ_r \right) + \left(DTV_r * tv01_{c,r} \right) \qquad \forall r \in rgn, c \in \left(cd \cup cm \right) \qquad (T1.5)$$

$$TX_{a,r} = \left(\left(txb_{a,r} + dabtx_{a,r} \right) * TXADJ_r \right) + \left(DTX_r * tx01_{a,r} \right) \qquad \forall r \in rgn \qquad (T1.6)$$

$$TYF_{f,r} = \left(\left(tyfb_{f,r} + dabtyf_{f,r} \right) * TYFADJ_r \right) + \left(DTYF_r * tyf01_{f,r} \right) \qquad \forall r \in rgn \qquad (T1.7)$$

$$TYH_{h,r} = \left(\left(tyhb_{h,r} + dabtyh_{h,r} \right) * TYHADJ_r \right) + \left(DTYH_r * tyh01_{h,r} \right) \qquad \forall r \in rgn \qquad (T1.8)$$

$$TF_{ff,a,r} = \left(\left(tfb_{ff,a,r} + dabtf_{ff,a,r} \right) * TFADJ_r \right) + \left(DTF_r * ty01_{f,a,r} \right) \qquad \forall r \in rgn, f \in ff \qquad (T1.9)$$

$$TF_{ff,a,r} = \sum_{cf \$ map\_fc\_cf} TSDD_{cf,a,r} \qquad \forall r \in rgn, fc \in ff \qquad (T1.10)$$

$$\begin{aligned} TSDD_{c,a\_h,r} = & \left(\left(tsddb_{c,a\_h,r} + dabtssd_{c,a\_h,r} \right) * TSDDADJ_r \right) \\ & + \left(DTSDD_r * tsdd01_{c,a\_h,r} \right) \\ & + \left(\begin{array}{l} \left(dabtssd\_G_{c,a\_h,r} * TSDD\_GADJ \right) \\ + \left(DTSDD\_G * tsdd01_{c,a\_h,r} \right) \end{array} \right) \end{aligned} \qquad \forall r \in rgn \qquad (T1.11)$$

1. If $TMADJ_r$ for one region is made a variable, which requires the fixing of another variable for that region, and all other initial conditions hold then the solution value for $TMADJ_r$ yields the optimum equiproportionate change in the import duty rates necessary to satisfy model constraints, e.g., if $TMADJ_r$ equals

1.1 then all applied (non-zero) import duty rates (that are found in the base data) for the specified region are increased by 10%.

2. If any element of $dabtm_{w,c,r}$ is not zero, and all the other initial conditions hold, then an absolute change in the initial import tariff rate for the relevant commodity is imposed, e.g., if $tmb_{w,c,r}$ for one element of *c* is 0.1 (a 10% import duty) and $dabtm_{w,c,r}$ for that element is 0.05, then the *applied* import tariff rate is 0.15 (15%).
3. If $TMADJ_r$ for one region is made a variable, which requires the fixing of another variable for that region, any elements of $dabtm_{w,c,r}$ are non-zero and all other initial conditions hold, then the solution value for $TMADJ_r$ yields the optimum equiproportionate change in the *applied* import duty rates, i.e., $tmb_{w,c,r} + dabtm_{w,c,r}$ (and these applied rates can be different from those in the base in which $dabtm_{w,c,r}$ is set at zero).
4. If DTM_r for one region is made a variable, which requires the fixing of another variable for that region, AND at least one element of $tm01_{w,c,r}$ is ONE then the subset of elements of $tm0_{w,c,r}$ identified by $tm01_{w,c,r}$ are allowed to (additively) increase by an equal absolute amount determined by the solution value for DTM_r and the initial values of the import duty rates, e.g., if food products are to be excluded from the change in tax rates then the elements of $tm01_{w,c,r}$ corresponding to the food commodities are (left as) zeros. Note how in this case it is necessary to both 'free' a variable and give values to a parameter for a solution to emerge. If the change in the applied tax rates is to be other than equal absolute values then values of $tm01_{w,c,r}$ other than one can be applied, e.g., if the changes in the import duties on food products are to be half those on other sectors then the elements of $tm01_{w,c,r}$ corresponding to the food commodities can be set to 0.5 and for the other commodities can be set to 1.

This combination of alternative adjustment methods covers the range of common tax rate adjustments used in the majority of applied applications while being flexible and easy to use. However, experience has shown that when working with GTAP data it is very important to check the tax rates that are applied in the base solution; in some aggregations some of the database applied tax rates only differ marginally from zero but are a mix of negatives and positives, in such cases it may be necessary to exogenously reset the tax rates to avoid apparently odd results. All the model parameters are exported to a GDX file and can be easily accessed using GAMS facilities for viewing GDX files.[36]

All tax rate equations follow the same structure. Specific import duties ($TMS_{c,r}$) are defined in in (T1.2), export taxes ($TE_{c,r}$) are defined in (T1.3), sales taxes ($TS_{c,r}$) are defined in (T1.4), VAT ($TV_{c,r}$) rates are defined in (T1.5), production tax rates ($TX_{a,r}$) are defined in (T1.6), factor income tax rates ($TYF_{f,r}$) are defined in (T1.7), and household income rates ($TYH_{h,r}$) are defined in (T1.8). Factor use tax rates ($TF_{ff,r}$) are defined in (T1.9 and T1.10): two assignments are needed to encompass factor use taxes paid on natural factors (T1.9) and commodity factors (T1.10). Agent specific sales taxes ($TSDD_{c,a\_h,r}$) are defined in (T1.11).

Note that the value added tax (VAT) is specified as additional to other 'general' sales taxes (*TS*, *TSDD*) levied on domestic demand; as such it is a simplification because it does not allow for differential realised rates of rebate of VAT on intermediate input demand – the implicit presumption is that the rebate rates are equal to 100 percent or that the rebate rate does not change in any simulation[37]. While this is a strong assumption, the increasing importance of VAT systems means that ignoring the VAT issue is arguably unwise since the tax incidence will be mis specified; moreover, it is recommended in the ANARRES model that the GST rates (*TS*) are NOT used as a tax replacement instrument and that they are not changed in experiments.[38]

Tax Rates on Emissions

The tax rates for emissions are slightly more complex so as to capture the agent specific dimension and the rate of emissions associated with consumption by activities and households. For each form of emission, a region-specific emission tax rate is defined and then this rate is used to define the tax rates applied to each agent for each emission type and region. The region-specific tax rates, TC for CO2, TN2O for nitrous oxide and TCH4 for methane, are assigned in equations T2.1a, T2.2a and T2.3a respectively, and follow the standard pattern for tax rates. Each region and agent specific rate is then define as the product of the respective

36 See Section 6 on complementary programmes.

37 If all other purchases by activities are correctly valued one possibility is that the production taxes levied on activities will include unrebated VAT.

38 *De facto* this advice means that sales taxes rates should be treated as an unchanging distortion in the underlying data and simulations should assume that the degree of distortion is unchanged. This is not an issue for intermediate inputs if the Leontief assumption is made because composition of intermediate inputs invariant to price changes. This model relaxes this assumption because energy input uses can be made price sensitive given enhanced data for sales taxes on energy.

rate and the emissions coefficients (*io_emit*$_{ghg,c,a\_h,r}$) to produce the applied rates: *TCARB*, *TNOX* and *TMETH* (T2.1b, T2.2b and T2.3b respectively). Final a variable, TEMIT, is defined as the sum of the emissions tax rates (T2.4). This is useful as a summary measure of the total emission taxes applied and simplifies some coding. Note however that care is needed to ensure the component tax rates are calibrated consistently – this is primarily a matter of scaling the data consistently – and that there is no adjustment mechanism associated with this tax rate, all adjustments are applied through the components.

Emission Tax Rates Equations

$$TC_r = \left(\left(tcb_r + dabtc_r\right)*TCADJ_r\right) + \left(DTC_r * tc01_r\right) + \left(\left(tcgb + dabtcg\right)*TCGADJ * tc01_r\right) \quad \forall r \in rgn \tag{T2.1a}$$

$$TCARB_{c,a\_h,r} = TC_r * \sum_{ghg\_c} io\_emit_{ghg\_c,c,a\_h,r} \quad \forall r \in rgn; io\_emit_{ghg\_c,c,a\_h,r} > 0 \tag{T2.1b}$$

$$TN2O_r = \left(\left(tn2ob_r + dabtn2oc_r\right)*TN20ADJ_r\right) + \left(DTN20C_r * tn2o01_r\right) + \left(\left(tn2ogb + dabtn2og\right)*TN20GADJ\right) \quad \forall r \in rgn \tag{T2.2a}$$

$$TNOX_{c,a\_h,r} = TN2O_r * io\_emit_{'n2o',c,a\_h,r} \quad \forall r \in rgn \tag{T2.2b}$$

$$TCH4_r = \left(\left(tch4b_r + dabtch4_r\right)*TCH4ADJ_r\right) + \left(DTCH4_r * tch401_r\right) + \left(\left(tch4gb + dabtch4g\right)*TCH4GADJ\right) \quad \forall r \in rgn \tag{T2.3a}$$

$$TMETHX_{c,a\_h,r} = TCH4_r * io\_emit_{'ch4',c,a\_h,r} \quad \forall r \in rgn \tag{T2.3b}$$

$$TEMIT_{c,a\_h,r} = TCARB_{c,a\_h,r} + TNOX_{c,a\_h,r} + TMETH_{c,a\_h,r} \quad \forall r \in rgn \tag{T2.4}$$

The choice of emissions tax instruments is driven by the availability of emissions data. If greater details are reported for emissions these instruments can be readily extended. It is straightforward to relax the resumption that the emission tax rates are region specific; the equations for *TSDD* (T1.11) provide an example of one option.

Tax Revenue Equations (‘conventional’ taxes)

The government tax revenue equations simply sum the revenues from each different tax instrument for each region. TheYe are not necessary for the model since the equations could be collapsed into a single government income equation (see G1 below) and values calculated after the model solution is achieved. However, it is useful to carry around these additional variables since they can then be used in model closures to accommodate specific government tax revenue objectives and they are useful sources of information when analysing simulation results.

Conventional Tax Revenues Equations

$$MTAX_r = \sum_w \sum_c \left(TM_{w,c,r} * PWM_{w,c,r} * ER_r * QMR_{w,c,r} \right) \qquad \forall r \in rgn \qquad (T3.1)$$

$$MSTAX_r = \sum_w \sum_c \left(TMS_{w,c,r} * QMR_{w,c,r} \right) \qquad \forall r \in rgn \qquad (T3.2)$$

$$ETAX_r = \sum_w \sum_c \left(TE_{c,w,r} * PWE_{c,w,r} * ER_r * QER_{c,w,r} \right) \quad \forall r \in rgn \qquad (T3.3)$$

$$STAX_r = \sum_c \left(TS_{c,r} * PQS_{c,r} * \left(QINTD_{c,r} + \sum_h QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} \right) \right) \qquad (T3.4)$$
$$\forall r \in rgn$$

$$VTAX_r = \sum_c \sum_h \left(TV_{c,r} * PQD_{c,r} * QCD_{c,h,r} \right) \qquad \forall r \in rgn \qquad (T3.5)$$

$$ITAX_r = \sum_a \left(TX_{a,r} * PX_{a,r} * QX_{a,r} \right) \qquad \forall r \in rgn \qquad (T3.6)$$

$$FYTAX_r = \sum_f \left(TYF_{f,r} * \left(YF_{f,r} - \left(deprec_{f,r} * YF_{f,r} \right) \right) \right) \qquad \forall r \in rgn \qquad (T3.7)$$

$$HTAX_r = \sum_h \left(TYH_{h,r} * YH_{h,r} \right) \qquad \forall r \in rgn \qquad (T3.8)$$

$$FTAX_r = \sum_f \sum_a \left(TF_{f,a,r} * WF_{f,r} * WFDIST_{f,a,r} * FD_{f,a,r} \right) \qquad \forall r \in rgn \qquad (T3.9)$$

$$SDDTAX_r = \sum_{c,a} \left(TSDD_{c,a,r} * PQD_{c,r} * QINTD_{c,a,r} \right)$$
$$\sum_{c,h} \left(TSDD_{c,h,r} * PQCD_{c,r} * QCD_{c,h,r} \right) \qquad \forall r \in rgn \qquad (T3.10)$$

Ad valorem Import duty revenues (*$MTAX_r$*) are defined as total *ad valorem* import duty revenue in region *r* (T3.1); *Specific* Import duty revenues (*$MSTAX_r$*) are defined as total specific import duty revenues in region *r* (T3.2) Export tax revenues (*$ETAX_r$*) are defined as total export tax revenue in region *r* (T3.3); sales tax revenues (*$STAX_r$*) are defined as total sales tax revenue in region *r* (T3.4); VAT revenues (*$VATAX_r$*) are defined as total VAT revenue in region *r* (T3.5); production tax revenues (*$ITAX_r$*) are defined as total production tax revenue in region *r* (T3.6); factor income tax revenues (*$FYTAX_r$*) are defined as total factor income tax revenue in region *r* (T3.7); household income tax revenues (*$HTAX_r$*) are defined as total household income tax revenue in region *r* (T3.8) and factor use tax revenues (*$FTAX_r$*) are defined as total factor use tax revenue in region *r* (T3.9); and agent specific sales tax revenues ($SDDTAX_r$) are defined as total agent specific sales tax revenue in region *r* (T3.10).

Emissions Tax Revenues

In a similar fashion the tax revenues from emissions taxes are calculated for CO2 emissions (*CARBTAX*, T3.11), NOX emission (*NOXTAX*, T3.12) and methane emissions (*METHTAX*, T3.13). The total revenues from applied emissions taxes (*EMITTAX*) are also calculated (T3.14).

Emissions Tax Revenues Equations

$$CARBTAX_r = \sum_c \sum_a \left(TCARB_{c,a,r} * QINTD_{c,a,r} \right) + \sum_c \sum_h \left(TCARB_{c,h,r} * QCD_{c,h,r} \right) \quad \forall r \in rgn \qquad (T3.11)$$

$$NOXTAX_r = \sum_c \sum_a \left(TNOX_{c,a,r} * QINTD_{c,a,r} \right) + \sum_c \sum_h \left(TNOXB_{c,h,r} * QCD_{c,h,r} \right) \quad \forall r \in rgn \qquad (T3.12)$$

$$METHTAX_r = \sum_c \sum_a \left(TMETH_{c,a,r} * QINTD_{c,a,r} \right) + \sum_c \sum_h \left(TMETH_{c,h,r} * QCD_{c,h,r} \right) \quad \forall r \in rgn \qquad (T3.13)$$

$$EMITTAX_r = CARBTAX_r + NOXTAX_r + METHTAX_r \quad \forall r \in rgn \qquad (T3.14)$$

4.2.9 Government Block Equations

Government income (YG_r) is defined as the sum of government tax revenues (G1), where the tax revenues are treated as expenditures by the accounts paying the taxes and hence are defined in the tax block. One exception to the simple adding up formula is the treatment of emissions tax revenues (*EMITTAX*): for this tax, the share of emissions tax revenues that are transferred abroad (*emit_transfer*) and directed into domestic investment (*emit_inv*) are subtracted before adding the residual to government income. The implicit presumption is that shares of emissions tax revenues are earmarked for international transfer or domestic investment, even though governments rarely hypothecate tax revenues. This approach simplifies the implementation of certain scenarios, especially by simplifying some macroeconomic closure options. While this approach adds equations, it has the arguable advantage of being more transparent and easier to modify. Note how there is no provision for government to receive incomes from non-tax sources in this version; this reflects the fact that in the base GTAP database no such incomes are recorded.

Other Government Equations

$$\begin{aligned} YG_r = {} & MTAX_r + MSTAX_r + ETAX_r + STAX_r + VTAX_r + SDDTAX_r \\ & + ITAX_r + FYTAX_r + HTAX_r + FTAX_r \\ & + \left\{ EMITTAX_r * \left[1 - \left(\sum_{w,g} emit\_transfer_{w,g,r} + emit\_inv_r \right) \right] \right\} \\ & + \sum_{g,w} AID\_B\_IN_{g,w,r} + \sum_{g,w} AID\_M\_IN_{g,w,r} \\ & - \sum_{w,g,w} AID\_B\_OUT_{w,g,r} - \sum_{w,g} AID\_M\_OUT_{w,g,r} \qquad \forall r \in rgn \end{aligned} \tag{G1}$$

$$QGD_{c,r} = qgdconst_{c,r} * QGDADJ_r \qquad \forall r \in rgn \tag{G2}$$

$$EG_r = \left[\sum_c PQD_{c,r} * QGD_{c,r} \right] + \sum_h \left[\left(HGADJ_r * hogovconst_{h,r} \right) * CPI_r \right] \qquad \forall r \in rgn \tag{G3}$$

Government demand for commodities (G2) is assumed fixed in real terms, i.e., the volume is fixed,[39] but can be scaled or allowed to vary using an adjustment factor ($QGDADJ_r$). The precise specification depends upon the choice of closure rule (see below). Thereafter Government consumption expenditure (EG_r) is defined as the sum of commodity consumption (G3) plus the values of transfers to households ($hogovconst_{c,h,r}$). The advantage of separately expressing the volume and value of government expenditures is the increased flexibility it provides in the choice of closure rules for the government account. This

[39] This assumption means that the implicit utility function for the governments has a Leontief form; in the context of the IOT base for the GTAP SAM this 'utility' function could be thought of a the production function for government services. Other options are possible – and easily implemented, e.g., Cobb-Douglas, CES, LES, etc. The literature does not provide a clear theoretical rationale for identifying the appropriate form.

arrangement allows adjustment of government demand either through the volume, or the expenditure or the value share of final demand (see below).

Note that government expenditures are defined without aid transfers, i.e., they are defined as DOMESTIC expenditures.

4.2.10 International Transfer Block Equations

The equations in this block relate to international transfers on the current account. The standard GTAP database does not report international transfer other than for bilateral trade balances, i.e., the net balances on trade in goods and services. The equations in this block therefore require that either the GTAP database is augmented or that scenarios are simulated in which international transfers are defined in terms of changes from a base set of zero net international transfers. In the latter instances the modeller is required to specify both the patterns of transfers by regions and the patterns for the destinations for those transfers.

Aid Transfers

Bilateral aid transfers (*AID_B_OUT*) are defined as fixed shares (*aid_b_outsh*) of *GDP* (G3) plus any shares of emission taxed subject to international transfer (*EMITTAX * emit_transfer*). Since the outward aid transfers are bilateral, the aid transfers from region *r* to region *w* are identical to the transfer received by region *r* from region *w* after adjustments for the exchange rates. Hence, the aid inflows (*AID_B_IN*) are equal to the matched outflows (*AID_B_OUT*) converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region (G4).

Multilateral aid transfers *(AID_M_OUT)* are defined as fixed shares of *GDP* for all non-globe regions (G5a) plus any shares of emission taxed subject to international transfer (*EMITTAX * emit_transfer*). Since the Globe region is a clearing house, and therefore does not have *GDP*, aid transfers from Globe are defined as fixed shares of government income *YG*. For Globe, government income only comes from bilateral or multilateral aid flows and there is no tax revenue (G1). Multilateral aid payments received in a region *(AID_M_IN)* are payments from globe. Hence the aid inflows (*AID_M_IN*) are equal to the matched outflows (*AID_M_OUT*) converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region (G6).

Aid Equations

$$AID\_B\_OUT_{w,g,r} = \left(aid\_b\_outsh_{w,g,r} * GDP_r\right) + \left(EMITTAX_r * emit\_transfer_{w,g,r}\right) \quad \forall r \in rgn, w \in wgn, g \in gt \tag{G3}$$

$$AID\_B\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w,r\$map\_w\_r} \frac{AID\_B\_OUT_{w,g,r}}{ER_r}\right) * ER_r \quad \forall r \in rgn, w \in wgn, g \in gt \tag{G4}$$

$$AID\_M\_OUT_{w,g,r} = \left(aid\_m\_outsh_{w,g,r} * GDP_r\right) + \left(EMITTAX_r * emit\_transfer_{w,g,r}\right) \quad \forall r \in rgn, g \in gt \tag{G5a}$$

$$AID\_M\_OUT_{w,g,r} = aid\_m\_outsh_{w,g,r} * YG_r \quad \forall r \notin rgn, g \in gt \tag{G5b}$$

$$AID\_M\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{AID\_M\_OUT_{w,g,r}}{ER_r}\right) * ER_r \quad \forall g \in gt \tag{G6}$$

4.2.11 Capital Account Block Equations

Income to the capital (savings and investment) account, total savings, comes from household savings, depreciation allowances, government savings ($KAPGOV_r$) and the surplus on the capital account of the balance of payments ($KAPWOR_r$), plus any hypothecated savings incomes.

In this model the household savings rates are declared as variables ($SHH_{h,r}$) that define the proportions of income saved after the payment of income taxes. The savings rate equations (K1.1) used the same adjustment structure as used for the tax rate equations; hence $shh0_{h,r}$ are the base solution savings rates, $dabshh_{h,r}$, are absolute changes in the base savings rates, $SADJ_r$ are multiplicative adjustment factors, $DSHH_r$ are additive adjustment factors and

$shh01_{h,r}$ is a matrix of zero and non-zero values that determine for which households and regions the savings rates can adjust additively. Total household savings (HHSAV) can then be defined as the share of household incomes saved after the payment of income taxes (K1.2).

Savings Block Equations

$$SHH_{h,r} = \left(\left(shhb_{h,r} + dabshh_{h,r}\right) * SADJ_r\right) + \left(DSHH_r * shh01_{h,r}\right) \qquad \forall r \in rgn \qquad \text{(K1.1)}$$

$$HHSAV_{h,r} = \left(YH_{h,r} * \left(1 - TYH_{h,r}\right)\right) * \left(SHH_{h,r}\right) \qquad \forall r \in rgn \qquad \text{(K1.2)}$$

$$DEPRSAV_{f,r} = deprec_{f,r} * YF_{f,r} \qquad \forall r \in rgn \qquad \text{(K1.3)}$$

$$\begin{aligned} TOTSAV_r = & \sum_h HHSAV_{h,r} + \sum_f DEPRSAV_{f,r} + KAPGOV_r + \left(KAPWOR_r * ER_r\right) \\ & + \left[EMITTAX_r * emit\_inv_r\right] \\ & + \sum_{g,w\$g\_aid_g} AID\_B\_IN_{g,w,r} + \sum_{g,w\$g\_aid_g} AID\_M\_IN_{g,w,r} \\ & \forall r \in rgn \end{aligned} \qquad \text{(K1.4)}$$

Depreciation is a form of saving, which in this instance is presumed to be a fixed share ($deprec_{f,r}$) of factor incomes ($YF_{f,r}$); by indexing on f this allows for the introduction of multiple factors that can be subject to depreciation.. An alternative assumption may be that the value of depreciation is related to the value of the stocks of factors subject to depreciation. The justification for using factor incomes is that as the intensity of the use of a factor increases so, ceteris paribus, might the extent of depreciation.

Government savings are calculated as residual (see the $KAPGOV_r$ equations, MC3.1, below). The surplus on the capital account ($KAPWOR_r$) is defined in terms of the foreign currency (see MC3.4 and MC3.3) and therefore the exchange rate appears in this equation (this is a matter of preference).

Total savings can then be defined by summing the components (K1.4). The novelty in K1.4 is the inclusion of hypothecated savings. In this case it is assumed that a share of emissions tax revenues (*emit_inv*) is dedicated to investment, and that some ‘aid’ payments are received that must be dedicated to investment (*AID_B_IN* and *AID_M_IN*) where the transfers so constrained are defined by the set *g_aid*. The presumption of hypothecation is adopted for purposes of scenario specification not as a representation of how governments manage their budgets.

Investment demand is modeled in a similar way to government demand. Demand for commodities (K2.1) used in investment is assumed to be in fixed volume[40] ($qinvdconst_{c,r}$) multiplied by an investment-scaling variable ($IADJ_r$) that can accommodate changes in the exogenously determined level of investment and/or changes in the availability of funds for investment. The second stage (K2.2) captures the price effect by identifying the total value of investment ($INVEST_r$) and includes a terms to allow for fixed stock changes ($dstocconst_{c,r}$).[41] This arrangement allows adjustment of investment demand either through the volume, or the expenditure or the value share of final demand (see below).

Investment Block Equations

$$QINVD_{c,r} = IADJ_r * qinvdconst_{c,r} \qquad \forall r \in rgn \qquad \text{(K2.1)}$$

$$INVEST_r = \sum_c \left(PQD_{c,r} * QINVD_{c,r} \right) + \sum_c \left(PQD_{c,r} * dstocconst_{c,r} \right) \qquad \forall r \in rgn \qquad \text{(K2.2)}$$

4.2.12 Market Clearing Block Equations

In the base solution to the model the market clearing, or equilibrium, conditions are relatively straightforward. Factor supplies must equal factor demands plus any unemployed labour in a region (MC1). Thus, the factor supplies cannot be simply equated to the sum of factor demands in the base period but must equal the sum of factor demand plus any unemployed factors. As such the default specification of the model is NOT full employment but rather that there is the possibility of unemployment.

40 This assumption means that the implicit production function for the investment demand has a Leontief form; other options are possible – and easily implemented, e.g., Cobb-Douglas, CES, LES, etc. In fact, the composition of investment demand defines the 'technology' used to produce new capital goods.

41 Stock changes are not identified in the GTAP database but are useful in GLOBE to account for rounding 'errors' in the database.

Factor Account Equation

$$FS_{f,r} = \sum_{a} FD_{f,a,r} + UNEMP_{f,r} \qquad \forall r \in rgn \qquad \text{(MC1.1)}$$

$$UNEMP_{f,a} \geq 0 \qquad \forall r \in rgn \qquad \text{(MC1.2)}$$

It is necessary to ensure clearing of the production of differentiated commodities by activities when activities can adjust their output mixes in response to changes in relative commodity prices; this is done in equation (MC2.1). Market clearing for the composite commodity markets requires that the supplies of the composite commodity (*QQ*) are equal to total of domestic demands for composite commodities, which consists of intermediate demand (*QINTD*), household (*QCD*), and government (*QGD*) and investment (*QINVD*) final demands (C13). Note how the market clearing condition with respect to final demand by households is formulated to avoid double counting by ensuring that no aggregate commodities enter the definition (RHS) of domestic demand (MC2.2). It appears that there is no equilibrium condition for the supply of domestic output to the domestic market. In fact this is achieved through the commodity output equation (P1.9), which could have been treated as a market clearing equation.

Commodity Account Equations

$$QXAC_{a,c,r} - IOQXACQXV_{a,c,r} * QX_{a,r} \qquad \forall r \in rgn \qquad \text{(MC2.1)}$$

$$QQ_{c,r} = QINTD_{c,r} + \sum_{h} QCD_{c,h,r} + QGD_{c,r} + QINVD_{c,r} + dstocconst_{c,r}$$
$$\forall c \in (cd \cup cm), r \in rgn \qquad \text{(MC2.2)}$$

The commodity trade accounts define the market clearing conditions for bilateral trade; these ensure consistency of trade prices and quantities. The *fob* prices for imports ($PWMFOB_{w,c,r}$) for all imports by destination and source must be equal to the *fob* prices for exports ($PWE_{c,w,r}$) by source and destination (MC3.1). In addition the quantities of imports

($QMR_{w,c,r}$) for all imports by destination and source must be equal to the quantities of exports ($QER_{c,w,r}$) by source and destination (MC3.2). These equations are not completely straightforward since it is necessary in their implementation to employ mappings between exporting and importing regions that require the 'switching' of labels on accounts within the equation.

Commodity Trade Account Equations

$$PWMFOB_{w,c,r} = PWE_{c,w,r} \qquad \forall c \in cmr, map\_r\_w, map\_w\_r \qquad \text{(MC3.1)}$$

$$QMR_{w,c,r} = QER_{c,w,r} \qquad \forall c \in cmr, map\_r\_w, map\_w\_r \qquad \text{(MC3.2)}$$

The trade consistency equations do not however deal with the requirements for market clearing with respect to the trade transactions undertaken by the GILD region. However similar conditions apply for the margins trade. These require that the total demand for each and every trade and transport service ($QT_{w,c,r}$) is exactly equal to the exports of that service by GILD ($QER_{c,w,"glo"}$) and the import prices of margin services ($PT_{c,r}$) must equal the export prices (*fob*) from GILD; these conditions are imposed by (MC4.1) and (MC4.2) respectively.

Margin Trade Account Equations

$$\sum_{w} QT_{w,c,r} = \sum_{w} QER_{c,w,"glo"} \qquad \forall c \in ct2, r \in rgn \qquad \text{(MC4.1)}$$

$$PT_{c,r} = PWE_{c,w,"glo"} \qquad \forall c \in ct2, r \in rgn \qquad \text{(MC4.2)}$$

4.2.13 Macroeconomic Closure Block Equations

The government account is cleared by defining government savings ($KAPGOV_r$) as the difference between government income and government expenditure on consumption and transfers; hence government savings are explicitly treated as a residual, (MC5.1). The deficit/surplus on the current account is computed in two-stages. First the bilateral trade balances ($KAPREG_{w,r}$) are calculated as the difference in the values of imports and exports, for first the trade flows between regions other than GILD (MC5.2) and then for trade between GILD and all other regions (MC5.3) – the latter being the trade balances on margins trade. These transactions are valued in terms of the global numéraire. Then the overall balance of trade ($KAPWOR_r$) is computed for each region, (MC5.4).

Investment and Savings Account Equations

$$KAPGOV_r = YG_r - EG_r \qquad \forall r \in rgn \qquad \text{(MC5.1)}$$

$$KAPREG_{w,r} = \begin{pmatrix} \left(PWMFOB_{w,r} * QMR_{w,r}\right) \\ +\sum_g \frac{AID\_B\_OUT_{w,g,r}}{ER_r} + \sum_g \frac{AID\_M\_OUT_{w,g,r}}{ER_r} \end{pmatrix} - \begin{pmatrix} \left(PWE_{w,r} * QER_{w,r}\right) \\ +\sum_g \frac{AID\_B\_IN_{g,w,r}}{ER_r} + \sum_g \frac{AID\_M\_IN_{g,w,r}}{ER_r} \end{pmatrix} \qquad \forall w \in wgn$$

$$KAPREG_{w,r} = \left(\left(\sum_w PT_r * QT_{w,r}\right) + \sum_g \frac{AID\_M\_OUT_{w,g,r}}{ER_r}\right) - \left(\left(PWE_{w,r} * QER_{w,r}\right) + \sum_g \frac{AID\_M\_IN_{g,w,r}}{ER_r}\right) \qquad \text{(MC5.2)}$$

$$\forall w \notin wgn$$

$$KAPWOR_r = \sum_w KAPREG_{w,r} \qquad \forall r \qquad \text{(MC5.4)}$$

The absorption closure equations exist to support closure rule choices relating to the distribution of final demand between the institutions within regions. The value of final demand by region ($VFDOMD_r$) is simply the sum of the value of final demand across the three sources of final demand – households, government and investment (MC6.1). The value share of final demand accounted for by investment ($INVESTSH_r$) is defined in (MC6.2), and the value share of final demand accounted for by government ($VGDSH_r$) is defined in (MC6.3). No similar expression exists for the households because household expenditures are defined after taxes and savings and it is difficult to justify setting the household shares relative to the value of total final demand since this would nullify the distributional consequences of a policy shock. If the expenditure shares by investment and government are fixed, then the expenditure share of ALL households will by definition be fixed and $HEXP_r$ could be fixed for individual households if there was good reason.

Absorption Closure Equations

$$VFDOMD_r = \sum_{c,h}\left(PQHD_{c,h,r} * QGD_{c,h,r}\right) + \sum_{c}\left(PQD_{c,r} * \left(QGD_{c,r} + QINVD_{c,r}\right)\right) \quad \forall r \in rgn \qquad \text{(MC6.1)}$$

$$INVESTSH_r * VFDOMD_r = INVEST_r \quad \forall r \in rgn \qquad \text{(MC6.2)}$$

$$VGDSH_r * VFDOMD_r = \sum_{c} PQD_{c,r} * QGD_{c,r} \quad \forall r \in rgn \qquad \text{(MC6.3)}$$

4.2.14 Macroeconomic Aggregates Block Equations

Strictly the model does not require the specification of any macroeconomic aggregates since such variables, in the context of a CGE model, are summary measures that do not play an active role in the behavioural relationships, i.e., they could all be computed post simulation as summary measures. However, in the context of a dynamic model it may be useful to endogenously compute various macroeconomic variables to facilitate the calibration of the dynamic baseline.

Two such variables are included in ANARRES[42]. GDP is defined from the expenditure side (MC7.1) as is Absorption (ABSORP) in MC7.2.

4.2.15 Slacks Block Equations

The final equations are for slacks. The satisfaction of Walras's law requires that if all other accounts balance, then so must the final account; thus, the difference between savings and investment for all regions that save should always be zero and hence the variable $WALRAS_r$ should equal zero in (M87.1). That these variables equal zero is a good check on the correct specification of the model. Finally, the global trade balance must be zero and hence so must *KAPWORSYS* (MC8.2). The commodity trade balance for GILD must be zero and hence the

42 Other summary measures of total economic activity are specified in the System of National Accounts, e.g., GNI, NDP, etc. These summary measures could be used in preference to GDP and Absorption.

slack variable (*GLB_SL_M*) must be zero (MC8.3). Similarly, the balance on multilateral aid transfers must be zero and hence slack variable (*GLB_SL_AID*) must be zero (MC8.4).

Macroeconomic Aggregates Equations

$$GDP_r = \sum_{c,h}\left(PQHD_{c,h,r} * QCD_{c,h,r}\right) + \left(\sum_{c} PQD_{c,r} * \left(QGD_{c,r} + QINVD_{c,r} + dstocconst_{c,r}\right)\right) + \left(\sum_{c,w} PWE_{c,w,r} * ER_r * QER_{c,w,r}\right) - \left(\sum_{w,c} PWM_{w,c,r} * ER_r * QMR_{w,c,r}\right) \quad \forall r \in rgn \qquad \text{(MC7.1)}$$

$$ABSORP_r = \sum_{c,h}\left(PQHD_{c,h,r} * QCD_{c,h,r}\right) + \left(\sum_{c} PQD_{c,r} * \left(QGD_{c,r} + QINVD_{c,r} + dstocconst_{c,r}\right)\right) \forall r \in rgn \qquad \text{(MC7.2)}$$

Slack Equations

$$TOTSAV_r = INVEST_r + WALRAS_r \qquad \forall r \in rgn \qquad \text{(MC8.1)}$$

$$KAPWORSYS = \sum_r KAPWOR_r \qquad \forall r \in rgn \qquad \text{(MC8.2)}$$

$$\left[\sum_{w} PWM_{w,c,r} * QMR_{w,c,r}\right] = \left[\sum_{w} PWE_{c,w,r} * QER_{c,w,r}\right] + GLB\_SL\_M \quad \forall c \in ct2, r \notin rgn \qquad \text{(MC8.3)}$$

$$\sum_{w} AID\_M\_OUT_{w,g,r} = \left[\sum_{w} AID\_M\_IN_{g,w,r}\right] + GLB\_SL\_AID \quad \forall r \in rgn \qquad \text{(MC8.4)}$$

5: Model Closure Conditions or Rules[43]

In mathematical programming terms the model 'closure' conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However, the economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model 'closure' rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The discussion of 'closure' rules here distinguishes between macroeconomic closure conditions and factor market clearing conditions.

The macroeconomic closure conditions can be perceived as operating on two levels; on a general level whereby the 'closure' rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific macroeconomic closure conditions. The discussion below provides information about some of the options available with this formulation of the model by reference to the accounts to which the conditions refer. However, as will be apparent there are many permutations available, and hence this discussion deals with the general principles rather than trying to define all possible permutations. The philosophy adopted in the implementation of this model is to define a (minimal) base closure for the replication of the base case and then impose the relevant macroeconomic closure conditions within a loop in the experiment/simulation file. This approach allows substantial flexibility in the definition of policy simulations while simultaneously allowing sensitivity testing of the chosen model closures. For the base solution the choice of model closure is, typically, a balanced macroeconomic closure[44]; the reason for choosing this closure is NOT a statement about its appropriateness, rather it is a closure that allows flexibility in model solution and hence simplifies the process of verifying

43 The term model 'closure' is used generically here to cover all permutations including variations in market clearing conditions and macroeconomic closure conditions.

44 Changes in absorption are even distributed across domestic final demand institutions.

that the model is correctly specified and calibrated. For brevity this is referred to as the "default" closure settings; the file with this closure is called "`glb_CC_cl_balm.inc`".

5.1 Macroeconomic Closure Conditions

5.1.1 Foreign Exchange Account Closure

For the world numéraire the exchange rate index for the reference regions (*EPRI*) is fixed (C1a), although a parameter, *numerchk*, is attached to allow for ease of checking the homogeneity of the model.[45] At the same time the exchange rate for GILD is fixed as equal to the world numéraire (C1b).

Foreign Exchange Account Closure

$$EPRI = \overline{EPRI0} * numerchk \qquad \text{(C1a)}$$

$$ER_{"glo"} = \overline{EPRI0} * numerchk \qquad \text{(C1b)}$$

$$KAPWOR_{rgn} = \overline{KAPWOR_{rgn}}\,. \qquad \text{(C1c)}$$

$$ER_{rgn} = \overline{ER}_{rgn} \qquad \text{(C1d)}$$

The default closure for the non-GILD regions involves an assumption that the nominal exchange rate is flexible. Hence the trade balances ($KAPWOR_{rgn}$) for all non-GILD regions are fixed (C1c) and the exchange rate is flexible. The most straightforward alternative is to fix the exchange rates for all non-GILD regions (C1d) and unfix the trade balances. However, there are clearly a range of permutations whereby the exchange rates for some regions are flexible while for others they are fixed.[46]

45 In the base and model simulations, numerchk equals one. It is a parameter attached to the world numéraire and the regional numéraire (see C5a and C5b). To check for homogeneity, a value other than one should have no effect on real variables in the model.

46 The option is provided in the programme to control choice of these, and several other, closures by selection of regions to assign to a set and its complement.

5.1.2 Capital Account Closure

This discussion of the closure rules for the capital account begins by abstracting from some of the complexities into interactions between different closure conditions. To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. The default closure for this account requires the assumption that savings rates are exogenously determined which involves fixing both the multiplicative savings rates adjusters ($SADJ_r$) (C2a) and the additive savings rates adjusters ($DSHH_r$) (C2b) and allowing the savings rates to be fixed – typically at their base rates. With such a savings-driven closure the value and volume of investment must be free to vary.

Choosing an investment-driven closure requires making two distinct choices; first, is the level investment to be determined in terms of volume, value or expenditure share of final demand, and second, will savings adjust in a multiplicative or additive mode. Clearly, again, it is possible that the choices will vary by region. If the investment volume adjuster ($IADJ_r$) is fixed (C2c), then the value of investment expenditure might change due to changes in the prices of investment commodities ($PQD_{c,r}$). If the value of investment ($INVEST_r$) is fixed (C2d) then both the value of savings must be free to adjust as must the actual volume of investment. The same applies if the shares of investment expenditures in final demand ($INVESTSH_r$) are fixed (C2e).

$$SADJ_r = \overline{SADJ_r} \qquad \text{(C2a)}$$

$$DSHH_r = \overline{DSHH_r} \qquad \text{(C2b)}$$

$$IADJ_r = \overline{IADJ}_r \qquad \text{(C2c)}$$

$$INVEST_r = \overline{INVEST}_r \qquad \text{(C2d)}$$

$$INVESTSH_r = \overline{INVESTSH}_r \qquad \text{(C2e)}$$

Fixing savings, and thus deeming the economy to be savings-driven, could be termed a Neo-Classical approach. Closing the economy by fixing investment however makes the model reflect a Keynesian investment-driven assumption for the operation of an economy.

However, there are potentially important interaction effects. Note that there are other sources of potential savings for region – the government and the trade balances. The magnitudes of these other savings sources can also be controlled through the closure rules (see below). Consequently, there will clearly be an important interdependence between the choices of closure rules for different accounts; the most obvious one to be aware of is the interaction between household savings rates and household income tax rates when tax rates are made flexible and the level of government savings/deficit is fixed.

5.1.3 Government Account Closure

The closure rules for the government account are slightly more complex because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income, i.e., tax rates, remittances etc., and expenditure are 'fixed', government savings must be free to adjust.

Thus, in the default specification all the tax rates are fixed by declaring the tax rates as parameters and then fixing all the tax rate scaling factors (C3.1a – C3.1q). Consequently, any changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (T2.1 to T2.6).

Controlling the volumes of commodity demand ($QGD_{c,r}$) in the base specification government expenditure is then achieved by fixing the volumes of commodity demand ($QGD_{c,r}$) through the government demand adjuster ($QGDADJ_r$) (C3.2a), while the government transfer adjuster ($HGADJ_r$) is fixed in (C3.2b). Notice however that since prices are typically free to change it is necessary to ensure that both government expenditure (EG_r) in (C3.2c) and the share of government expenditure in final demand ($VGDSH_r$) in (C3.2d) must be flexible.

$$TMADJ_r = \overline{TMADJ}_r \quad \text{(C3.1a)}$$

$$TEADJ_r = \overline{TEADJ}_r \quad \text{(C3.1b)}$$

$$TSADJ_r = \overline{TSADJ}_r \quad \text{(C3.1c)}$$

$$TVADJ_r = \overline{TVADJ}_r \quad \text{(C3.1d)}$$

$$TSDDADJ_r = \overline{TSDDADJ}_r \quad \text{(C3.1e)}$$

$$TXADJ_r = \overline{TXADJ}_r \quad \text{(C3.1f)}$$

$$TYFADJ_r = \overline{TYFADJ}_r \quad \text{(C3.1g)}$$

$$TYHADJ_r = \overline{TYHADJ}_r \quad \text{(C3.1h)}$$

$$TFADJ_r = \overline{TFADJ}_r \quad \text{(C3.1i)}$$

$$DTM_r = \overline{DTM}_r \quad \text{(C3.1j)}$$

$$DTE_r = \overline{DTE}_r \quad \text{(C3.1j)}$$

$$DTS_r = \overline{DTS}_r \quad \text{(C3.1l)}$$

$$DTSDD_r = \overline{DTSDD}_r \quad \text{(C3.1m)}$$

$$DTX_r = \overline{DTX}_r \quad \text{(C3.1n)}$$

$$DTYF_r = \overline{DTYF}_r \quad \text{(C3.1o)}$$

$$DTYH_r = \overline{DTYH}_r \quad \text{(C3.1p)}$$

$$DTF_r = \overline{DTF}_r \quad \text{(C3.1q)}$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not directly control. Hence the equilibrating condition is that government savings, the internal balance ($KAPGOV_r$), is not fixed in (C3.2e).

$$TCADJ_r = \overline{TCADJ_r} \tag{C3.1r}$$

$$DTC_r = \overline{DTC_r} \tag{C3.1s}$$

$$TCGADJ = \overline{TCGADJ} \tag{C3.1t}$$

$$TN20ADJ_r = \overline{TN20ADJ_r} \tag{C3.1u}$$

$$DTN20C_r = \overline{DTN20C_r} \tag{C3.1v}$$

$$TN20GADJ = \overline{TN20GADJ} \tag{C3.1w}$$

$$TCH4ADJ_r = \overline{TCH4ADJ_r} \tag{C3.1x}$$

$$DTCH4_r = \overline{DTCH4_r} \tag{C3.1y}$$

$$TCH4GADJ = \overline{TCH4GADJ} \tag{C3.1z}$$

Government Expenditure and Savings Closure

$$QGDADJ_r = \overline{QGDADJ_r} \tag{C3.2a}$$

$$HGADJ_r = \overline{HGADJ_r} \tag{C3.2b}$$

$$EG_r = \overline{EG_r} \tag{C3.2c}$$

$$VGDSH_r = \overline{VGDSH_r} \tag{C3.2d}$$

$$KAPGOV_r = \overline{KAPGOV_r} \tag{C3.2e}$$

The number of possible permutations for closing the government account for each region is consequently substantial. Practical experience indicates that great care is needed when adjusting the government closure rules to avoid both unbalancing the model and imposing closure rules that are contradictory.[47] A common change is to require that the government's savings/deficit is fixed; this requires fixing $KAPGOV_r$ in C3.2e, in which case either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

In its simplest form this can be achieved by allowing one of the previously fixed tax rate adjusters (C3.1a to C3.1n) to vary. Thus, if the sales tax adjuster ($TSADJ_r$) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables $TMADJ_r$ and $KAPGOV_r$ would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the import duties mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates, it is necessary to change the relevant tax parameters. Such changes would be implemented in policy experiment files by changing the values of the parameters that determine relative tax rates.

Note also that as with the investment account, there is a needed care over setting the constraints on government demand. If the government demand volume adjuster ($QGDADJ_r$) is fixed (C3.2a), then the value of government expenditure might change due to changes in the prices of commodities ($PQD_{c,r}$). If the value of government expenditure (EG_r) is fixed (C3.2b) then both government savings must be free to adjust as must the actual volume of expenditure, when tax rates are exogenous. The same applies if the shares of government expenditures in final demand ($VGDSH_r$) are fixed (C3.2e).

47 The most common problems with contradictory government closure rules relate to the interactions between the government and capital (investment) accounts, in particular when a combination of flexible savings and tax rates producing mutually contradictory effects.

5.1.4 Numéraire

The model specification allows for a choice between two price normalisation equations, the consumer price index (CPI_r) and a producer price index (PPI_r). A *numéraire* is needed for each region, to serve as a base since the model is homogenous of degree zero in prices for each region, and hence only defines relative prices.

Numéraire Closure

$$CPI_r = \overline{CPI_r} * numerchk \quad \text{(C5a)}$$

$$PPI_r = \overline{PPI_r} * numerchk \quad \text{(C5b)}$$

5.1.5 Technology and Efficiency

The default assumption in the model is that efficiency, commonly called technology, is fixed. Hence the shift parameters are assumed to be constant.

$$ADXADJ_r = \overline{ADXADJ0_r} \quad (C4.3a)$$

$$DADX_r = \overline{DADX0_r} \quad (C4.3b)$$

$$ADVAADJ_r = \overline{ADVAADJ0_r} \quad (C4.4c)$$

$$DADVA_r = \overline{DADVA0_r} \quad (C4.4d)$$

$$ADFDfADJ_f = \overline{ADFDfADJ0_f} \quad (C4.5e)$$

$$ADFDaADJ_a = \overline{ADFDaADJ0_a} \quad (C4.5f)$$

$$ADFDrADJ_r = \overline{ADFDrADJ0_r} \quad (C4.5g)$$

Changes in efficiency can either be imposed exogenously or a technology variable can be freed up to satisfy some other condition, e.g., the required efficiency gain needed to satisfy a predetermined increase in GDP.

5.2 Factor Market Clearing Conditions

The factor market closure rules can be more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

5.2.1 Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply of and total demand for factors equate, and since the total supplies of each factor are determined exogenously this condition is satisfied by the equilibrium condition (C6.1a). The demands for factor *f* by activity *a* in region *r* and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages ($WF_{f,r}$), by allowing for the possibility that the payments to notionally identical factors might vary across activities, due to activity specific 'efficiency' factors ($WFDIST_{f,a,r}$) through the variable that captures the 'sectoral proportions for factor prices'. Since these proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, they are therefore assumed fixed (C6.1b). And finally bounds are placed upon the average factor prices, (C6.1c), so that meaningful results are produced.

Full Employment and Mobile Factors

$$FS_{f,r} = \overline{FS}_{f,r} \tag{C6.1a}$$

$$WFDIST_{f,a,r} = \overline{WFDIST}_{f,a,r} \,. \tag{C6.1b}$$

$$\begin{aligned} &\text{Min } WF_{f,r} = 0 \\ &\text{Max } WF_{f,r} = +\text{infinity} \end{aligned} \tag{C6.1c}$$

<u>5.2.2 Factor Immobility and/or Unemployment Closures</u>

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as fixed. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations (C6.2a) where *fact* indicates the specific factor and *activ* a specific activity. This block of equations includes all the variables that were declared for the model with reference to factors plus extra equations for $WFDIST_{fact,actv,r}$, i.e., $WFDIST_{fact,activ,r} = \overline{WFDIST}_{fact,activ,r}$, whose role will be defined below. The choice of which equations are binding, and which are not imposed will determine the factor market closure conditions.

Factor Unemployment Closure

$$
\begin{aligned}
FS_{fact,r} &= \overline{FS}_{fact,r} \\
WFDIST_{fact,a,r} &= \overline{WFDIST}_{fact,a,r} \\
\text{Min } WF_{fact,r} &= 0 \\
\text{Max } WF_{fact,r} &= +\text{infinity} \\
FD_{fact,a,r} &= \overline{FD}_{fact,a,r} \\
WF_{fact,r} &= \overline{WF}_{fact,r} \\
WFDIST_{fact,activ,r} &= \overline{WFDIST}_{fact,activ,r} \\
\text{Min } FS_{fact,r} &= 0 \\
\text{Max } FS_{fact,r} &= +\text{infinity}
\end{aligned}
\qquad \text{(C6.2a)}
$$

As can be seen the first four equations in the block (C6.2a) are the same as those in the 'Full Factor Mobility and Employment Closure'; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the 'Full Factor Mobility and Employment Closure'. Assume that this set of conditions represents the starting points, i.e., the first four equations are binding, and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically, this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, thus (C6.3a) must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., (C6.3b) must now be relaxed.

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition (C6.3a) is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e., (C6.3d), which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.[48]

Activity Specific and Immobile Factor

$$FD_{fact,a,r} = \overline{FD}_{fact,a,r} \tag{C6.3a}$$

$$WFDIST_{fact,a,r} = \overline{WFDIST}_{fact,a,r} \tag{C6.3b}$$

$$FS_{fact,r} = \overline{FS}_{fact,r} \tag{C6.3c}$$

$$WFDIST_{fact,activ,r} = \overline{WFDIST}_{fact,activ,r} \tag{C6.3d}$$

Start again from the closure conditions for full factor mobility and employment and then assume that there is unemployment of one or more factors in the economy; typically, this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing a condition that fixes the wage rate, (C6.4a) and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing (C6.4b).

It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions (C6.4c) can be imposed.[49]

48 It can be important to ensure a sensible choice of reference activity. This is important if a factor is not used, or little used, by the chosen activity.

49 If the total demand for the unemployed factor increases unrealistically in the policy simulations, then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

$$WF_{fact,r} = \overline{WF}_{fact,r} \qquad (C6.4a)$$

$$FS_{fact,r} = \overline{FS}_{fact,r} \qquad (C6.4b)$$

$$\text{Min } FS_{fact,r} = 0 \qquad \text{Max } FS_{fact,r} = +\text{infinity} \qquad (C6.4c)$$

5.2.3 Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance, it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

If all factors used by an activity are fixed, this requires imposing the conditions C6.5a, where *activ* refers to the activity of concern. But the returns to these factors in these activities must now be allowed to vary, i.e., the conditions (C6.5b) must now be relaxed. In this case the condition fixing the total supply of the factor is not redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Fixed Factor Use

$$FD_{f,activ,r} = \overline{FD}_{f,activ,r} \qquad \text{(C6.5a)}$$

$$WFDIST_{f,activ,r} = \overline{WFDIST}_{f,activ,r} \qquad \text{(C6.5b)}$$

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to manage the model by gathering together factor market conditions that are inspired by activity characteristics after the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supersedes previous conditions.

7: Concluding Comments

This paper has described a version of the ANARRES global CGE model. The model described here contains relatively simple behavioural relationships although it requires a programme that contains a number of technically sophisticated features. Moreover, it is capable of being implemented with any aggregation of the GTAP database.

This CGE model is subject to on-going revisions, some seek to extend the capabilities of the model to support economic analyses while others concentrate upon improving the functionality of the model. As with all CGE models technical documents can only provide an overview of the model; ultimately the only truly accurate documentation of a CGE model is the computer code.

Appendices

A1. ANARRES Model Genealogy

The ANARRES model derives from the GLOBE model. The GLOBE model project started in June 2002 at a conference on *Poverty, Trade & Tools for Development*, which was hosted by the International Food Policy Research Institute in honour of Sherman Robinson's 60th birthday. During that conference Karen Thierfelder and Scott McDonald discussed, and agreed on, the creation of a SAM based global CGE model calibrated using the GTAP database. Karen and Scott had both 'learnt' CGE models starting with versions of a single country CGE model developed for the US Department of Agriculture's (USDA) Economic Research Service (ERS) under the leadership of Sherman Robinson (Robinson *et al*., 1990; Kilkenny, 1991). Karen had worked on the NAFTA project, also led by Sherman Robinson, and had subsequently used a global model that was descended from the NAFTA model. Scott had been introduced to GTAP by Terrie Walmsley and had used SAMs derived from the GTAP database for teaching single country CGE modelling. These teaching models and the PROVIDE project model (McDonald, 2003) provided the core single country code used to develop GLOBE.[50]

The first full versions of the model were in use by late 2004 (the womod7*.gms series) while the first formal documentation (McDonald, Robinson and Thierfelder, 2007) appeared in May 2007 (based on the womod9*.gms series) as a working paper from the US Naval Academy (working paper 14). Subsequent developments continued until late 2009 when the technical documentation and user guide (McDonald and Thierfelder) were revised and an open source version of the model, GLOBE v1, was made available (glb1.gms). Model developments since late 2009 were consolidated to produce GLOBE v2, which also formed the base for GLOBE_DYN in c 2014. The first version of the GLOBE v3 model was compiled in early 2020. GLOBE v2 provided the basis for the ANARRES_t (teaching) model that is used by the Global CGE course offered by cgemod (www.cgemod.org.uk).

50 The IFPRI standard model (Lofgren *et al*., 2001) was also a descendant of the USDA's ERS model. Both Karen and Scott were in close contact with the team at IFPRI during the development of the IFPRI standard model and aspects of that model clearly influenced both the GLOBE and STAGE models.

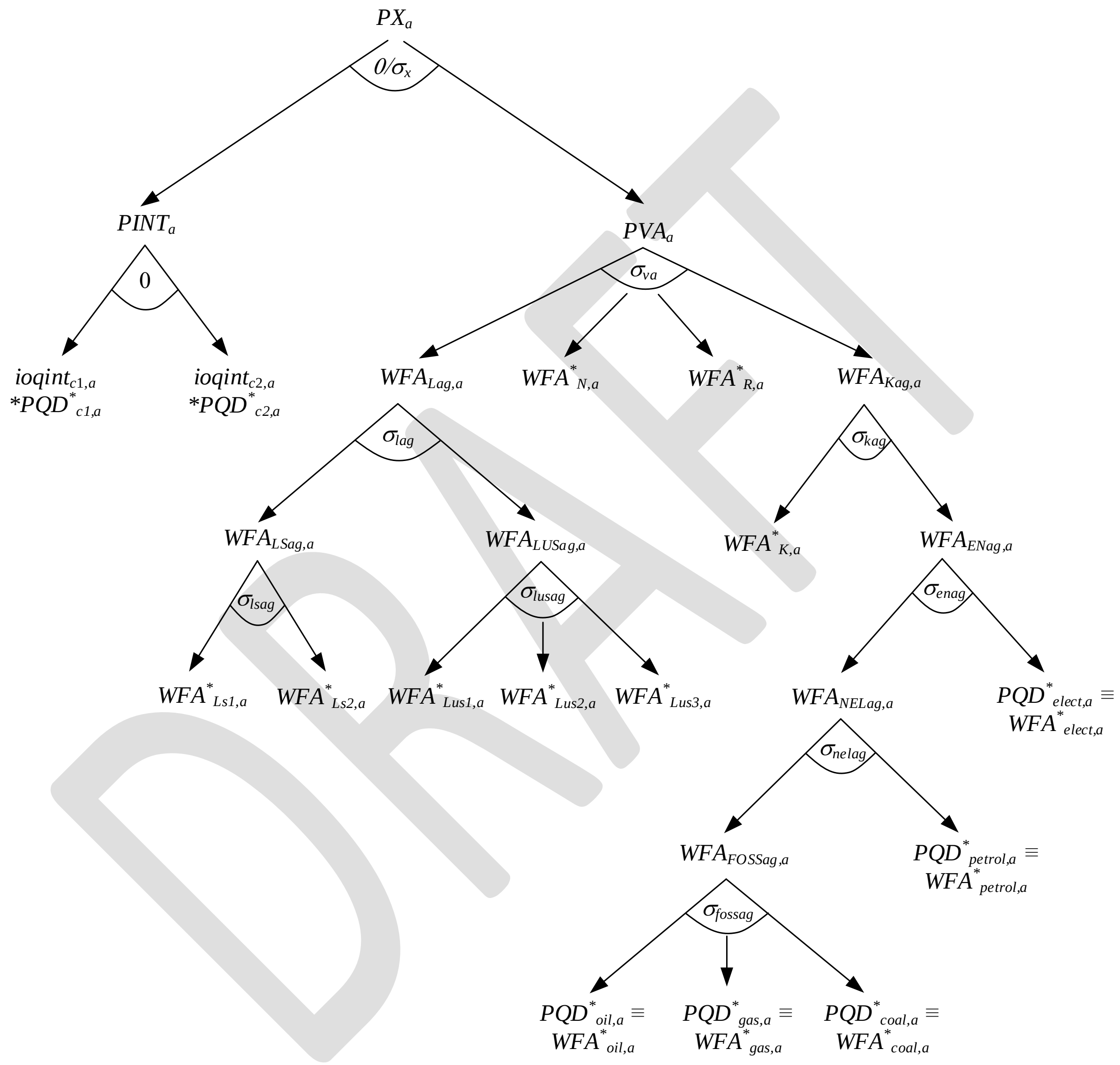
Variants of the GLOBE model have been produced: GLOBE_EN (energy model), GLOBE_MIG (migration model), GLOBE_IMP (imperfect competition) and other less formal variants developed as parts of various research activities, e.g., GLOBE_LAB.

The ANARRES model is a descendant of GLOBE v3. It involves a consolidation of behavioural relationships from previous variants and the addition of new ideas, together with a closer integration of the codes used in the STAGE (single country) and global models. It is part of a suite of models that include a single country model (STAGE – a development of the PROVIDE model that was developed from the USDA ERS model) and a range of teaching models – the SMOD suite. All these model use a (overwhelmingly) common set of notation and formats.

A2 Production System for an Energy Model

This is an illustration of a production system for a 'standard' model using the accounts available in the GTAP database.

Figure A2.1 A Production Price System for an 'Energy' Model



where

$WFA =$ activity a specific factor price paid to the factor

$WFA^{*} =$ activity a specific factor price paid by the activity (tax inclusive)

$PQD^{*} =$ activity a specific factor commodity price paid by the activity (tax inclusive)

$PVA =$ aggregate price of value added in activity a

$PINT =$ aggregate price NON commodity factor intermediates in activity a

$PX =$ activity a output price (exclusive of taxes paid on output)

$ioq\text{int}_{c,a} =$ Leontief input coefficient fro commodity c in a (based on $QINT$)

$N =$ land

$R =$ land

$K =$ land

$Ls* =$ types of skilled labour

$Lus* =$ types of unskilled labour

$*ag =$ an aggregate based on the arguments in the nest below

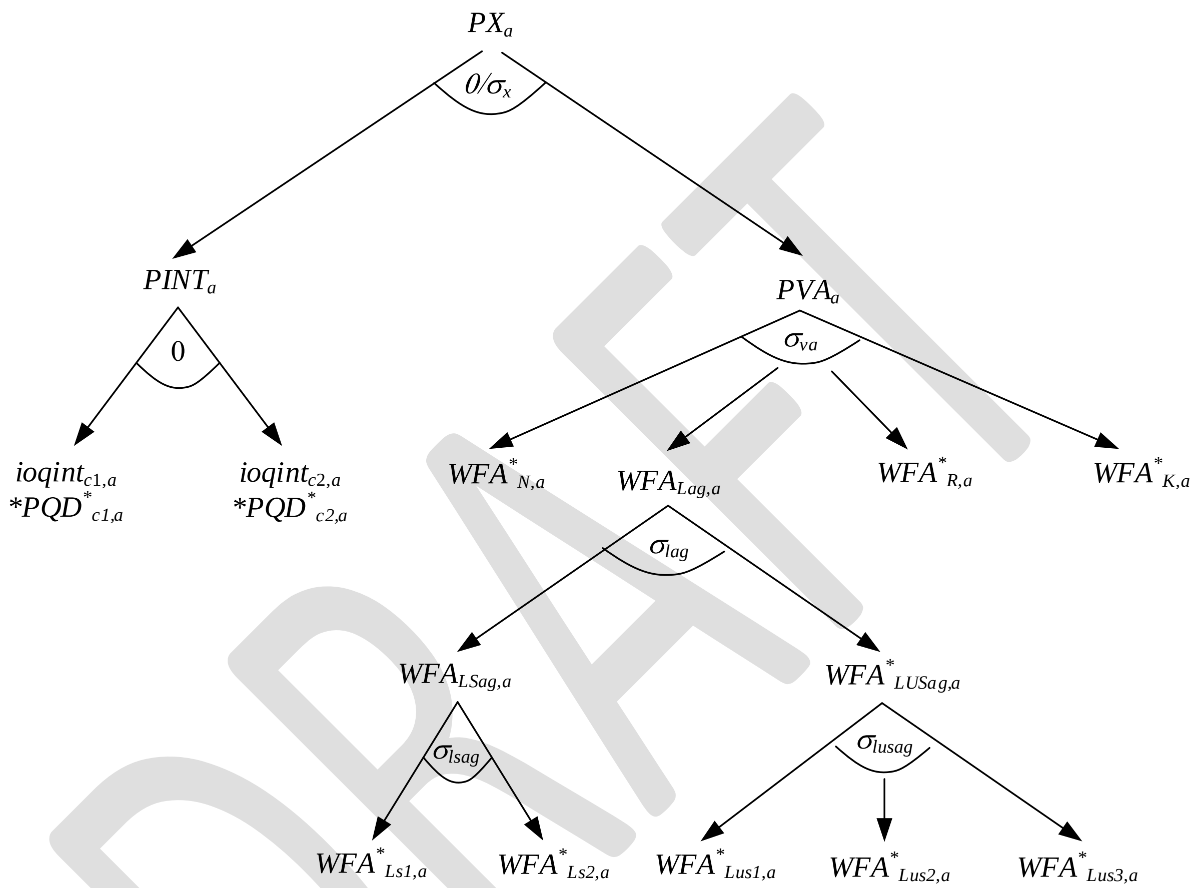
$\sigma_{*} =$ CES substitution elasticity specific to region r activity a and level in the nest

and the labels for energy types – electricity, oil, gas, coal and petrol – and the labels at various levels of the system are based on the arguments included below, e.g., ‘ENag’ identifies aggregate energy inputs.

A3 Production System for a 'Standard' Model

This is an illustration of a production system for a 'standard' model using the accounts available in the GTAP database.

Figure A3.1 A Production Price System for a 'Standard' Model



where the labels are the same as those for the Figure A2.1 in Appendix A2.

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